

# Kaon Flavour Physics Strikes Back

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**Abstract.** In this short presentation I emphasize the increased importance of kaon flavour physics in the search for new physics (NP) that we should witness in the rest of this decade and in the next decade. The main actors will be the branching ratios for the rare decays  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ , to be measured by NA62 and KOTO, and their correlations with the ratio  $\varepsilon'/\varepsilon$  on which recently progress by lattice QCD and large  $N$  dual QCD approach has been made implying a new flavour anomaly. Further correlations of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and  $\varepsilon'/\varepsilon$  with  $\varepsilon_K$ ,  $\Delta M_K$ ,  $K_L \rightarrow \mu^+ \mu^-$  and  $K_L \rightarrow \pi^0 \ell^+ \ell^-$  will help us to identify indirectly possible NP at short distance scales. This talk summarizes the present highlights of this fascinating field including some results from concrete NP scenarios.

## 1. Introduction

In three recent reports [1, 2, 3] I have stressed the increased importance of kaon flavour physics in the search for new physics (NP) which we should witness in the near future. Indeed after years of silence I expect that kaon flavour physics will strike back providing new insights in the dynamics at very short distance scales. The following pages can be considered as an express review of this fascinating field. Further details and in particular numerous references can be found in [1, 2, 3, 4].

## 2. Important Messages

### 2.1. $\varepsilon'/\varepsilon$

Presently in kaon flavour physics the most exciting appears to be the anomaly in  $\varepsilon'/\varepsilon$ . The present status of  $\varepsilon'/\varepsilon$  in the SM can be summarized as follows. The RBC-UKQCD lattice collaboration calculating hadronic matrix elements of all contributing operators but not including isospin breaking effects finds [5, 6]

$$(\varepsilon'/\varepsilon)_{\text{SM}} = (1.38 \pm 6.90) \times 10^{-4}, \quad (\text{RBC} - \text{UKQCD}). \quad (1)$$

Using the hadronic matrix elements of QCD-penguin ( $Q_6$ ) and EW-penguin ( $Q_8$ )  $(V - A) \otimes (V + A)$  operators from RBC-UKQCD lattice collaboration but extracting the matrix elements of penguin  $(V - A) \otimes (V - A)$  operators from the data on CP-conserving  $K \rightarrow \pi\pi$  amplitudes and including isospin breaking effects one finds [7]

$$(\varepsilon'/\varepsilon)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}, \quad (\text{BGJJ}) \quad (2)$$

that is confirmed within the errors by the recent analysis in [8]

$$(\varepsilon'/\varepsilon)_{\text{SM}} = (1.1 \pm 5.1) \times 10^{-4}, \quad (\text{KNT}). \quad (3)$$



All these results are significantly below the experimental world average from NA48 [9] and KTeV [10, 11] collaborations,

$$(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}, \quad (4)$$

suggesting that models providing enhancement of  $\varepsilon'/\varepsilon$  are favoured.

These results are based on NLO calculations of the Wilson coefficients of the relevant operators [12, 13, 14, 15, 16, 17]. Partial NNLO calculations have been performed in [18, 19, 20]. Complete NNLO result from Maria Cerda-Sevilla, Martin Gorbahn, Sebastian Jäger and Ahmet Kokulu should be available soon.

While these results, based on the hadronic matrix elements from RBC-UKQCD lattice collaboration, suggest some evidence for NP in  $\varepsilon'/\varepsilon$ , the large uncertainties in the hadronic matrix elements in question do not yet preclude that eventually the SM will agree with data. Therefore the upper bounds on the relevant hadronic matrix elements of  $Q_6$  and  $Q_8$  from large  $N$  dual QCD approach [21] are important as they give presently the strongest support to the anomaly in question, certainly stronger than present lattice results.

