IOP Conf. Series: Journal of Physics: Conf. Series **802** (2017) 012009 doi:10.1088/1742-6596/802/1/012009

Energy density of light quark jet using AdS/CFT

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Abstract.
We study the energy loss rate of light quarks via the AdS/CFT correspondence in both a static and an expanding plasma. Unlike heavy quarks, light quark energy loss in AdS/CFT is surprisingly dependent on both the string initial conditions and the very definition of the jet itself in the gravity theory. We aim to more closely match the string initial conditions to those expected from perturbative quantum chromodyanics (pQCD)–the theory known to describe the physics of high-momentum particles at early times in heavy ion collisions–by computing the energy-momentum tensor associated with the propagation of the classical string solution. The jet energy-momentum tensor in a strongly-coupled calculation can be found by a superposition of contributions from a collection of point particles whose paths approximate the evolution of the string world-sheet. My results show that some times after creation the pair of quark-anti quark, the energy density is not time dependent. This means that the corresponding jet does not lose energy and the associated nuclear modification factor would be one as expected. Also, the results reveal the virtuality dependency of energy density distribution over space. As expected, the energy of a more virtual jet is spread over wider angles.

1. Introduction

The spectacular measurements from the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) provide compelling evidence that the matter produced in heavy ion collision is a deconfined state of QCD, Quark-Gluon Plasma (QGP) [1, 2, 3, 4], at temperatures above ∼160 MeV which appears to be nearly perfect, with an extremely low viscosity-to-entropy ratio $\eta/s \sim 1/4\pi$ [5, 6]. Jets are produced within the expanding fireball and probe the QGP. Analyses the energy loss of these energetic partons as they travel throw QGP may reveal extremely valuable information about the dynamics of the plasma and exhibit distinctive properties such as jet-quenching which can clearly be observed at RHIC [1, 2, 3, 4] and more recently LHC [7, 8, 9].

While lattice QCD is the proper tool for understanding the static equilibrium thermodynamics of such strongly coupled plasma, it does not allow us to calculate its dynamics evolution on heavy-ion collision. Recently, a novel tool called "the AdS/CFT correspondence" [10, 11, 12, 13, 14] provide valuable insight into the strongly coupled plasma. In this paper, we review the string setup corresponding to a light quark jet in AdS/CFT and the nuclear modification factor comes from this setup. The result demonstrates the dynamics of jet depends on the initial conditions of the string profile [15]. It is obviouse that any further progress in underestanding the jet in AdS/CFT needs a better underestanding of the string initial conditions. The only way to constraint the string initial conditions is calculation of a gauge invariant quantity in the field theory which is stress tensor. We obtain the energy density of light quark jet in the vacuum using the null string approximation in AdS_5.

2. Light quark jet energy loss

According to the AdS/CFT correspondence [13], the $\mathcal{N} = 4$ SYM theory in the large N_c and large 't Hooft coupling is dual to classical supergravity on ten-dimensional $AdS_5 \times S^5$ geometry [10]. In order to study the theory at finite temperature, one can add black hole (BH) to the geometry [11] which yields to the AdS-Sch metric

$$ds^2 = \frac{L^2}{u^2}\left[-f(u)\,dt^2 + d\mathbf{x}^2 + \frac{du^2}{f(u)}\right], \tag{1}$$

where $f(u) \equiv 1 - (u/u_h)^4$ is the blackening factor and L is the AdS curvature radius. Four dimensional Minkowski coordinates are denoted by x_μ and the coordinate u is an inverse radial coordinate. So, the boundary of the AdS-Sch spacetime is at $u = 0$ and the event horizon is located at $u = u_h$. The temperature of the equilibrium SYM plasma relates to the event horizon as $T \equiv \frac{1}{(\pi u_h)}$. Fundamental quarks are described by open strings attached to the D7 flavor branes. These branes extend along the radial coordinate from the boundary at $u = 0$ down to maximal coordinate at $u = u_m$ as well as fill the whole 4d Minkowski space. Also, they wrap on S^3 from the S^5 sphere. The bare mass M of quark is proportional to $1/u_m$ [16], so for massless quarks the D7 brane should fill the whole radial direction. In the $5d$ geometry these strings can fall unimpeded toward the event horizon until their end points reach the radial coordinate u_m where the D7 brane ends. Since for sufficiently light or massless quarks, open string end points can fall into the horizon. In order to study the back-to-back jets, we consider the configurations in which the two endpoints of string move away from each other as the total spatial momentum of the string vanishes. By choosing the appropriate frame, one half of the string has a large spatial momentum in x direction, and the other half of the string carries a large negative spatial momentum. We will limit our attention to strings which create at a point (x_c, u_c) at initial time $t_c = 0$, and then move in one direction in the R^3 space, x direction. By time evolution, the string evolves from a point into an extended object and the string endpoints fall toward the horizon.

