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Status and Prospect of ϵ'/ϵ from a Lattice perspective

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Abstract. An ab-initio determination of the hadronic matrix elements relevant for a theoretical determination of $K \rightarrow \pi\pi$ decays has been a goal for the lattice QCD community for about three decades. These matrix elements are key to the theoretical determination of the CP violation parameters ϵ and ϵ' . This is not only relevant to challenge the Standard Model but also to provide stringent constraints on New-Physics scenarios. Although such a computation is very challenging, impressive results have been reported over the last few years. The most spectacular is the first realistic computation of ϵ'/ϵ with physical kinematics by RBC-UKQCD [1]. The novelty is the $\Delta I = 1/2$ channel, whereas the $\Delta I = 3/2$ contribution is now computed at several values of the lattice spacing and extrapolated to the continuum limit. After a short overview, I will report on the status and the prospect of direct and indirect violation in kaon decays from a lattice perspective.

Direct CP violation in kaon decays is a very small effect which is well measured experimentally, and as such provides a crucial test for the SM. Besides, since it comes mainly from virtual loops, one expects an important sensitivity to new physics. However the relevant weak matrix elements receive huge QCD contributions, which have to be computed non-perturbatively in a reliable way. This is why lattice QCD has a crucial role to play.

1. $K \rightarrow \pi\pi$ decays and CP violation

Various nice reviews are available on the subject, see for example [2]. I just recollect here some basic facts about $K \rightarrow \pi\pi$ phenomenology. Assuming isospin symmetry, the decays $K \rightarrow \pi\pi$ can be written in terms of the amplitudes $A[K \rightarrow (\pi\pi)_I] = A_I e^{i\delta_I}$, where I denotes the isospin of the two-pion state, either 0 or 2, and δ_I is the corresponding strong phase. The parameters of indirect (resp. direct) CP violation, ϵ (resp. ϵ') are given by

$$\epsilon = \frac{A[K_L \rightarrow (\pi\pi)_0]}{A[K_S \rightarrow (\pi\pi)_0]}, \quad \frac{\epsilon'}{\epsilon} = \frac{1}{\sqrt{2}} \left(\frac{A[K_L \rightarrow (\pi\pi)_2]}{A[K_L \rightarrow (\pi\pi)_0]} - \frac{A[K_S \rightarrow (\pi\pi)_2]}{A[K_S \rightarrow (\pi\pi)_0]} \right). \quad (1)$$

The first measurement of ϵ is the well-known discovery of indirect CP violation due to Christenson, Cronin, Fitch and Turlay [3] in 1964, for which Cronin and Fitch were awarded a Nobel prize. ϵ' has a long experimental history as it took tremendous

efforts to measure direct CP violation. The final measurements are due to KTeV at Fermilab and NA48 at CERN [4, 5], the averages read

$$|\epsilon| = 2.228(11) \times 10^{-3}, \quad \text{Re} \left(\frac{\epsilon'}{\epsilon} \right) = 16.6(2.3) \times 10^{-4}. \quad (2)$$

In a theoretical approach, the standard framework to study $K \rightarrow \pi\pi$ decay is the $\Delta S = 1$ effective Hamiltonian obtained after integrating out the heavy degrees of freedom [6, 7]. In the three-flavour theory, it reads (see for example [8] for a review)

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu). \quad (3)$$

2. Matching Lattice QCD with phenomenology

The physical amplitudes are given by $\langle \pi\pi | H_W | K \rangle$. In Eq. 3, the short-distance effects, computed in perturbation theory are factorised into the so-called Wilson coefficients, y_i, z_i , whereas V_{ij} are CKM matrix elements, $\tau = V_{ts}^* V_{td} / V_{us}^* V_{ud}$ and μ is an energy scale which can be thought as a cut-off. G_F is the Fermi constant. If we neglect the operators which emerge from the electric and magnetic dipole part of the electromagnetic and QCD penguins (see the talk by V.Lubicz at Lattice'14 and [9] for a recent lattice study by the ETM collaboration.), naively we find that there are ten four-quark operators Q_i which contribute. However, in four dimensions and in the three-flavour theory, they do not form a basis of the $\Delta S = 1$ four-quark operators as they are not linearly independent. Following for example [10], one can build a seven-operator basis, we denote them by Q'_i . Obtaining a reliable evaluation of the matrix elements $\langle \pi\pi | Q'_i | K \rangle$ is the most difficult part of the computation. Many attempts have been in the last decades using either effective theories or lattice simulations (or combinations of both), see for example [11, 12, 13, 14, 15, 16, 17] and reference therein.

