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On stable exponential cosmological solutions with non-static volume factor in the Einstein-Gauss-Bonnet model

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Abstract. A $(n+1)$-dimensional gravitational model with cosmological constant and Gauss-Bonnet term is studied. The ansatz with diagonal cosmological metrics is adopted and solutions with exponential dependence of scale factors: $a_i \sim \exp{(v^i t)}$, $i = 1, \ldots, n$, are considered. The stability analysis of the solutions with non-static volume factor is presented. We show that the solutions with $v^1 = v^2 = v^3 = H > 0$ and small enough variation of the effective gravitational constant G are stable if certain restriction on (v^i) is obeyed. New examples of stable exponential solutions with zero variation of G in dimensions $D = 1 + m + 2$ with $m > 2$ are presented.

1. Introduction

This paper deals with gravitational model governed by the action

$$S = \int_M d^D z \sqrt{|g|}\{\alpha_1(R[g] - 2\Lambda) + \alpha_2 \mathcal{L}_2[g]\}, \tag{1}$$

where $g = g_{MN} dz^M \otimes dz^N$ is the metric defined on the manifold M, $\dim M = D$, $|g| = |\det(g_{MN})|$,

$$\mathcal{L}_2 = R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^2 \tag{2}$$

is the quadratic "Gauss-Bonnet term" and Λ is cosmological term. Here α_1 and α_2 are non-zero constants. The appearance of the Gauss-Bonnet term was motivated by string theory [1].

At present, the so-called Einstein-Gauss-Bonnet (EGB) gravitational model which is governed by the action (1) and its modifications are intensively used in cosmology, e.g. for explanation of accelerating expansion of the Universe.

Here we consider the cosmological solutions with diagonal metrics governed by n scale factors depending upon one variable, where $n > 3$; $D = n + 1$. We study the solutions with exponential dependence of scale factors with respect to the synchronous time variable t

$$a_i(t) \sim \exp{(v^i t)}, \tag{3}$$

$i = 1, \ldots, n$.

For possible physical applications solutions describing an exponential isotropic expansion of 3-dimensional flat factor-space, i.e. with

$$v^1 = v^2 = v^3 = H > 0, \tag{4}$$

and small enough variation of the effective gravitational constant G are of interest.

At present, the variation of G is allowed at the level of 10^{-13} per year and less. The most stringent limitation on G-dot was obtained in reference [2]

$$\dot{G}/G = (0.16 \pm 0.6) \cdot 10^{-13} \text{ year}^{-1} \tag{5}$$

allowed at 95% confidence (2-σ).

For our model $G = G^J_{eff}(t) \sim (\prod_{i=4}^n a_i(t))^{-1}$ is four-dimensional effective gravitational constant which appears in multidimensional analogue of the so-called Brans-Dicke-Jordan (or simply Jordan) frame [3].

Here we study the stability of exponential cosmological solutions (3) with non-static volume factor in the EGB model [4], e.g. those describing the isotropic accelerated expansion of 3d flat space obeying (4) with small enough variation of G. We restrict ourselves by a class of perturbations which depend on t and do not disturb the diagonal form of the metric.

2. The model

Here we consider the manifold

$$M = (t_-, t_+) \times M_1 \times \ldots \times M_n, \tag{6}$$

with the metric

$$g = -dt \otimes dt + \sum_{i=1}^{n} e^{2\beta^i(t)} dy^i \otimes dy^i, \tag{7}$$

where $M_1, ..., M_n$ are one-dimensional manifolds (either $\mathbf{R}$ or S^1) and $n > 3$. The functions $\beta^i(t)$, $i = 1, \ldots, n$, are smooth on (t_-, t_+). For physical applications we put $M_1 = M_2 = M_3 = \mathbf{R}$.