In the strict large  $N$  limit [22, 23, 24] the parameters  $B_6^{(1/2)}$  and  $B_8^{(3/2)}$  that represent the relevant hadronic matrix elements of the QCD penguin operator  $Q_6$  and the electroweak penguin operator  $Q_8$ , respectively, are simply given by

$$B_6^{(1/2)} = B_8^{(3/2)} = 1, \quad (\text{large } N \text{ Limit}). \quad (5)$$

But RBC-UKQCD results [6, 5] imply [7, 25]

$$B_6^{(1/2)} = 0.57 \pm 0.19, \quad B_8^{(3/2)} = 0.76 \pm 0.05, \quad (\text{RBC-UKQCD}), \quad (6)$$

and the suppression of  $B_6^{(1/2)}$  below unity is the main origin of the strong suppression of  $\varepsilon'/\varepsilon$  below the data within the SM. Yet in view of the large error in  $B_6^{(1/2)}$  one could be sceptical about any claims that there is NP in  $\varepsilon'/\varepsilon$ . Future lattice results could in principle raise  $B_6^{(1/2)}$  towards its large  $N$  value and above  $B_8^{(3/2)}$  bringing the SM result for  $\varepsilon'/\varepsilon$  close to its experimental value.

However, the analyses of  $B_6^{(1/2)}$  and  $B_8^{(3/2)}$  within the dual QCD approach in [21, 26] show that such a situation is rather unlikely. Indeed, in this approach going beyond the strict large  $N$  limit one can understand the suppression of  $B_6^{(1/2)}$  and  $B_8^{(3/2)}$  below the unity as the effect of the meson evolution from scales  $\mu = \mathcal{O}(m_\pi, m_K)$  at which (5) is valid to  $\mu = \mathcal{O}(1 \text{ GeV})$  at which Wilson coefficients of  $Q_6$  and  $Q_8$  are evaluated [21]. This evolution has to be matched to the usual perturbative quark evolution for scales higher than 1 GeV and in fact the suppressions in question and the property that  $B_6^{(1/2)}$  is more strongly suppressed than  $B_8^{(3/2)}$  are consistent with the perturbative evolution of these parameters above  $\mu = \mathcal{O}(1 \text{ GeV})$ . Thus we are rather confident that [21]

$$B_6^{(1/2)} < B_8^{(3/2)} < 1 \quad (\text{dual QCD}). \quad (7)$$

For further details, see [21, 2].

Additional support for the small value of  $\varepsilon'/\varepsilon$  in the SM comes from the recent reconsideration of the role of final state interactions (FSI) in  $\varepsilon'/\varepsilon$  [26]. Already long time ago the chiral perturbation theory practitioners put forward the idea that both the amplitude  $\text{Re}A_0$ , governed by the current-current operator  $Q_2 - Q_1$  and the  $Q_6$  contribution to the ratio  $\varepsilon'/\varepsilon$  could be enhanced significantly through FSI in a correlated manner (see e.g. [27] and other reference in [2]). However, as shown recently in [26] FSI are likely to be important for the  $\Delta I = 1/2$  rule, in agreement with these papers but much less relevant for  $\varepsilon'/\varepsilon$ .

It should finally be noted that even without lattice results, varying all input parameters, the bound in (7) implies the upper bound on  $\varepsilon'/\varepsilon$  in the SM

$$(\varepsilon'/\varepsilon)_{\text{SM}} < (8.6 \pm 3.2) \times 10^{-4}, \quad (\text{BG}). \quad (8)$$

On the other hand employing the lattice value for  $B_8^{(3/2)}$  in (6) and  $B_6^{(1/2)} = B_8^{(3/2)} = 0.76$ , one obtains  $(6.0 \pm 2.4) \times 10^{-4}$  instead of (8), well below the data.

All these findings give strong motivation for searching for NP which could enhance  $\varepsilon'/\varepsilon$  above its SM value. We will summarize the present efforts in this direction below.