The dynamics of string is governed by the Polyakov action

$$S_P = -\frac{T_0}{2}\int d^2\sigma\,\sqrt{-\eta}\,\eta^{ab}\,\partial_a X^\mu \partial_b X^\nu\, G_{\mu\nu}\,. \tag{2}$$

Variation of the Polyakov action with respect to the embedding functions X^μ leads to the equation of motion which can be solved numerically by considering the appropriate initial conditions for the string profile.

In order to study jets in AdS/CFT we need to define the proper objects in the dual string theory that corresponds to a jet, a slippery object even in field theory; jets are truly only defined by the algorithm used to measure them. Presumably the ideal way to compute jet observables in AdS/CFT is to compute the energy momentum tensor associated with falling string on the dual field theory by solving the gravitational bulk-to-boundary problem and then"run" a jet finding algorithm on the result. However, there are lots of attempts to define jet in the string theory side itself [17, 15]. The authors of [17] are motivated by the localization of the baryon density on the boundary which is of scale of order $\Delta x \sim 1/\pi T$ and defined jet as a part of string which is in the Δx spatial distance from the endpoint. We called this as the "$\Delta x - prescription$" of jet [17]. In [15], we proposed a jet prescription, Δu prescription, motivated by the separation of energy scales in, e.g., thermal field theory. In our prescription the portion of the string above some cutoff $u = u^*$ in the radial direction is considered part of the jet; the portion of the string

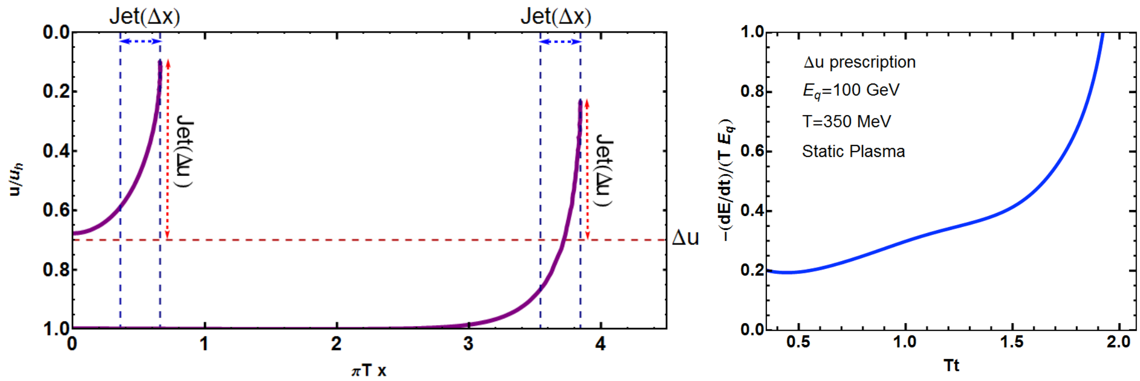


Figure 1. (a) Illustration of the Δx and Δu prescriptions of a jet in the string theory; see text for details. (b) The instantaneous energy loss of a light quark jet as a function of time in the AdS-Sch using the Δu prescription. The normalization constant $E_q = 100$ GeV is the initial energy of the jet, which has a virtuality of 175 GeV^2, and $T = 350$ MeV is the temperature of the plasma.

below the cutoff is considered part of the thermalized medium. By choosing any value of u above the black hole horizon as the cutoff, we regain the natural result that a jet that is thermalized no longer has detectable energy or momentum. We evaluated the energy loss rate of jet using this prescription which shows a Bragg peak at late times figure 1.

