IOP Conf. Series: Journal of Physics: Conf. Series **798** (2017) 012112 doi:10.1088/1742-6596/798/1/012112

Neutrino-electron scattering in a dense magnetized plasma

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Abstract. The process of neutrino-electron scattering in a dense plasma and magnetic field of arbitrary strength, where electrons can occupy the states corresponding to excited Landau levels, is analyzed. The total probability of this process, summarized over all initial states of the plasma electrons which is only physically meaningful, is calculated. Possible astrophysical applications are discussed.

1. Introduction

Neutrino physics plays a decisive role in astrophysical cataclysms such as supernova explosions and coalescence of neutron stars, and also in the early universe. Consequently, studies of neutrino interactions and in particular neutrino-electron processes in an external active medium are of considerable interest. At the same time, an investigation of neutrino processes under such extreme physical conditions is interesting from the conceptual viewpoint since it affects fundamental problems of quantum field theory.

A correct analysis of the neutrino propagation process in a hot dense plasma in the presence of a strong magnetic field requires to consider the complete set of neutrino-electron processes. Then only the probability of the process summarized over all initial states of the plasma electrons is physically meaningful. In Ref. [1], numerical calculations of the differential cross-section of the neutrino-electron scattering in dense magnetized plasma were performed in the limit of rather weak magnetic field B, $eB < \mu E$, where μ is the plasma chemical potential, E is the typical neutrino energy.[1] In Refs. [2, 3], the probability of the $\nu e \to \nu e$ process and the volume density of the neutrino energy and momentum losses, summarized over all initial states of the plasma electrons, were evaluated under the physical situation where the magnetic field is moderate, while the density of plasma is large, so the conditions are satisfied: $\mu^2 > eB \gg (T^2, E^2) \gg m_e^2$, where T is the plasma temperature, and $eB \gg \mu E$. Calculations were performed for the case when both initial and final electrons occupy the same Landau levels, because, as it was concluded in Refs. [2, 3], such transitions were dominating. The purpose of the present research is to calculate analytically the probability of this process for a more general case when the initial and final electrons could occupy any physically allowed Landau levels. Some details of the calculation technique can be found e.g. in Refs. [4, 5, 6].

[1] We use natural units in which $c = \hbar = 1$, $e > 0$ is the elementary charge.

2. Solutions of the Dirac equation for an electron in a magnetic field

It appears to be convenient to use the electron wave functions as the eigenstates of the operator $\hat{\mu}_z$ [7, 8]:

$$\hat{\mu}_z = m_e\Sigma_z - \mathrm{i}\gamma_0\gamma_5[\boldsymbol{\Sigma}\times\boldsymbol{P}]_z\,, \tag{1}$$

where $\boldsymbol{P} = -\mathrm{i}\boldsymbol{\nabla} + e\boldsymbol{A}$. We take the frame where the field is directed along the z axis, and the Landau gauge where the four-potential is: $A^\lambda = (0, 0, xB, 0)$. In this approach, the electron wave functions have the form

$$\Psi^s_{p,n}(X) = \frac{\mathrm{e}^{-\mathrm{i}(\varepsilon_n t - p_y y - p_z z)}\; U^s_n(\xi)}{\sqrt{4\varepsilon_n M_n(\varepsilon_n + M_n)(M_n + m_e)L_y L_z}}\,, \tag{2}$$

where $\varepsilon_n = \sqrt{M_n^2 + p_z^2}\,,\quad M_n = \sqrt{m_e^2 + 2\beta n}\,,\quad \beta = eB\,,\quad \xi = \sqrt{\beta}\,(x + p_y/\beta)$. The functions $\Psi^s_{p,n}(X)$ satisfy the equation: $\hat{\mu}_z\,\Psi^s_{p,n}(X) = s\,M_n\,\Psi^s_{p,n}(X)\,(s = \pm 1)$. The bispinors $U^s_n(\xi)$ in Eq. (2) take the form:

