

# Modelling graph dynamics of flaring active regions using SDO/HMI data

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**Abstract.** Large solar flares define parameters of space weather near the Earth and normal operation of spacecrafts substantially depends on the cosmic particles flux of solar wind. Therefore, search for large flares predictors is an important problem. We propose to approximate topology of the magnetic field of active region by graph, whose vertices are minima and maxima of a scalar field and edges form a so-called critical net. Localization and number of critical points change during the process of evolution and, therefore, it is possible to track dynamic regimes of active region by considering dynamics of graphs. Numerical characteristic of the critical net is a spectrum of eigenvalues of its discrete Laplacian. We present examples which show that, apparently, Laplacian spectrum is closely related to flaring activity. It will allow us to use critical net in prognostic systems for prediction of large solar flares.

## 1. Introduction

Evolution of flaring active regions (AR) shows high spatial complexity and nontrivial changes of observed patterns over time. It is practically impossible to give detailed description of a magnetogram morphology. This is why signatures are quite often used: critical points, neutral lines, unsigned and signed areas of magnetic textures. However, when comparing two magnetograms adjacent in time, it is difficult to find or measure significant signature changes because of high patterns variability. This is why it is more convenient to use approximating simplicial structures like graphs, namely set of vertices connected by edges. As nodes we can use Morse critical points, which are obtained by consecutively convolving the magnetogram and gaussian kernel. Such convolution is equivalent to the solution of second boundary problem where kernel width replaces time. Let us choose the smoothing level such that the number of extrema in Scale-Space remains constant in chosen range. Then the difference of two consecutive convolutions gives an estimation of Laplacian, which contains stable maxima and minima.

We compute so-called critical nets, which consist of those singular points connected by ascending paths in terms of Morse theory [1]. Critical net approximates singular manifold of AR with graph which is rebuilt during evolution. Dynamics regimes can be conveniently tracked using methods of spectral geometry [2]. For that we use discrete Laplacian and its spectrum. We present examples of critical nets for flaring AR and estimations of their spectra using HMI/SDO magnetograms of flaring active regions. Examples show explicit relation of spectra with flaring productivity.



## 2. Scale-Space, stable Laplacian and critical net

Scale-Space represents a sequence of images (or pyramid) with decreasing level of detalization (or size). The first level of Scale-Space is the image itself, the next image in pyramid can be obtained by smoothing previous image in the sequence. Such pyramids of images are used to analyze images at different scales, where larger scales correspond to bigger objects in images. It can be particularly useful for localization of spot-like structures.

Laplacian is now widely used in different detectors for stable keypoints localization and it has been shown that it provides one of the most stable features compared to others ([3], [4]). It is known that Laplacian can be approximated by computing difference of consecutive convolutions of image with gaussian kernel  $G_t$  (where  $t$  is the kernel width):  $L_t = \Delta I_t \approx I * G_{t+1} - I * G_t$ .

Laplacian contains maximally convex regions (i.e. connected components of pixels for which  $L_t > 0$ ) and maximally concave regions ( $L_t < 0$ ). Scale  $t$  is called  $\beta$ -stable if  $t$  is the smallest integer for which the number of maximally convex regions in Laplacian  $L_\xi$  for all  $\xi \in [t - \beta, t)$  does not change. Critical net is a directed acyclic graph, whose vertices are represented by local extrema of a Laplacian  $f$  and where edges goes from minima to maxima according to ascending paths [1].

## 3. Discrete Laplacian

Let  $G = (V, E)$  be a graph with  $n$  vertices and  $\mathbf{x} = (x_1, \dots, x_n)$  be a distribution of weights defined on  $V$  then:

**Definition 1 (Graph Laplacian)** *Laplacian of a graph  $G$  is the sum:*

$$\text{Lap}(\mathbf{x}) = \sum_{\{i,j\} \in E} (x_i + x_j)^2 \quad (1)$$

Matrix of this quadratic form can be written as  $L = D - A$ , then:

$$\text{Lap}(\mathbf{x}) = \mathbf{x}^T L \mathbf{x}$$

where  $D = \text{diag}(d_1, \dots, d_n)$  is a diagonal matrix and  $d_i$  is a degree of  $i$ -th vertex,  $A$  is adjacency matrix. Laplacian (1) in such form may be thought of as a measure of heterogeneity of distribution  $\mathbf{x}$ .