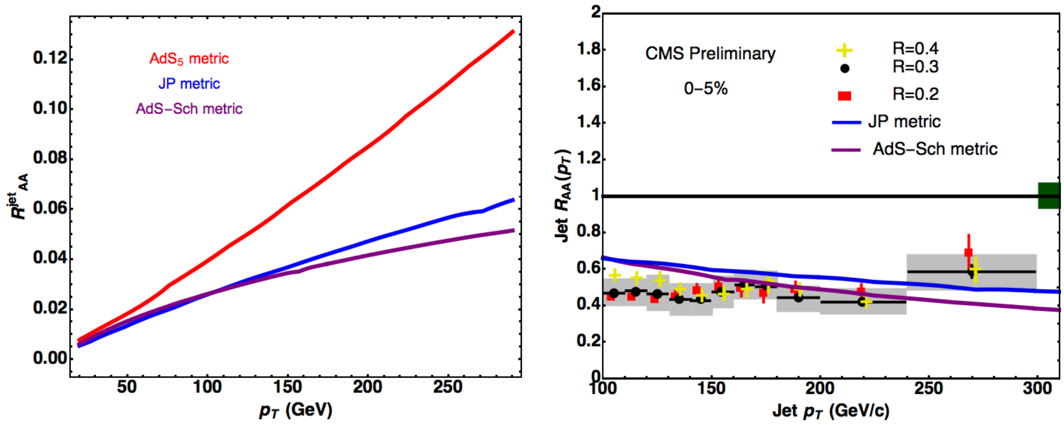
Then, we calculated an approximation of the nuclear modification factor R_{AA} for jet using our energy loss model of jet in three different metrics AdS_5, AdS-Sch, and JP metric corresponding to the vacuum, static and expanding plasma respectively. Our results are shown in figure 2. The purple curve shows the R_{AA} obtained from the AdS-Sch metric, while the blue curve is the R_{AA} obtained from the JP metric. We expect that the red line which is the R_{AA} obtained from the AdS_5 would be one since we expect that jets produced in the pp collision do not loss their energy.

Since the definition of R_{AA} measured the difference between the medium and vacuum effects on jet, we define a renormalized R_{AA} in AdS/CFT by dividing the medium R_{AA} by the vacuum R_{AA}, plotted in figure 2(b). Results are in a good agreement with the CMS preliminary data for the most central Pb-Pb collision at $\sqrt{s_{NN}} = 2.76\,GeV$ [18]. In our calculations, we have used the value of t Hooft coupling $\lambda = 5.5$ comes from lattice calculation for quark-anti-quark potential.

3. SYM stress-tensor

According to AdS/CFT dictionary, presence of a source in the bulk perturb the geometry. The behaviour of the metric perturbation near the boundary determine the energy-momentum tensor of jet on the boundary. The metric perturbation can be obtained by solving the linearized Einstein equation. The metric perturbation contains 15 degrees of freedom which couple to each other via the linearized Einstein equations. On the other hand, the 4d SYM stress tensor is traceless and conserved and consequently containes 5 independent degrees of freedom. These two quantities must be contrasted with each other. So, not all of these degrees of freedom in the metric perturbation are physical.

It is possible to construct gauge invariant quantities out of linear combinations of metric perturbations and its derivatives [19] and calculate the gauge invariant variables directly from



(a) (b)

Figure 2. (a) Jet R_{AA} as a function of p_T in the most central Pb-Pb collision obtained via AdS/CFT in AdS_5 (red), JP (blue) and AdS-Sch (purple) metrics. (b) AdS/CFT jet R_{AA} as a function of p_T compared with the preliminary CMS data in different effective cone sizes for anti-k_T jets using the Bayesian unfolding method for most central Pb-Pb collision at the LHC with $\sqrt{s} = 2.76$ TeV per nucleon [18]. The value of t Hooft coupling used in these calculations is $\lambda = 5.5$.