## 2.2. Tensions between $\varepsilon_K$ and $\Delta M_{s,d}$ in the SM and CMFV Models

In [28] we have pointed out a significant tension between  $\varepsilon_K$  and  $\Delta M_{s,d}$  within the SM and models with constrained MFV (CMFV) implied by new lattice QCD results from Fermilab Lattice and MILC Collaborations [29] on  $B_{s,d}^0 - \bar{B}_{s,d}^0$  hadronic matrix elements. Even if this tension is certainly not as large as is the case of the  $\varepsilon'/\varepsilon$  anomaly the plots in [28], in particular in Fig. 5 of that paper, show that there is a clear tension between  $\varepsilon_K$  and  $\Delta M_{s,d}$  in the SM and CMFV models. Moreover this tension persists independently of the values of CKM parameters. For smaller (exclusive) values of  $|V_{cb}|$  one finds  $\Delta M_{s,d}$  to agree well with the data, while  $\varepsilon_K$  is roughly 25% below its experimental value. For  $|V_{cb}|$  in the ballpark of inclusive determinations one finds  $\varepsilon_K$  to agree with the data, while  $\Delta M_{s,d}$  are then typically by 15% larger than their experimental values. These numbers are for the SM, in all other CMFV models the situation gets worse.

The improved  $\Delta B = 2$  hadronic matrix elements from other lattice collaborations and improved values of  $|V_{cb}|$  and  $|V_{ub}|$  will tell us one day whether this tension persists and if this will turn out to be the case, whether there is a  $\varepsilon_K$  anomaly and/or a  $\Delta M_{s,d}$  anomaly.

## 2.3. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in the SM

These two rare decays allow to test the short distance scales far beyond the reach of the LHC. Even scales of  $\mathcal{O}(100)$  TeV can be probed in this manner [30]. The present status of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  within the SM has been presented in [25] with the result

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11}, \quad (9)$$

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.4 \pm 0.6) \times 10^{-11}. \quad (10)$$

But the most important outcome of this paper are parametric expressions for the branching ratios of these two decays in terms of the CKM input and the correlations between  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $B_s \rightarrow \mu^+ \mu^-$  and between  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $\varepsilon_K$  in the SM. These formulae should allow to monitor the numerical values for these branching ratios within the SM when the CKM input improves. Interesting correlations between  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and various observables are found in simplified models with flavour violating couplings of the SM  $Z$  and of a heavy  $Z'$  [31].

## 2.4. Strategy for $\varepsilon'/\varepsilon$ and Lessons

In order to investigate the implications of  $\varepsilon'/\varepsilon$  anomaly on rare decays  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  in a systematic fashion a strategy has been proposed in [32]. While  $\varepsilon'/\varepsilon$  plays the dominant role in this strategy it was useful to assume that there is also a modest  $\varepsilon_K$  anomaly.

Then  $\varepsilon'/\varepsilon$  and  $\varepsilon_K$  in the presence of NP contributions are given by

$$\frac{\varepsilon'}{\varepsilon} = \left( \frac{\varepsilon'}{\varepsilon} \right)^{\text{SM}} + \left( \frac{\varepsilon'}{\varepsilon} \right)^{\text{NP}}, \quad \varepsilon_K \equiv e^{i\varphi_\varepsilon} [\varepsilon_K^{\text{SM}} + \varepsilon_K^{\text{NP}}] \quad (11)$$

with NP contributions parametrized as follows:

$$\left(\frac{\varepsilon'}{\varepsilon}\right)^{\text{NP}} = \kappa_{\varepsilon'} \cdot 10^{-3}, \quad 0.5 \leq \kappa_{\varepsilon'} \leq 1.5, \quad \varepsilon_K^{\text{NP}} = \kappa_{\varepsilon} \cdot 10^{-3}, \quad 0.1 \leq \kappa_{\varepsilon} \leq 0.4. \quad (12)$$

The ranges for  $\kappa_{\varepsilon'}$  and  $\kappa_{\varepsilon}$  indicate the required size of this contribution but can be kept as free parameters. They will be determined one day when the theory on  $\varepsilon'/\varepsilon$  and the CKM input improve.

