

Anisotropic flow analyses with multiparticle azimuthal correlations

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Abstract. Multiparticle azimuthal correlations are nowadays utilized regularly by all major collaborations worldwide which are analyzing heavy-ion data. Most notably, correlation techniques are used to explore the collective properties of the new state of matter, the Quark-Gluon Plasma, by performing measurements of anisotropic flow phenomenon in heavy-ion collisions. In these proceedings we highlight the theory of multiparticle azimuthal correlations and summarize briefly the most important recent physical results obtained with them. Some unresolved problems and next future steps in their development are discussed as well.

1. Introduction

The primary objective of heavy-ion program at ultrarelativistic colliders is to explore the properties of a new state of matter, the Quark-Gluon Plasma (QGP). The QGP consists of asymptotically free quarks and gluons interacting dominantly via the strong nuclear force, one of the four fundamental forces in nature. The QGP does not exist at ordinary temperatures and energy densities, where the building blocks of matter are composite particles, baryons (made up of three quarks) and mesons (made up of one quark and one antiquark). However, it is believed that the QGP existed a few microseconds after the Big Bang. Therefore, by producing and studying the properties of QGP in heavy-ion collisions we are essentially recreating and studying the conditions which existed in the distant past of our universe and shedding light on its evolution.

1.1. Anisotropic flow

The key measurement in the exploration of QGP properties is the measurement of collective anisotropic flow phenomenon [1]. In non-central heavy-ion collisions the initial volume containing the deconfined nuclear matter is anisotropic in the coordinate space, due to the geometry of non-central collisions. Multiple interactions within this anisotropic volume cause the anisotropy to be transferred from the coordinate space into the momentum space. In essence, this transfer is the collective anisotropic flow phenomenon, and can for instance occur if the produced nuclear matter manages to thermalize. In order to reach thermalization a large number of mutual interactions among constituents is a necessary condition, which can be achieved if a large number of interacting particles are confined to a small volume. Clearly, such conditions can be easier established in heavy-ion collisions than in the collisions of lighter objects, like proton-proton collisions.



If anisotropic flow has developed, the resulting anisotropy in momentum space will cause an anisotropic distribution of particles recorded in the detector after each heavy-ion collision, and is therefore an observable quantity. Anisotropic flow is quantified via so-called flow harmonics v_n and symmetry planes Ψ_n , which are independent degrees of freedom in the Fourier series expansion [2] of resulting anisotropic distribution in the momentum space:

$$f(\varphi) = \frac{1}{2\pi} \left[1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)] \right]. \quad (1)$$

Using just the orthogonality properties of trigonometric functions, one can derive that

$$v_n = \langle \cos[n(\varphi - \Psi_n)] \rangle, \quad (2)$$

where angular brackets denote an average over all particles in an event. Due to only mathematical steps involved in the derivation of equation (2), we remark that v_n harmonics calculated solely from this equation have a priori no physical meaning. In addition, it is impossible to measure reliably in an experiment the orientation of symmetry planes Ψ_n event-by-event. Both of these issues can be overcome with the usage of correlation techniques, which we describe next.

1.2. Multiparticle azimuthal correlations

In order to make a statement on whether the harmonics v_n in equation (1) are dominated by contributions from collective anisotropic flow or by some other processes which are non-collective in nature, we can use correlation techniques involving two or more particles. When only collective anisotropic flow is present, all produced particles are independently emitted, and are correlated only to some common reference planes. This can be easily understood by considering an analogous example from gravity: Trajectories of two falling bodies in the gravitational field of Earth appear to be correlated, but the only correlation in the system is an individual correlation of each falling body to the common center of gravity. Therefore, whether two bodies are falling simultaneously, or one after another, their trajectories will be exactly the same. In the context of heavy-ion collisions, anisotropic pressure gradients correspond to gravitational field, and emitted particles to falling bodies. This physical observation translates into the following mathematical statement:

$$f(\varphi_1, \dots, \varphi_n) = f_{\varphi_1}(\varphi_1) \cdots f_{\varphi_n}(\varphi_n). \quad (3)$$