The equations of motion for the action (1) and the metric (7) may be rewritten as follows [5, 6, 7]

$$E = G_{ij} h^i h^j + 2\Lambda - \alpha G_{ijkl} h^i h^j h^k h^l = 0, \tag{8}$$

$$Y_i = \frac{dL_i}{dt} + (\sum_{j=1}^{n} h^j) L_i - \frac{2}{3}(G_{ij} h^i h^j - 4\Lambda) = 0. \tag{9}$$

where $h^i = \dot{\beta}^i$, $\alpha = \alpha_1/\alpha_2$, $G_{ij} = \delta_{ij} - 1$, $G_{ijkl} = G_{ij} G_{ik} G_{il} G_{jk} G_{jl} G_{kl}$,

$$L_0 = G_{ij} h^i h^j - 2\Lambda - \frac{1}{3}\alpha G_{ijkl} h^i h^j h^k h^l, \tag{10}$$

and

$$L_i = L_i(h) = 2 G_{ij} h^j - \frac{4}{3}\alpha G_{ijkl} h^j h^k h^l, \tag{11}$$

$i = 1, \ldots, n$.

Let us consider the following solutions to eqs. (8) and (9)

$$h^i(t) = v^i, \tag{12}$$

with constant v^i, which correspond to the solutions $\beta^i = v^i t + \beta^i_0$, where β^i_0 are constants, $i = 1, \ldots, n$. We obtain for the metric

$$g = -dt \otimes dt + \sum_{i=1}^{n} B_i e^{2v^i t} dy^i \otimes dy^i, \tag{13}$$

where $B_i > 0$ are arbitrary constants.

For the fixed point $v = (v^i)$ we have the set of polynomial equations

$$E = G_{ij} v^i v^j + 2\Lambda - \alpha G_{ijkl} v^i v^j v^k v^l = 0, \tag{14}$$

$$Y_i = (\sum_{j=1}^{n} v^j) L_i(v) - \frac{2}{3} G_{kj} v^k v^j + \frac{8}{3}\Lambda = 0, \tag{15}$$

where L_i is defined in (11), $i = 1, \ldots, n$.

3. Stability of fixed point solutions

Here we study the stability of static solutions $h^i(t) = v^i$ to eqs. (8) and (9) in linear approximation in pertubations. We put

$$h^i(t) = v^i + \delta h^i(t), \tag{16}$$

$i = 1, \ldots, n$. By substitution (16) into eqs. (8) and (9) we obtain in linear approximation the following relations for perturbations δh^i

$$C_i(v) \delta h^i = 0, \tag{17}$$

$$L_{ij}(v) \delta \dot{h}^j = B_{ij}(v) \delta h^j, \tag{18}$$

where

$$C_i(v) = 2v_i - 4\alpha G_{ijks} v^j v^k v^s, \tag{19}$$

$$L_{ij}(v) = 2G_{ij} - 4\alpha G_{ijks} v^k v^s, \tag{20}$$

$$B_{ij}(v) = -(\sum_{k=1}^{n} v^k) L_{ij}(v) - L_i(v) + \frac{4}{3} v_j. \tag{21}$$

Here $v_i = G_{ij} v^j$, $L_i(v) = 2v_i - \frac{4}{3}\alpha G_{ijks} v^j v^k v^s$ and $i, j, k, s = 1, \ldots, n$.

We put the following restriction on the matrix $L = (L_{ij}(v))$

$$\text{(R)} \quad \det(L_{ij}(v)) \neq 0, \tag{22}$$

i.e. the matrix L should be invertible.

Here we consider exponential solutions (13) with non-static volume factor, which is proportional to $\exp(\sum_{i=1}^{n} v^i t)$, i.e. we put

$$K(v) = \sum_{i=1}^{n} v^i \neq 0. \tag{23}$$

It was proved in reference [8] that the set of linear equations on perturbations (17), (18) has the following solution

$$\delta h^i = A^i \exp(-K(v)t), \tag{24}$$

$$\sum_{i=1}^{n} C_i(v) A^i = 0, \tag{25}$$

$i = 1, \ldots, n$. Due to (24) the following proposition is valid (see also an equivalent criteria in [9]).