In the simplest NP scenarios with tree-level  $Z$  and  $Z'$  exchanges, the imaginary parts of flavour-violating  $Z$  or  $Z'$  couplings to quarks are then determined as functions of  $\kappa_{\varepsilon'}$ . As  $\varepsilon_K$  is governed by the product of imaginary and real parts of these couplings, invoking it allows then to determine the corresponding real parts as functions of  $\kappa_{\varepsilon'}$  and  $\kappa_{\varepsilon}$ .

Having fixed the flavour violating couplings of  $Z$  or  $Z'$  in this manner, one can express NP contributions to the branching ratios for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ ,  $K_L \rightarrow \mu^+ \mu^-$  and to  $\Delta M_K$  in terms of  $\kappa_{\varepsilon'}$  and  $\kappa_{\varepsilon}$ . Explicit formulae can be found in [32]. In this manner one can directly study the impact of  $\varepsilon'/\varepsilon$  and  $\varepsilon_K$  anomalies in  $Z$  and  $Z'$  scenarios on these four observables.

In [32] numerous plots for the ratios

$$R_+^{\nu\bar{\nu}} \equiv \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}}}, \quad R_0^{\nu\bar{\nu}} \equiv \frac{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} \quad (13)$$

as functions of  $\kappa_{\varepsilon'}$  and  $\kappa_{\varepsilon}$  within the models with tree-level  $Z$  and  $Z'$  exchanges have been presented. We will list the most important lessons from this study that depend on the flavour violating couplings  $\Delta_{L,R}^{sd}(Z)$  and  $\Delta_{L,R}^{sd}(Z')$  [33]. Moreover, we will use the abbreviations: (LHS  $\equiv$  left – handed scenario) and (RHS  $\equiv$  right – handed scenario) for NP scenarios in which only left-handed (LH) or right-handed (RH) flavour-violating couplings are present. The first six lessons deal with tree-level  $Z$  exchanges, the remaining four with  $Z'$  tree-level exchanges.

**Lesson 1:** In the LHS, a given request for the enhancement of  $\varepsilon'/\varepsilon$  determines the coupling  $\text{Im}\Delta_L^{sd}(Z)$ .

**Lesson 2:** In LHS an enhanced  $\varepsilon'/\varepsilon$  implies uniquely *suppression* of  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ . This property is known from NP scenarios in which NP to  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and  $\varepsilon'/\varepsilon$  enters dominantly through the modification of  $Z$ -penguins.

**Lesson 3:** The imposition of the  $K_L \rightarrow \mu^+ \mu^-$  constraint in LHS determines the range for  $\text{Re}\Delta_L^{sd}(Z)$  which with the already fixed  $\text{Im}\Delta_L^{sd}(Z)$  allows to calculate the shifts in  $\varepsilon_K$  and  $\Delta M_K$ . They are very small.

**Lesson 4:** With fixed  $\text{Im}\Delta_L^{sd}(Z)$  and the allowed range for  $\text{Re}\Delta_L^{sd}(Z)$ , the range for  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  can be obtained. Both an enhancement and a suppression of  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  are possible.  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  can be enhanced by a factor of 2 at most.

**Lesson 5:** Analogous pattern is found in RHS, although the numerics is different. See Fig. 1 in [32]. In particular the suppression of  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  for a given  $\kappa_{\varepsilon'}$  is smaller. Moreover, an enhancement of  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  up to a factor of 5.7 is possible.

**Lesson 6:** In a general  $Z$  scenario with LH and RH flavour-violating couplings the pattern of NP effects changes because LR operators dominate NP contributions to  $\varepsilon_K$  and  $\Delta M_K$ . One can then enhance simultaneously  $\varepsilon'/\varepsilon$ ,  $\varepsilon_K$ ,  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  and  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  which is not possible in LHS and RHS. The correlations between  $\varepsilon'/\varepsilon$  and  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  depend sensitively on the ratio of real and imaginary parts of the flavour-violating couplings involved. Moreover large departures from SM predictions for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  are possible and  $\varepsilon'/\varepsilon$  anomaly can be explained.