The left-hand side of equation (3) is a joint multivariate probability density function (p.d.f.) of n observables $\varphi_1, \dots, \varphi_n$. The right-hand side of equation (3) is the product of the normalized marginalized p.d.f., $f_{\varphi_i}(\varphi_i)$, where $1 \leq i \leq n$. The functional forms of all p.d.f.'s $f_{\varphi_i}(\varphi_i)$ are the same and are given by equation (1). Therefore, when all particles are emitted independently, as is the case for collective anisotropic flow, the joint p.d.f. for *any* number of particles will factorize as in equation (3). Based on this reasoning, one can build up, in principle, infinitely many independent azimuthal observables sensitive to various combinations of flow harmonic moments and corresponding symmetry planes by adding more and more particles to the correlators.

In general, the first moment of an arbitrary multiparticle correlator can be related analytically to the flow degrees of freedom, via the following result first derived in [3]:

$$\mu_{\langle m \rangle_{n_1, n_2, \dots, n_m}} \equiv \langle \cos(n_1 \varphi_1 + \cdots + n_m \varphi_m) \rangle = v_{n_1} \cdots v_{n_m} \cos(n_1 \Psi_{n_1} + \cdots + n_m \Psi_{n_m}). \quad (4)$$

The key point in the above derivation was the assumption that the anisotropic flow is the only source of correlations between produced particles. The next important point comes from

observation that experimentally from the measured azimuthal angles $\varphi_1, \varphi_2, \dots$ one can estimate reliably the correlator $\langle \cos(n_1\varphi_1 + \dots + n_m\varphi_m) \rangle$ only if all effects of self-correlations are *exactly* removed. This problem was only recently resolved for the most general case in [4, 5].

Due to the presence of non-flow correlations, which typically involve only few particles and which do not have geometrical origin, the factorization in equation (3) will break down in reality. As a consequence, the multiparticle correlators are not any longer reliable estimators of flow degrees of freedom via equation (4). The next improvement came with cumulants, which can be expressed in terms of multiparticle correlations, and which systematically suppress contributions from unwanted non-flow correlations [6, 7]. For instance, anisotropic flow harmonics v_n estimated with two- and four-particle cumulants, $c_n\{2\}$ and $c_n\{4\}$, are denoted as $v_n\{2\}$ and $v_n\{4\}$, respectively, and are defined as:

$$v_n\{2\} = \sqrt{c_n\{2\}}, \quad (5)$$

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}, \quad (6)$$

where above cumulants are expressed in terms of multiparticle correlations in the following way:

$$c_n\{2\} = \langle\langle 2 \rangle\rangle, \quad (7)$$

$$c_n\{4\} = \langle\langle 4 \rangle\rangle - 2 \langle\langle 2 \rangle\rangle^2. \quad (8)$$

In above equations, $\langle 2 \rangle$ and $\langle 4 \rangle$ are the only isotropic two- and four-particle correlators for which all harmonics coincide in absolute values; written explicitly: $\langle 2 \rangle \equiv \langle \cos[n(\varphi_1 - \varphi_2)] \rangle$ and $\langle 4 \rangle \equiv \langle \cos[n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)] \rangle$. The double angular brackets indicate that the averaging procedure has been performed in two distinct steps—first over all distinct pairs and quadruplets in an event, and then in the second step the single-event averages were weighted with ‘number of combinations’ weight [4]. Flow observables $v_n\{2\}$ and $v_n\{4\}$ were used to constrain the transport properties of QGP, most notably its shear viscosity to entropy density ratio (η/s). Their measurements have demonstrated that the QGP produced at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC) is one of the best examples of a perfect fluid, i.e. its η/s is very close to a universal lower bound of $1/4\pi$ which is obtained from theoretical arguments based on the AdS/CFT conjecture.