Error propagation. The four-quark operators and the Wilson coefficients entering Eq 3 have to be computed in the same scheme and at the same scale. The Wilson coefficients are traditionally computed in $\overline{\text{MS}}$ in the ten-operator pseudo-basis Q_i . The lattice results given [1] have been obtained in the seven-operator basis Q'_i and - for convenience - have been converted to the ten-operators pseudo-basis. However, one has to keep in mind that the errors given for the latter are obviously highly correlated, therefore a naive propagation of the lattice errors in quadrature will over-estimate the errors. Instead, since the hadronic errors dominate, I would advise to convert the Wilson coefficients to the basis used for the lattice computation. Eventually, when the determination of the hadronic matrix elements becomes more precise, one would have to take into account the correlations between the various operators: although the Q'_i are linearly independent in the algebraic sense, they are clearly correlated - mainly because the operators mix under renormalisation - but also because they come from the same Monte-Carlo simulation (they have been evaluated on the same gauge ensembles).

3. Lattice computation of $K \rightarrow \pi\pi$ amplitudes

From the lattice point of view, the computation of these amplitudes is very challenging for various reasons. First, chiral symmetry is crucial in order to prevent unwanted mixing with lower-dimension operators. Such a mixing comes with coefficients which

diverge in inverse powers of the lattice spacing (eg $1/a^2$) and would need to be subtracted exactly before the continuum limit can be taken. However it is well known that maintaining both chiral and flavour symmetry at finite lattice spacing is numerically very expensive. With the RBC-UKQCD collaboration, we use the so-called Domain-Wall formulation, which respects chiral-flavour symmetry “almost exactly” but with numerical cost much higher than other discretisations of the Dirac operator. Furthermore, simulating light quarks is also very demanding, their propagators are difficult to compute and require large physical volumes, because finite volume artefacts are dominated by light pions. One possibility is to work on relatively coarse lattice lattice spacing relatively coarse. But if one wants to simulate finer lattices (and eventually take the continuum limit) one needs a lot of point in each space-time direction. Hence, simulating physical light quarks with a discretisation which preserves chiral-flavour symmetry is already a challenge by itself.

However, besides these technical difficulties, the main obstacle is to simulate the kinematic situation, in particular the final state made of two hadrons with non-vanishing momenta (because the phase-shift is not easily accessible from traditional Euclidean simulations, this is known as the Maiani-Testa no-go theorem [18]). The RBC-UKQCD collaboration follows the method proposed by Lellouch and Lüscher [19] which circumvent in a very elegant way this problem and allows us to access the physical amplitude. However, there is another technical problem: the physical state of interest appears as an excited state, which is much more difficult to extract because an excited state is statistically more noisy. Since we are already fighting with the noise in this project, we want to “eliminate” the unphysical states. We use two different techniques, depending on the isospin of the $\pi\pi$ state

- $K \rightarrow (\pi\pi)_2$, ie $\Delta I = 3/2$: taking advantage of the Wigner-Eckart theorem and using a peculiar choice of boundary condition, we compute the “unphysical process” $K^+ \rightarrow \pi^+\pi^+$, which is related to the physical one for pure QCD in the isospin limit,
- $K \rightarrow (\pi\pi)_{I=0}$, ie $\Delta I = 1/2$: we implement G-parity boundary conditions [20, 21], see for also [22, 23].