them. In [20], the energy density of stress tensor on the boundary at a space-time point $(t_b, \mathbf{r_b})$ has been evaluated as

$$\begin{aligned}\mathcal{E}(t_b, \mathbf{r_b}) &= \frac{2L^3}{\pi} \int d^4r \, \frac{du}{u^2} \, \Theta(t_b - t) \, \delta''(W) \, [u(2t_{00} - t_{55}) - (t_b - t)t_{05} + (x_b - x)^i t_{i5}] \qquad (3) \\ &+ \frac{2L^3}{3\pi} \int d^4r \, \frac{du}{u} \, \Theta(t_b - t) \, \delta'''(W) \, [|\mathbf{r_b} - \mathbf{r}|^2 \, (2t_{00} - 2t_{55} + t_{ii}) - 3(x_b - x)^i \, (x_b - x)^j \, t_{ij}],\end{aligned}$$

where $W = -(t_b - t)^2 + (\mathbf{r_b} - \mathbf{r})^2 + u^2$ is the bulk to bounadry propagator and t_{MN} is the bulk stress tensor. At time t, the bulk excitation localized at $(t, \mathbf{r})$ emits a gravitational wave h_{MN} which propagates through AdS_5 at the respective speed of light up to the measurement point $(t_b, \mathbf{r_b})$ on the boundary. The δ function in the integrand represents the support of the retarded bulk-to-boundary propagator for the Einstein equations in AdS_5.

The falling string can be approximated with a null string such that each point of string follows a null geodesic in the bulk with the same velocity as string velocity, after the geometric optic expansion [21]. Then, the jet energy-momentum tensor in a strongly-coupled calculation can be relatively easily found by a superposition of contributions from a collection of point particles whose paths approximate the evolution of the string worldsheet. The point particle energy density in vacuum is obtained [22]

$$\mathcal{E}_{particle}(t_b, \mathbf{r_b}) = \frac{E_0}{2\pi\gamma^4} \frac{(t_b + u_c\gamma)^2}{(t_b + u_c\gamma - v(x_b + u_c v\gamma))^3} \, \delta(\,(t_b + u_c\gamma)^2 - x_{\perp b}^2 - (x_b + u_c v\gamma)^2\,)\,, \qquad (4)$$

where x_b is the direction along the motion of string in 4d Minkowski space and $x_{\perp b}$ is the transverse direction. In figure 3, the angular distribution of energy is plotted for two strings with the same energies but different virtualities. As we expected, after a short time after creation the enregy is highly concentrated at $\theta = 0, \pi$ correcponding to the position of the string endpoints and also it is no time dependent. This means that the corresponding jet does not lose energy and the associated nuclear modification factor would be one as expcted. Also, the

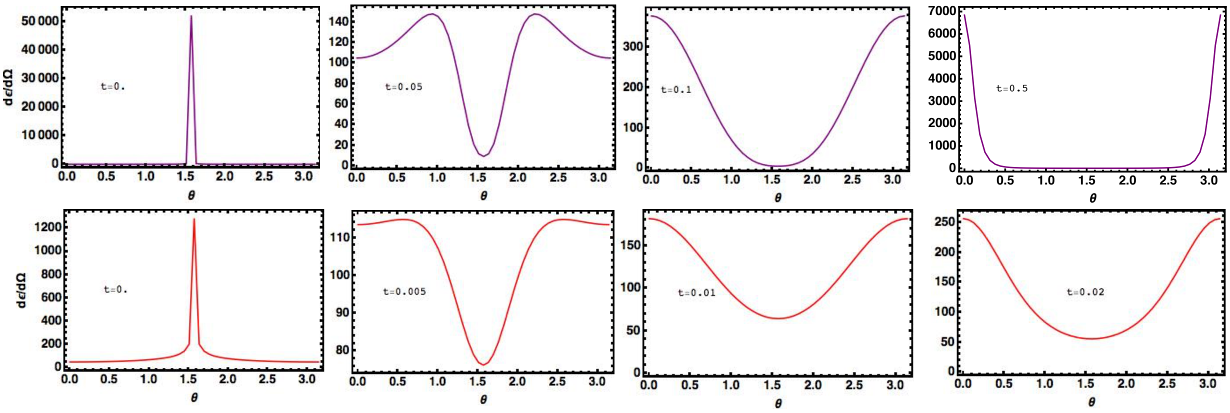


Figure 3. Angular distribution of light quark jet energy for two different jets with 100 GeV initial energy. The first falling string has $Q^2 \simeq 176\,GeV^2$ while the second string has $Q^2 \simeq 6000\,GeV^2$. θ is the angle with the direction of motion of the string endpoint x.