$Z'$  models exhibit quite different pattern of NP effects in the  $K$  meson system than the LH and RH  $Z$  scenarios. In  $Z$  scenarios only electroweak penguins (EWP) can contribute to  $\varepsilon'/\varepsilon$  in an important manner because of flavour dependent diagonal  $Z$  coupling to quarks. But in  $Z'$

models the diagonal quark couplings can be flavour universal so that QCD penguin operators (QCDP) can dominate NP contributions to  $\varepsilon'/\varepsilon$ . Interestingly, the pattern of NP in rare  $K$  decays depends on whether NP in  $\varepsilon'/\varepsilon$  is dominated by QCDP or EWP operators.

As demonstrated in [32] there is a large hierarchy between real and imaginary parts of the flavour violating couplings implied by  $\varepsilon'/\varepsilon$  anomaly in QCDP and EWP scenarios. In the case of QCDP imaginary parts dominate over the real ones, while in the case of EWP this hierarchy is opposite unless the  $\varepsilon_K$  anomaly is absent. Because of these different patterns there are striking differences in the implications of the  $\varepsilon'/\varepsilon$  anomaly for the correlation between  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  in these two NP scenarios if significant NP contributions to  $\varepsilon'/\varepsilon$  are required. The plots in [32] and in particular analytic derivations presented there illustrate these differences in a spectacular manner. The main lessons are as follows.

**Lesson 7:** In the case of QCDP scenario the correlation between  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  takes place along the branch parallel to the Grossman-Nir bound [34].

**Lesson 8:** In the EWP scenario the correlation between  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  is very different from the one of the QCDP case. NP effects in rare  $K$  decays turn out to be modest in this case unless the diagonal quark couplings are  $\mathcal{O}(10^{-2})$  and then the requirement of shifting upwards  $\varepsilon'/\varepsilon$  implies large effects in  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  also in the EWP scenario.

**Lesson 9:** For fixed values of the neutrino and diagonal quark couplings in  $\varepsilon'/\varepsilon$  the predicted enhancements of  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  are much larger when NP in QCDP is required to remove the  $\varepsilon'/\varepsilon$  anomaly than it is the case of EWP.

**Lesson 10:** In QCDP scenario  $\Delta M_K$  is *suppressed* and this effect increases with increasing  $M_{Z'}$  whereas in the EWP scenario  $\Delta M_K$  is *enhanced* and this effect decreases with increasing  $M_{Z'}$  as long as real couplings dominate. Already on the basis of this property one could differentiate between these two scenarios when the SM prediction for  $\Delta M_K$  improves.

### 3. Results in specific NP models

#### 3.1. Preliminaries

The latest analyses of NP contributions to  $\varepsilon'/\varepsilon$  in models with tree-level  $Z$  and  $Z'$  exchanges like 331 models, Littlest Higgs model with T-parity can be found in [35, 31, 36, 32, 37]. The analyses in supersymmetric models can be found in [38, 39, 40]. In view of space limitations we will only briefly summarize the results in 331 models and models with vector-like quarks.

#### 3.2. 331 Flavour News

The 331 models are based on the gauge group  $SU(3)_C \times SU(3)_L \times U(1)_X$  [41, 42, 43, 44, 45]. In these models new contributions to  $\varepsilon'/\varepsilon$  and other flavour observables are dominated by tree-level exchanges of a  $Z'$  with non-negligible contributions from tree-level  $Z$  exchanges generated through the  $Z - Z'$  mixing. The size of these NP effects depends on  $M_{Z'}$ , on a parameter  $\beta$ , which distinguishes between various 331 models, on fermion representations under the gauge group and a parameter  $\tan \bar{\beta}$  present in the  $Z - Z'$  mixing. Extensive recent analyses in these models can be found in [46, 47, 48, 36, 37]. References to earlier analyses of flavour physics in 331 models can be found there and in [49, 50].