What remains completely unknown is how the η/s of QGP depends on temperature, and this study has been just initiated by the theorists [8]. This question triggered among the experimentalists the development of new flow observables, since the individual flow harmonics v_n turned out to be insensitive to the details of temperature dependence. In the next section we describe in detail these new flow observables.

2. New flow observables: Symmetric Cumulants

In addition to the created volume’s spatial anisotropy, originating solely from the collision geometry and which will predominantly generate elliptic flow, v_2 , there are also the anisotropies stemming from the fluctuations in the initial positions of participating nucleons within the created volume. Such fluctuations can in principle generate any type of anisotropy in the coordinate space, which will be also via mutual interactions transferred to momentum space, where they can give rise to any harmonic v_n . Therefore, one is also interested in their mutual relations—for instance, fluctuations of which flow harmonics are correlated or anti-correlated to each other?

Such information can be extracted from the new multiparticle observables, the so-called Symmetric Cumulants (SC), recently published in Section IV.C of [5]. These observables are

defined as follows:

$$\begin{aligned}
 \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle_c &= \\
 &= \langle\langle \cos(m\varphi_1 + n\varphi_2 - m\varphi_3 - n\varphi_4) \rangle\rangle \\
 &- \langle\langle \cos[m(\varphi_1 - \varphi_2)] \rangle\rangle \langle\langle \cos[n(\varphi_1 - \varphi_2)] \rangle\rangle \\
 &= \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle, \tag{9}
 \end{aligned}$$

with the condition $m \neq n$ for two positive integers m and n . The four-particle cumulant in equation (9) is less sensitive to non-flow correlations than any two- or four-particle correlator on the right-hand side taken individually [7]. It is zero in the absence of flow fluctuations, or if the magnitudes of harmonics v_m and v_n are uncorrelated [5]. It is also unaffected by relationship between symmetry plane angles Ψ_m and Ψ_n . Finally, it can be shown that the four-particle cumulant in equation (9) is proportional to the linear correlation coefficient $c(a, b)$ introduced in [9], with $a = v_m^2$ and $b = v_n^2$. The new SC observables, as defined above, are the cleanest experimental way to quantify the relationship between event-by-event fluctuations of magnitudes of two different harmonics v_m and v_n [5].

The first results for SC observables were recently released by ALICE Collaboration in [10]. It was found that fluctuations of v_2 and v_3 are anti-correlated in all centralities, however the details of the centrality dependence differ in the fluctuation-dominated (most central) and the geometry-dominated (mid-central) regimes. On the other hand, fluctuations of v_2 and v_4 are correlated in all available centralities. These measurements were used to discriminate between the inputs of state-of-the-art hydro model stemming from different parameterizations of η/s temperature dependence, for all of which the centrality dependence of elliptic, triangular and quadrangular flow are nearly insensitive at the LHC [8]. In particular, the centrality dependence of SC(4, 2) cannot be captured with the constant η/s . These results were used as well to discriminate between two different parameterizations of initial conditions and it was demonstrated that in the fluctuation-dominated regime MC-Glauber initial conditions with binary collisions weights are favored over wounded nucleon weights [10].

3. Unresolved problems

The recent flow measurements with multiparticle cumulants in the collisions of light and heavy nuclei, like p-Pb at LHC or p-Au, d-Au and ^3He -Au at RHIC, have caused a lot of controversy. Since to leading order these measurements resemble the features observed in the heavy-ion collisions, which are attributed to collective anisotropic flow, it is very tempting to interpret them the same way for smaller systems. This interpretation is challenged by the outcome of Monte Carlo studies for e^+e^- systems [11] in which neither the QGP existence nor collective effects are expected, where to leading order multiparticle cumulants exhibit yet again the similar universal trends, both for v_2 and v_3 harmonics.