results reveal the virtuality dependency of energy density distribution over space. It should be mentioned that we have used the particle physics sign convention for the virtuality of quark defined as $Q^2 = E_q^2 - p_q^2$. Actually, energy of a more virtual jet spread over a wider angle.

4. Conclusions

We have purposed a novel prescription of jet based on energy scale separation in AdS/CFT. We have shown that using our prescription of jet, the light quark energy loss exhibit the Bragg peak again at late times in both static and expanding plasmas. This late time behavior of jet energy lost implies that after traveling substantial distances through the plasma, the thermalization of light quark ends with a large amount of energy transferring to the plasma. We considered a brick of plasma and calculated the nuclear modification factor of jet in both AdS-Sch and JP metric. We assumed that the temperature of plasma around 350 MeV at AdS-Sch metric and at initial time in JP metric. Our results show an over suppression of jet of order of ten respect to the data. We investigated that it is because of the falling string setup at AdS space. In fact, R_{AA} of jet using the falling string in AdS_5 which is dual to jets in vacuum is not one, even though it is less than one. We introduced a renormalized R_{AA}^{renorm} by dividing the R_{AA} in the medium to the R_{AA} in the vacuum. Surprisingly, our ratio shows good agreement with the experimental data on the R_{AA}^{jet} of most central Pb-Pb collision at LHC figure 2 (b).

Our results on jets using AdS/CFT correspondence reveals two important issues: 1) The dynamics of jet highly depends on the initial conditions of falling string figure 4, while there is no known map between the string initial conditions and parameters of the dual state in the field theory. Further progress in describing experimental results will require significant advances in the understanding of string initial conditions. 2) That the results of our simple model are in such good agreement with data suggests that we attempt to better define the jet in AdS/CFT and constrain the possible string initial conditions. We can likely accomplish both goals by computing the energy-momentum tensor associated with the propagation of the classical string solution. With the energy- momentum tensor in hand, we should be able to compute directly from the string theory the actual quantities measured experimentally.

One can solve Einstein's equations for the perturbation in the $5d$ geometry due to the presence of the string and according to the bulk to boundary map interpret the near boundary behavior of the metric perturbation as the perturbation in the SYM energy-momentum tensor by the

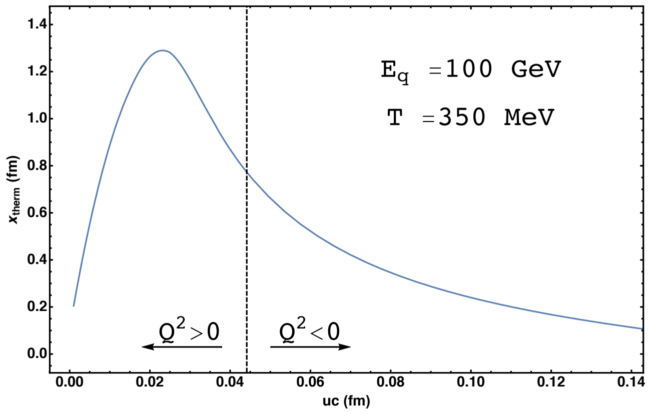


Figure 4. The maximum stopping distance of jet versus the virtuality of quark Q^2. Our result show a nontrivial dependency of the thermalization time to the initial condition of the string.

presence of jet. Our studies show that the energy of jet in vacuum is distributed about the string endpoints on the boundary, see figure 3. Also, after a short time after creation its angular distribution is not time dependent which means that the corresponding jet does not lose energy in vacuum and the associated nuclear modification factor would be one as expcted.

In order to compute the R_{AA} for jet, we need to calculate the boundary energy density of falling string in $AdS - Sch$ and then compare with the energy density in AdS_5 which we left for future work.

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