$$U^-_n(\xi) = \begin{pmatrix} -\mathrm{i}\sqrt{2\beta n}\,p_z V_{n-1}(\xi) \\ (\varepsilon_n + M_n)(M_n + m_e)V_n(\xi) \\ -\mathrm{i}\sqrt{2\beta n}(\varepsilon_n + M_n)V_{n-1}(\xi) \\ -p_z(M_n + m_e)V_n(\xi) \end{pmatrix},\; U^+_n(\xi) = \begin{pmatrix} (\varepsilon_n + M_n)(M_n + m_e)V_{n-1}(\xi) \\ -\mathrm{i}\sqrt{2\beta n}\,p_z V_n(\xi) \\ p_z(M_n + m_e)V_{n-1}(\xi) \\ \mathrm{i}\sqrt{2\beta n}(\varepsilon_n + M_n)V_n(\xi) \end{pmatrix}. \tag{3}$$

Here, $V_n(\xi)\,(n = 0, 1, 2, \ldots)$ are the well-known normalized harmonic oscillator functions, which are expressed in terms of the Hermite polynomials $H_n(\xi)$.

In this case, the process amplitude will have an explicit Lorentz invariant structure.

3. The process of the $\nu e \to \nu e$ scattering

The effective local Lagrangian is:

$$\mathcal{L} \;=\; -\frac{G_\mathrm{F}}{\sqrt{2}}[\bar{e}\gamma_\alpha(C_V - C_A\gamma_5)e]\,[\bar{\nu}\gamma^\alpha(1-\gamma_5)\nu],\; C_V = \pm\frac{1}{2} + 2\sin^2\theta_\mathrm{W}\,,\; C_A = \pm\frac{1}{2}\,, \tag{4}$$

where the upper signs correspond to ν_e and the lower signs correspond to $\nu_{\mu,\tau}$.

The S matrix element of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$ takes the form

$$\begin{aligned} S \;&=\; \mathrm{i}\,\frac{G_\mathrm{F}}{\sqrt{2}}\,\frac{(2\pi)^3\,\delta(\varepsilon'_n - \varepsilon_\ell - q_0)\,\delta(p'_y - p_y - q_y)\,\delta(p'_z - p_z - q_z)}{\sqrt{2EV\,2E'V\,2\varepsilon_\ell L_y L_z\,2\varepsilon'_n L_y L_z}}\mathrm{e}^{-q_\perp^2/4eB - \mathrm{i}q_x(p_y + p'_y)/2eB} \\ &\times\; [\bar{u}(p')\,\hat{\jmath}(C_V - C_A\gamma_5)\,u(p)]\,, \end{aligned} \tag{5}$$

where $q = P - P' = p' - p$, ε_ℓ and ε'_n are the energies of the initial and final electrons, $q_\perp$ is the projection of the vector $\boldsymbol{q}$ on the plane perpendicular to the vector $\boldsymbol{B}$, $q_\perp^2 = q_x^2 + q_y^2$, and $j_\alpha = \bar{\nu}(P')\gamma_\alpha(1-\gamma_5)\nu(P)$ is the Fourier tranform of the current of the left-handed neutrinos.

The total process probability per unit time can be presented in the form

$$W(\nu e^- \to \nu e^-) = \frac{1}{\mathcal{T}}\sum_\ell\sum_n\sum_{s,s'}\int \mid S \mid^2\; \mathrm{d}n_{e^-}\,\mathrm{d}n'_{e^-}\,\frac{\mathrm{d}^3P'\,V}{(2\pi)^3}\,(1 - f(E'))\,, \tag{6}$$

where $\mathrm{d}n_{e^-} = f(\varepsilon_\ell)\,\mathrm{d}p_y\,\mathrm{d}p_z\,L_y\,L_z/(2\pi)^2\,,\; \mathrm{d}n'_{e^-} = (1 - f(\varepsilon'_n))\,\mathrm{d}p'_y\,\mathrm{d}p'_z\,L_y\,L_z/(2\pi)^2\,,\; f(\varepsilon_\ell)$ is the fermion distribution function, e.g. for initial electrons it is: $f(\varepsilon_\ell) = [\mathrm{e}^{(\varepsilon_\ell - \mu)/T} + 1]^{-1}$; $\mathcal{T}$ is the total time of interaction, $V = L_x\,L_y\,L_z$ is the total volume of the interaction region.