In practice this implies that for $\Delta I = 3/2$ we can use already existing gauge configurations, but for $\Delta I = 1/2$ we have to generate dedicated ensembles. $\Delta I = 3/2$ is also much easier for other reasons: only three operators contribute and there is no “disconnected diagram” (in which no quark line connects the initial kaon and the final two-pion states; these diagrams are numerically very demanding). In 2012, we performed the computation of the $\Delta I = 3/2$ channel [24, 25], with three flavours of dynamical quarks, physical kinematics and nearly-physical pion mass. However we had only one rather coarse lattice spacing $a \sim 1.4$ fm. Therefore, we renormalised the relevant four-quark operators at a rather low energy scale of $\mu \sim 1.4$ GeV and run non-perturbatively to $\mu = 3$ GeV ‡ using a step-scaling technique [27, 28] At $\mu = 3$ GeV, we match to $\overline{\text{MS}}$ using perturbation theory, using [29]. Meanwhile there has been a very important breakthrough for Domain-Wall fermions, with a new formulation which preserves the same amount of chiral symmetry for a much lower cost [30]. Thanks to this formulation, the RBC-UKQCD collaboration could simulate new ensembles, still with three flavours and physical quark masses, but with finer lattice spacing (and

‡ The renormalisation is done with the Rome-Southampton method [26], where the renormalisation scale is the momentum given to the quarks. On the lattice, non-zero momenta imply discretisation artefacts, typically of order $(ap)^2$, therefore one keeps the scale below the cutoff: $\mu^2 = p^2 \ll (\pi/a)^2$.

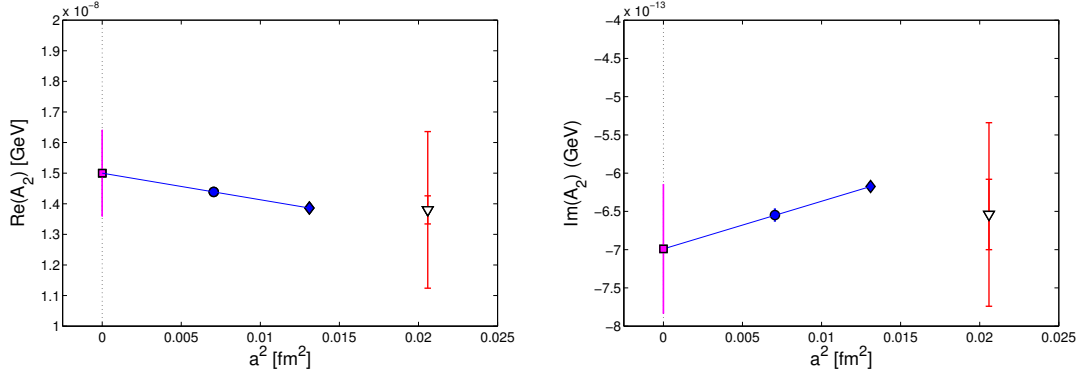


Figure 1. Real and Imaginary part of $A_2 = A[K \rightarrow (\pi\pi)]_{I=2}$. The triangle represents the 2012 computation on the coarse lattice and the blue points the 2014 determinations on the new ensembles (statistical error only), from which a continuum limit is extracted and shown in magenta (statistical and systematic errors combined). For the 2012 computation, we show both the statistical and the systematic error, largely dominated by the discretisation artefacts.

roughly the same physical volume) [31], namely $a \sim 0.11$ and $a \sim 0.84$ fm. In [32], these new lattices have been used to improve the determination of A_2 : by performing a continuum extrapolation we were able to eliminate the main source of uncertainty of the previous computation, reducing the systematic error by roughly a factor 2 for the real part and a factor 1.5 for the imaginary part. Thanks to these new lattice determinations, the current errors on the theoretical determination of A_2 are of the order of 10% (see figure 1), we find :

$$Re(A_2) = 1.50(4)_{\text{stat}}(14)_{\text{syst}} \times 10^{-8} \text{GeV} , \quad (4)$$

$$Im(A_2) = -6.99(20)_{\text{stat}}(84)_{\text{syst}} \times 10^{-13} \text{GeV} . \quad (5)$$