The underlying difficulty stems from the fact that when anisotropic flow harmonic v_n is estimated with k -particle correlator, the statistical spread of that estimate scales to leading order as $\sigma_{v_n} \sim \frac{1}{\sqrt{N}} \frac{1}{M^{k/2}} \frac{1}{v_n^{k-1}}$, where M is the number of particles in an event (multiplicity) and N is total number of events. This generic scaling ensures that multiparticle correlations are precision method only in heavy-ion collisions, characterized both with large values of multiplicity and flow. On the other hand, for collisions of small systems the results obtained with correlation techniques are characterized with large spread, and resolution is typically too small to get reliable results (see Monte Carlo study in figure 1). In addition, the contribution from non-flow typically scales as an inverse of multiplicity (raised to power which depends on the order of correlator), implying that contribution from non-flow will not be diluted in correlators measured in small systems. Therefore, in their present form, correlation techniques are not suitable for anisotropic flow analyses in small colliding systems.

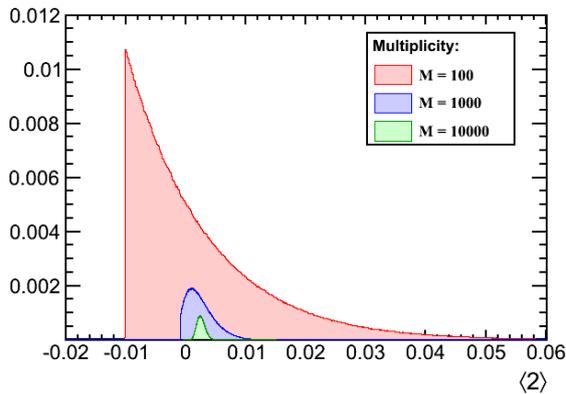


Figure 1. Three different Monte Carlo simulations were performed with the same input value of $v_2 = 0.05$, so that the expected average value of two-particle correlation is 2.5×10^{-3} in each case. On the other hand, multiplicity was varied, in order to demonstrate that only for its large values, of the order of 10000 (green color), resolution is good enough to determine the expected average value of 2.5×10^{-3} .

Another important open question related to correlation techniques stems from the fact that at the moment it is only known how their averages, i.e. the first moments, relate to the flow degrees of freedom. Only the analytic derivation of the p.d.f. of multiparticle azimuthal correlations for the most general case of anisotropic flow would enable for the first time a complete characterization of anisotropic flow with correlation techniques. Such derivation at present is still out of hand, even for the simplest case of two-particle correlations.

Anisotropies in the distribution of final-state particles can originate from inefficiencies in the detector's acceptance, which then typically lead to the most dominant source of systematic uncertainties in the flow estimates with correlation techniques. If there are no real holes in the acceptance, such effects can be corrected for with so-called particle weights [5]. However, for the case of real holes in the acceptance, the correction for such systematic biases is not available at present for higher order correlators.

4. Next steps

One of the most promising new tools for the study of the dynamics of heavy-ion collisions is the Event Shape Engineering [12]. Since this technique is based solely on the values of Q -vector, it is rather natural to generalize this idea and investigate whether it is possible to select events based on the values of various multiparticle azimuthal correlations, and sort them out in terms of underlying anisotropic flow quantified with harmonics v_n . Symmetric Cumulants were developed to quantify the relationship between event-by-event fluctuations of two different harmonics v_m and v_n , but it would be also interesting to investigate whether any genuine three-harmonic correlation can exist. This question can be also tackled straightforwardly with multiparticle correlations. In addition, symmetry plane correlations can be analyzed in the similar manner, and not exclusively with the Event Plane method or with the Scalar Product method, as it was done by the ATLAS Collaboration [13]. Finally, the femtoscopic correlations are mostly measured nowadays with two-particle correlations, and we foresee in the future that an independent information also in this field will be extracted from multiparticle correlations, building on pioneering work published in [14].

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