For the probability per unit time of the subprocess $\nu e^-_{(\ell)} \to \nu e^-_{(n)}$ one obtains

$$W_{\ell n} = \frac{\beta}{(2\pi)^4 16E} \int \frac{\mathrm{d}^3 P'}{E'} \left(1 - f(E')\right) \int \frac{\mathrm{d}p_z}{\varepsilon'_n \, \varepsilon_\ell} \, \delta(\varepsilon'_n - \varepsilon_\ell - q_0) \, f(\varepsilon_\ell) \left(1 - f(\varepsilon'_n)\right) \sum_{s,s'} |\mathcal{M}^{ss'}_{\ell n}|^2 \,. \quad (7)$$

Taking into account possible polarisation states of the initial and final electrons, there exist four invariant polarization amplitudes $\mathcal{M}^{ss'}_{\ell n}$, where $s, s' = \pm 1$, which can be presented in the form:

$$\mathcal{M}^{ss'}_{\ell n} = \eta \, \frac{G_\mathrm{F}}{2\sqrt{2}} \left\{ A^{ss'}_{\ell n} \frac{(j\widetilde{\varphi} q)}{q^2_\parallel} + B^{ss'}_{\ell n} \frac{(j\widetilde{\Lambda} q)}{q^2_\parallel} + C^{ss'}_{\ell n} \frac{(j\Lambda q)}{q^2_\perp} + D^{ss'}_{\ell n} \frac{\mathrm{i}\,(j\varphi q)}{q^2_\perp} \right\}, \quad (8)$$

where η is an inessential phase factor, $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ is the dimensionless tensor of the external magnetic field, $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\,\varepsilon_{\alpha\beta\mu\nu}\varphi^{\mu\nu}$ is the dual dimensionless tensor; the four-vectors with the indices $\perp$ and $\parallel$ belong to the Euclidean $\{1,\,2\}$-subspace and the Minkowski $\{0,\,3\}$-subspace, correspondingly, and $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta} = \mathrm{diag}(0,1,1,0)$, $\widetilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta} = \mathrm{diag}(1,0,0,-1)$. In Eq. (8), auxiliary functions are introduced: A, B, C, D. To illustrate, we give here their explicit form for one of the possible sets of polarization, $s = s' = -1$:

$$\begin{aligned} A^{--}_{\ell n} &= \sqrt{\left(1 + \frac{m}{M_\ell}\right)\left(1 + \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell,n} \left(C_V\,\tau + C_A\,\kappa\right) \\ &+ \sqrt{\left(1 - \frac{m}{M_\ell}\right)\left(1 - \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell-1,n-1} (C_V\,\tau - C_A\,\kappa)\,, \end{aligned} \quad (9)$$

$$\begin{aligned} B^{--}_{\ell n} &= \sqrt{\left(1 + \frac{m}{M_\ell}\right)\left(1 + \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell,n} \left(C_V\,\kappa + C_A\,\tau\right) \\ &+ \sqrt{\left(1 - \frac{m}{M_\ell}\right)\left(1 - \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell-1,n-1} (C_V\,\kappa - C_A\,\tau)\,, \end{aligned} \quad (10)$$

$$\begin{aligned} C^{--}_{\ell n} &= \sqrt{q^2_\perp}\Bigg[- \sqrt{\left(1 - \frac{m}{M_\ell}\right)\left(1 + \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell,n-1} \left(C_V\,u + C_A\,v\right) \\ &- \sqrt{\left(1 + \frac{m}{M_\ell}\right)\left(1 - \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell-1,n} \left(C_V\,u - C_A\,v\right)\Bigg]\,, \end{aligned} \quad (11)$$