Another interpretation of Laplacian is a transition to differential or  $\delta$ -coordinates [5]. Let  $M = (V, E, F)$  be a triangular mesh, where  $V$  is a set of  $n$  vertices,  $E$  is a set of edges and  $F$  is a set of faces. Each vertex is represented by Cartesian coordinates  $v_i = (x_i, y_i, z_i)$ , then differential coordinates can be defined as follows:

$$\delta_i = (\delta_i^x, \delta_i^y, \delta_i^z) = v_i - \frac{1}{d_i} \sum_{j \in N(i)} v_j$$

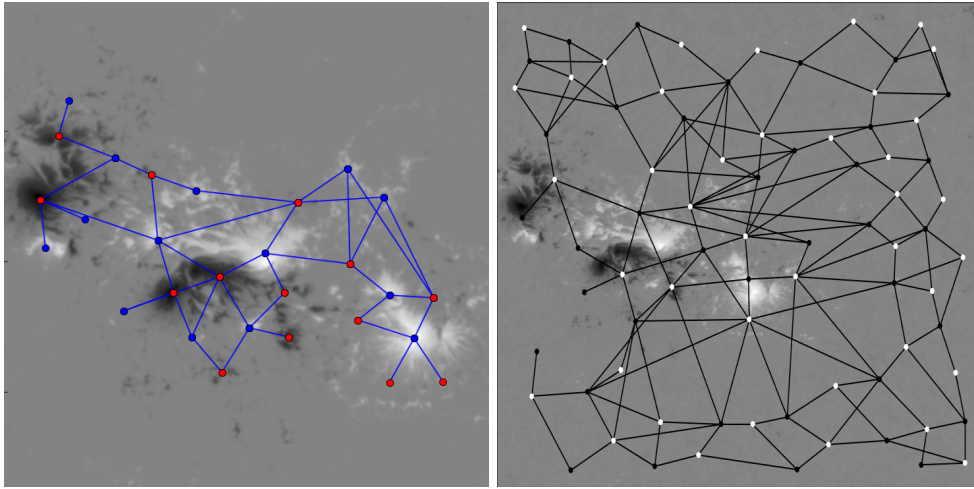
where  $N(i)$  is the set of immediate neighbours of vertex  $i$  and  $d_i = |N(i)|$ . The vector of differential coordinates approximates local normal direction and mean curvature at  $v_i$ , that is it encapsulates the local surface shape.

Such transformation can be written in matrix form:

$$L = I - D^{-1} A$$

where  $I$  is the identity matrix,  $A$  is the adjacency matrix and  $D$  is the matrix of vertices degrees. It can also be written in symmetric form:

$$L' = DL = D - A$$



**Figure 1.** Examples of critical nets constructed using magnetogram from active region 11158. Critical net on the left is constructed by using our approach, critical net on right is constructed by authors of work [7].

whose eigenvalues are bounded by  $2 \max d_i$ . However, we consider the symmetric normalized version of matrix  $L$ :

$$L_s = D^{-1/2} L' D^{-1/2} = I - D^{-1/2} A D^{-1/2}$$

The set of eigenvalues  $0 = \lambda_0 \leq \lambda_1 \leq \dots \lambda_{n-1}$  of  $L$  is called the spectrum of  $L$ . All eigenvalues of  $L_s$  lie in range  $[0, 2]$  in contrast to eigenvalues of nonnormalized  $L'$  [6]. There are some well-known facts about the spectrum of Laplacian matrix:

- the multiplicity of zero eigenvalue is the number of connected components of graph
- the first nonzero eigenvalue is called spectral gap
- the second nonzero eigenvalue is called algebraic connectivity

Spectral gap and algebraic connectivity often coincide when graph consists of several connected components. The intuition above spectral gap and algebraic connectivity is quite simple: the smaller the spectral gap the less connected graph is.