A detailed analysis of 331 models with different values of  $\beta$ ,  $\tan \bar{\beta}$  for two fermion representations  $F_1$  and  $F_2$ , with the third SM quark generation belonging respectively to an antitriplet and a triplet under the  $SU(3)_L$ , has been presented in [48]: 24 models in total. Requiring that these models perform at least as well as the SM, as far as electroweak tests are concerned, seven models have been selected for a more detailed study of FCNC processes. Recent updated analyses of these seven models, that address the  $\varepsilon'/\varepsilon$  anomaly, have been presented in [36, 37] and we summarize the main results of these two papers. The main findings of [36, 37] for  $M_{Z'} = 3 \text{ TeV}$  are as follows:

- Among seven 331 models singled out through electroweak precision study only three (M8, M9, M16) can provide significant shift of  $\varepsilon'/\varepsilon$  but not larger than  $6 \times 10^{-4}$ , that is  $\kappa_{\varepsilon'} \leq 0.6$ .
- The tensions between  $\Delta M_{s,d}$  and  $\varepsilon_K$  can be removed in these models.
- Two of them (M8 and M9) can simultaneously suppress  $B_s \rightarrow \mu^+ \mu^-$  and bring the theory within  $1\sigma$  range of the combined result from CMS and LHCb. The most recent result from ATLAS [51], while not accurate, appears to confirm this picture. On the other hand these models do not really help in the case of  $B_d \rightarrow K^* \mu^+ \mu^-$  anomalies [52, 53].
- In M16 the situation is opposite. The rate for  $B_s \rightarrow \mu^+ \mu^-$  can be reduced for  $M_{Z'} = 3 \text{ TeV}$  by only a small amount but the anomaly in  $B_d \rightarrow K^* \mu^+ \mu^-$  can be significantly reduced.
- For higher values of  $M_{Z'}$  the effects in  $B_s \rightarrow \mu^+ \mu^-$  and  $B_d \rightarrow K^* \mu^+ \mu^-$  are small. NP effects in rare  $K$  decays and  $B \rightarrow K(K^*) \nu \bar{\nu}$  remain small in all 331 models even for  $M_{Z'}$  of a few TeV. This could be challenged by NA62, KOTO and Belle II experiments in this decade.

All these results are valid for  $|V_{ub}| = 0.0036$ . For its inclusive value of  $|V_{ub}| = 0.0042$ , we find that for  $|V_{cb}| = 0.040$  the maximal shifts in  $\varepsilon'/\varepsilon$  are increased to  $7.7 \times 10^{-4}$  and  $8.8 \times 10^{-4}$  for  $M_{Z'} = 3 \text{ TeV}$  and  $M_{Z'} = 10 \text{ TeV}$ , respectively. Renormalization group effects are responsible for this enhancement of  $\varepsilon'/\varepsilon$  for increased  $M_{Z'}$ . A recent analysis in the MSSM in [39] identifies this effect as well. But as explained in [36] eventually for very high  $M_{Z'}$ , NP effects in  $\varepsilon'/\varepsilon$  will be suppressed.

Thus the main message from [36, 37] is that NP contributions in 331 models can simultaneously solve  $\Delta F = 2$  tensions, enhance  $\varepsilon'/\varepsilon$  and suppress either the rate for  $B_s \rightarrow \mu^+ \mu^-$  or  $C_9$  Wilson coefficient without any significant NP effects on  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and  $b \rightarrow s \nu \bar{\nu}$  transitions. While sizable NP effects in  $\Delta F = 2$  observables and  $\varepsilon'/\varepsilon$  can persist for  $M_{Z'}$  outside the reach of the LHC, such effects in  $B_s \rightarrow \mu^+ \mu^-$  will only be detectable provided  $Z'$  will be discovered soon.