The situation is more complicated for the $\Delta I = 1/2$ channel, as discussed above. First, to develop the numerical techniques needed to extract the signal, we did a pilot study with unphysical kinematics (at threshold and with heavy quark masses) but with all the diagrams included [33], so we had a first estimation for ImA_0 . Recently we have reported on the first complete computation with physical quark masses and physical kinematics in [1]. From a more technical point of view, this computation requires the evaluation of all-to-all propagators and noise reduction techniques. As it is the first computation of this kind and because it requires new (and expensive) ensembles, the computation is performed at a single value of the lattice spacing, again at $a \sim 0.14$ fm. The results read

$$Re(A_0) = 4.66(1.00)_{\text{stat}}(1.21)_{\text{syst}} \times 10^{-7} \text{GeV} \quad (6)$$

$$Im(A_0) = -1.90(1.23)_{\text{stat}}(1.04)_{\text{syst}} \times 10^{-11} \text{GeV} \quad (7)$$

and the corresponding theoretical value for ϵ'/ϵ $Re(\epsilon'/\epsilon) = 1.38(5.15)(4.43) \times 10^{-4}$, which is an approximate agreement ($\sim 2.1\sigma$) with the experimental value. The final result is compatible with 0 but we show in [1] that we are able to extract a non-vanishing number for 6 of the 7 relevant operators. Rather than concluding that a significant deviation of the Standard Model prediction has been found, we note that the total error is much larger than the experimental one. From a phenomenological

point of view, at this level of precision, these results do not invalidate the Standard Model, neither do they rule out the need for new-physics in $K \rightarrow \pi\pi$ decays. The important point is that for the first time ϵ'/ϵ has been computed with a full error budget, all the different contributions of the seven linearly independent operators are computed with controlled errors and with a precision which can be systematically improved. Furthermore, the individual contribution of each four-quark operator can be used in phenomenological applications. For example, the authors of [34] suggest to combine these contributions in a different way in the evaluation of ϵ'/ϵ in order to reduce the hadronic uncertainty, see also [35, 36, 37] at this conference. From a lattice perspective, now that the technology has been developed, reaching a precision of, say, 10% should be possible in the close future. In addition to reducing the statistical errors, the simulation can be done on finer lattices and extrapolated to the continuum limit, as done for A_2 (right now for A_0 we only have the points lying at the right-hand side of the plots shown in Fig. 1). Another dominant systematic error is due to the truncation of the perturbative series (needed to compute the Wilson coefficients). The renormalisation was performed at a scale of $\mu \sim 1.5$ GeV in order to keep the discretisation effects under control. Clearly this can be improved by running non-perturbatively to a higher scale, as done for the $\Delta I = 3/2$ channel, or again by performing the computation on finer lattice spacings. We should also improve on the normalisation of the weak matrix elements. In phenomenology it is a tradition to use the bag parameters, but it might not be the best choice since they are very sensitive to the quark mass and is not uniquely defined. An alternative could be to take the ratio of matrix elements, for example to divide $Q'_{i>1}$ by Q_1 . On the lattice one expects some systematic errors to cancel in the ratios and milder continuum extrapolation. In the future we will also include the matrix elements of the four-quark operators which are not present in the SM but appear in generic BSM theories. It is also worth noting that another computation (done at threshold) has been done with Wilson fermions [38].

4. Conclusions and outlook

This is an exciting time for Kaon physics (see for example [39]); to a great extent this is due to the impressive progress achieved recently by the lattice community. The computation of $K \rightarrow (\pi\pi)_{I=2}$ is reaching a mature stage and a first computation $K \rightarrow (\pi\pi)_{I=0}$ with physical kinematic and complete error budget has recently been reported by the RBC-UKQCD collaboration. The results of these computations have an important role to play in particle physics phenomenology. Most of the $\Delta I = 1/2$ enhancement (the $\Delta I = 1/2$ puzzle) seems to come from non-perturbative effects [40]. Although this has to be confirmed with a precise computation of A_0 , it certainly shows the importance of non-perturbative techniques. Regarding indirect CP violation, B_K is now known with an impressive precision. The various investigations of the $\Delta S = 2$ BSM operators are converging, the discrepancies observed by several collaborations are likely to be due to systematic errors affecting the non-perturbative renormalisation procedure in RI-MOM schemes [41]. Although a careful study is required, the solution could be provided by the SMOM schemes, which have a much better behaviour. Future improvements will also require the matching between lattice and phenomenology to be done at a much higher scale in order to decrease the error due to perturbation theory. I have presented here the new determinations of $K \rightarrow \pi\pi$ decay amplitudes and I have mentioned neutral kaon mixing matrix elements, but there are other new interesting and promising developments such as rare kaon decays and the $K_L - K_s$

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mass difference (see [42] and [43]).

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