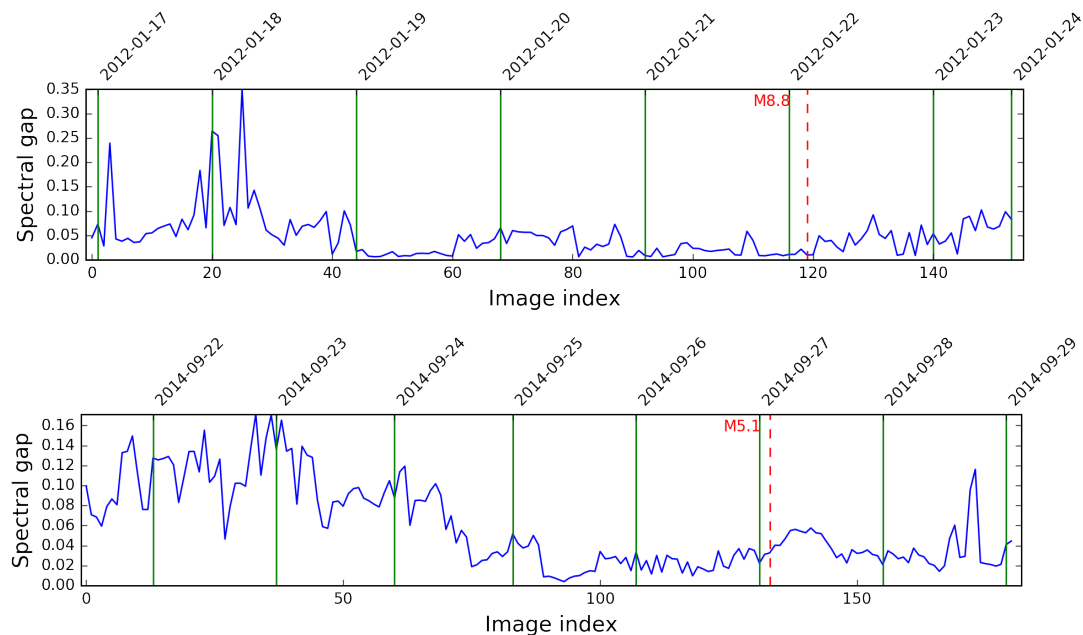
#### 4. Experimental results and conclusion

In our approach we replace dynamics of the magnetic structures by dynamics of graphs (critical nets): each magnetogram is replaced by critical net and then analyzed. We processed approximately 1400 images from 8 different active regions and computed graphs of spectral gap for each active region.

Although we use ideas and notion of critical net introduced in [1], it is not said how to select one particular level among all  $\beta$ -stable levels, because image may contain many such levels with different  $\beta$  values. In our case we compute median scale of a  $\beta$ -stable level among all such levels for each region and use its median scale  $s$  for construction of critical nets.

Additionally, when computing critical points of  $L_t$ , we use threshold for Laplacian value in order to remove points corresponding to dim and hardly seen structures lying far from compact groups of spots which can be considered as noise.

There is work [7] in which authors use the same ideas but consider the spectrum of ordinary symmetric graph Laplacian  $L = D - A$  and do not remove noisy points what results in worse approximation of active region. It is also unknown how they select smoothing scale depending



**Figure 2.** Graph of spectral gap for active regions 11402 and 12173.

on  $\beta$ -stable levels: too small smoothing scale will provide many keypoints corresponding to high frequency component and noisy structures, too big scale will only contain points corresponding to huge spots.

The result of constructing critical nets compared to critical net built in work [7] can be seen in figure 1. Our critical net better approximates active region and contains less noisy points.

Figure 2 shows graph of spectral gap for active regions 11402 and 12173. Green vertical lines mark days and red dashed line marks day when flare appears. It can be seen that there is a big increase in spectral gap value 4–5 days before the flare (compared to 1–2 days in [7]) — it can be interpreted as an increase in graphs connectivity. Obtained results show that Laplacian spectrum of the critical net and solar flare productivity are closely related and proposed method is capable of tracking flaring dynamic regimes of active regions.

## Acknowledgements

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## References

- [1] Gu S, Zheng Y and Tomasi C 2010 *Lecture Notes in Computer Science* **6313** 663–676
- [2] Reuter M *et al.* 2009 *Computers & Graphics* **33**(3) 381–390
- [3] Lowe D G 1999 *Proceedings of the Seventh IEEE International Conference on Computer Vision* **2** 1150–1157
- [4] Mikolajczyk K 2011 *Detection of local features invariant to affine transformations* (Grenoble: Institut National Polytechnique de Grenoble)
- [5] Sorkine O 2006 *Proceedings of the 4th international conference on Computer graphics and interactive techniques in Australasia and Southeast Asia* 381–389
- [6] Chung F 1997 *Spectral Graph Theory* (Providence: American Mathematical Society)
- [7] Knyazeva I S *et al.* 2016 *Journal of Physics: Conference Series* **738**(1) 012070