### 3.3. Models with vector-like quarks (VLQs)

A detailed analysis of flavour violation patterns in the  $K$  and  $B_{s,d}$  sectors in eleven models with VLQs has been presented in [54]. The simplest (five of them) are the ones in which the gauge group is the SM one and the only new particles are VLQs in a single complex representation under the SM gauge group. A general classification of such models and references to the rich literature can be found in [54, 55]. In these models  $\Delta F = 1$  FCNCs are dominated by tree-level  $Z$  exchanges, while  $\Delta F = 2$  transitions by box diagrams with VLQs and scalars provided  $M_{\text{VLQ}} \geq 5 \text{ TeV}$ . Otherwise tree-level  $Z$  contributions cannot be neglected.

The summary of patterns of flavour violation in these models can be found in three DNA tables (Tables 5, 6, 10 in [54]) and the numerical results in Tables 8 and 9 of that paper. Our extensive numerical analysis has shown that NP effects in several of these models can still be very large and that simultaneous consideration of several flavour observables should allow to distinguish between these models. In particular models with left-handed and right-handed flavour violating currents can be distinguished from each other in this manner. Here we list most important results of this paper.

- All tensions between  $\Delta M_{s,d}$  and  $\varepsilon_K$  can be easily removed in these models because the usual CMFV correlations between  $\Delta M_{s,d}$  and  $\varepsilon_K$  are not valid in them. The box diagrams with VLQs and Higgs scalar exchanges are dominantly responsible for it.
- Tree-level  $Z$  contributions to  $\varepsilon'/\varepsilon$  can be large so that significant upward shift in  $\varepsilon'/\varepsilon$  can easily be obtained bringing the theory to agree with data.
- Simultaneously the branching ratio for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  can be significantly enhanced over its SM prediction, but only in models with flavour violating RH currents. In models with

only LH currents  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  branching ratio can have at most its SM value because of the  $K_L \rightarrow \mu \bar{\mu}$  constraint. On the other hand the positive shift in  $\varepsilon'/\varepsilon$  implies uniquely suppression of the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  branching ratio with the suppression being smaller in models with RH currents. The fact that in models with RH currents  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  can be enhanced, while  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  suppressed is a clear signal of non-MFV sources at work. But also in models with pure LH currents the correlations between the branching ratios of these two decays differ from the MFV one.

- These features distinguish VLQ-models from 331 models, discussed above, in which NP effects are dominated by  $Z'$  exchanges with the maximal shift in  $\varepsilon'/\varepsilon$  amounting to  $0.8 \times 10^{-3}$  and NP effects in rare  $K$  decays being very small.
- Significant suppressions of the branching ratio for  $B_s \rightarrow \mu^+ \mu^-$ , in particular in models with LH currents are possible. While such effects are also possible in 331 models, they cannot be as large as in VLQ models.
- On the other hand while 331 models can provide solutions to some LHCb anomalies, this is not possible in the VLQs models with SM gauge group and future confirmation of these anomalies could turn out to be a problem for the latter models.

Having the latter possibility in mind we have considered also six VLQ models with a heavy  $Z'$  related to  $U(1)_{L_\mu - L_\tau}$  symmetry and extended scalar sector. Some of such models, considered already in [56], can explain LHCb anomalies but NP effects in other observables are in my view less interesting than in models based on the SM gauge group. We refer to [54] for details. Future experimental results on  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ ,  $B_s \rightarrow \mu^+ \mu^-$  and LHCb anomalies and improved theoretical results on  $\varepsilon'/\varepsilon$  will tell us which of these VLQ models, if any, is selected by nature.

While the discovery of VLQs at the LHC would give a strong impetus to the models considered by us, non-observation of them at the LHC would not preclude their importance for flavour physics. In fact we have shown that large NP effects in flavour observables can be present for  $M_{\text{VLQ}} = 10$  TeV and in the flavour precision era one could even be sensitive to higher masses. In this context we have pointed out that the combination of  $\Delta F = 2$  and  $\Delta F = 1$  observables in a given meson system allows to determine the masses of VLQs in a given representation independently of the size of Yukawa couplings.

In summary the future of kaon flavour physics looks great and the coming years should be very exciting.

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