Proposition [8]. *The fixed point solution $(h^i(t)) = (v^i)$ $(i = 1, \ldots, n;\ n > 3)$ to eqs. (8), (9) obeying restrictions (22), (23) is stable under perturbations (16) (as $t \to +\infty$) if $K(v) = \sum_{k=1}^{n} v^k > 0$ and it is unstable (as $t \to +\infty$) if $K(v) = \sum_{k=1}^{n} v^k < 0$.*

4. Solutions with small enough variation of G

The 4d effective gravitational constant in Jordan frame is proportional to inverse volume scale factor of the internal space, i.e. $G \sim \prod_{i=4}^{n} [a_i(t)]^{-1}$, where $a_i(t) = \exp(\beta^i(t))$.

For the solutions (13) we obtain the relation $G(t) = G(0) \exp(-K_{int} t)$ with $K_{int}(v) = \sum_{i=4}^{n} v^i$, which implies

$$\frac{\dot{G}}{G} = -K_{int}(v). \tag{26}$$

Now, let us consider a subclass of cosmological solutions (13) which obey restriction (22) and describe an exponential isotropic expansion of 3-dimensional flat factor-space with $v^1 = v^2 = v^3 = H > 0$ and variation of G obeying the bounds

$$|\dot{G}/G| = |K_{int}(v)| < 3H. \tag{27}$$

Then we get $K(v) = \sum_{i=1}^{n} v^i = 3H + K_{int}(v) > 0$. According to Proposition any solution from this subclass is stable if the restriction (22) is obeyed.

Certain examples of stable exponential solutions with small enough variation of G were considered in references [8, 10, 11, 12].

Here we present new examples of exponential solutions with zero variation of G for $n = m+2$, $m > 2$ and $\alpha < 0$. The solutions are given by relation (13) with $(v^i) = (H, \dots, H, h, h)$ (with m copies of H),

$$H = (2(m-1)|\alpha|)^{-1/2} > 0, \tag{28}$$

$$h = -\frac{1}{2}(m-3)H \leq 0, \tag{29}$$

and

$$\Lambda = \frac{1}{8}|\alpha|^{-1}(m-1)^{-1}(2m^2 - 5m + 9) > 0, \tag{30}$$

for $m \geq 3$.

These solutions are stable, since they obey restrictions (22). Indeed, according to reference [8] the restriction (22) is satisfied if $m > 1$,

$$S_{HH} = (m-2)(m-3)H^2 + 4(m-2)Hh + 2h^2 \neq -(2\alpha)^{-1} \tag{31}$$

and

$$S_{hh} = m(m-1)H^2 \neq -(2\alpha)^{-1}. \tag{32}$$

Both inequalities are valid due to (28), (29) and $m > 2$.

The special cases of these solutions were obtained earlier in references: [8] ($m = 3$) and [12] ($m = 4, 5$).

5. Conclusions

Here we have considered the $(n+1)$-dimensional Einstein-Gauss-Bonnet (EGB) model with the Λ-term. By using the ansatz with diagonal cosmological metrics, we have overviewed the stability analysis of the solutions with exponential dependence of scale factors $a_i \sim \exp(v^i t)$, $i = 1, \dots, n$, (t is a synchronous time variable) for $n > 3$ and $\sum_{k=1}^{n} v^k \neq 0$.

Here a set of equations for perturbations δh^i was considered (in linear approximation) and general solution to these equations under restrictions (22) and (23) obeyed was presented.

We have shown that exponential solutions with $v^1 = v^2 = v^3 = H > 0$ and small enough variation of the effective gravitational constant obeying (27) are stable if the restriction (22) is fulfilled. We have also presented new examples of stable exponential solutions with two different Hubble-like parameters: $H > 0$ (m-times) and $h \leq 0$ (2-times), and zero variation of G in dimensions $D = 1 + m + 2$ with $m > 2$.

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