$$\begin{aligned} D^{--}_{\ell n} &= \sqrt{q^2_\perp}\Bigg[\sqrt{\left(1 - \frac{m}{M_\ell}\right)\left(1 + \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell,n-1} \left(C_V\,u + C_A\,v\right) \\ &- \sqrt{\left(1 + \frac{m}{M_\ell}\right)\left(1 - \frac{m}{M_n}\right)}\, \mathcal{I}_{\ell-1,n} \left(C_V\,u - C_A\,v\right)\Bigg]\,, \end{aligned} \quad (12)$$

where, for $n \geqslant \ell$

$$\mathcal{I}_{n,\ell}(x) = \sqrt{\frac{\ell!}{n!}}\, \mathrm{e}^{-x/2} x^{(n-\ell)/2} L^{n-\ell}_\ell(x)\,, \quad \mathcal{I}_{\ell,n}(x) = (-1)^{n-\ell} \mathcal{I}_{n,\ell}(x)\,, \quad (13)$$

$L_n^k(x)$ are the generalized Laguerre polynomials, and also:

$$\kappa = u\,(M_n - M_\ell)\,,\ \tau = v\,(M_n + M_\ell)\,,\ u = \sqrt{(M_n + M_\ell)^2 - q_\parallel^2}\,,\ v = \alpha\,\zeta\,\sqrt{(M_n - M_\ell)^2 - q_\parallel^2}\,.$$

Here, $\alpha = q_0/|q_0|$ is the sign of q_0, $\zeta = \pm 1$ is the sign factor associated with the two roots of the equation, corresponding to the zeros of the δ function argument in Eq. (7). In the frame where $q_z = 0$, ζ is the sign of the p_z component, which is not fixed by the equation.

Calculations of the process probability per unit time (6) with Eqs. (7)–(13) should be performed numerically for different values of the physical parameters B, T, μ, E etc., and we plan to present the results in an extended paper. It should be noted, that, in contrast to calculations of Refs. [5, 6], in the present case no upper limits arise on the Landau level numbers ℓ, n from kinematics, and the suppression of the large number contributions is provided by the distribution functions of initial and final electrons.

As the numerical analysis shows, the assumption made in Refs. [2, 3], that the subprocesses were dominating where both initial and final electrons occupied the same Landau levels, was incorrect. This means that the results for the probability of the $\nu e \to \nu e$ process obtained in Refs. [2, 3], were underestimated.

4. Conclusions

The probability of the $\nu e \to \nu e$ process in a dense magnetized plasma is calculated analytically, for a general case when the initial and final electrons could occupy any physically allowed Landau levels. The analysis shows, that the assumption made in previous calculations, that the subprocesses were dominating where both initial and final electrons occupied the same Landau levels, was incorrect.

In astrophysical applications, the mean values of the neutrino energy and momentum losses could be more interesting: $Q^\alpha = E\int (P - P')^\alpha\,\mathrm{d}W = -E\,(\mathcal{I}, \boldsymbol{F})$. where $\mathrm{d}W$ is the total differential probability of the process. The zeroth component of Q^α is connected with the mean energy lost by a neutrino per unit time due to the process considered, $\mathcal{I} = \mathrm{d}E/\mathrm{d}t$. The space components of the four-vector Q^α are similarly connected with the mean neutrino momentum loss per unit time, $\boldsymbol{F} = \mathrm{d}\boldsymbol{P}/\mathrm{d}t$. An analysis of the four-vector Q^α in a general case for the magnetic field of arbitrary strength, where electrons can occupy the states corresponding to excited Landau levels, now is in progress. The force density $\boldsymbol{F}$ could lead to a very interesting consequences if a strong toroidal magnetic field is generated in the supernova envelope, providing an asymmetry of the supernova explosion and, in particular, it can explain the phenomenon of high pulsar kick-velocities, for details see Ref. [4].

Acknowledgments

The study was supported by the Russian Foundation for Basic Research (Project No. 14-02-00233-a).

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