Self-consistent description of charged particle beam propagation in terrestrial magnetic field

H Y Barminova

Plasma Physics Department, National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway 31, Moscow, 115409, Russia

E-mail: eebarminova@mephi.ru

Abstract. Self-consistent models describing the charged particle beam behavior in external magnetic field are presented. The model application to study the beam characteristics transformation in terrestrial magnetic field is discussed.

1. Introduction

Charged particle beams are the important source of the information about various phenomena in space as well as in terrestrial area. The measurements of the beam characteristics such as the beam densities, the particle energies, the beam energy spreads with the coordinates of the measurement points and the date and the time of the measurements taken into account give the knowledge about the cosmic processes far from the Earth as well as in the solar system and in terrestrial magnetosphere. The various techniques and the apparatus used for the measurements should have the appropriate interpretation of the data obtained. One should take into account some peculiarities of the charged particle beam behavior in magnetic fields which could affect the results of the measurements. The incorrect determination of the phase beam characteristics, especially the energy spread and angular beam size, results in the mistakes of the beam initial data determination, such as the place of the particle birth, the reasons and the scale of their acceleration, the reasons of the current sheath interruptions (the beam bunching) and other. The analytic consideration of the beam phase characteristics transformation may be carried out with the help of self-consistent models. Such models which represent themselves as the accurate solutions of the Vlasov equation are discussed in this paper with the aim of their application for the tasks of cosmic ray and solar wind study.

2. 2D model and the beam phase characteristics transformation in magnetic fields

The model for continuous beam may be applied while measuring the particle fluid during the time significantly less than the current pulse duration. The geometry of the beam may be various, and common case of such a beam considers the geometry with elliptical cross-section. The parameters of elliptical cross-section may be chosen quite arbitrary so that the case of a sheet beam may be studied too. To describe self-consistently such a beam the model is proposed [1]. The model is based on the approximation of the linearity of the fields acting on the particles, and it is quite valid for the study of the effect of phase characteristics transformation.

The density of the beam must be calculated as

$$n = \int f(I)\,d\dot{x}\,d\dot{y} = \frac{\pi\kappa}{uv}\varepsilon_1\varepsilon_2\sigma(1 - R_x^2 x^2 - R_y^2 y^2)\,, \qquad (1)$$

where the kinetic distribution function *f(I):*

$$f = \kappa\delta(I - 1), \tag{2}$$

and motion equation invariant *I*:

$$I = \frac{(u^{\cdot}x - ux^{\cdot})^2}{\varepsilon_x^2} + \frac{(v^{\cdot}y - vy^{\cdot})^2}{\varepsilon_y^2} + \frac{x^2}{u^2} + \frac{y^2}{v^2} + C_0(x^{\cdot}y - xy^{\cdot}). \tag{3}$$

The values R_x, R_y are the semi-axes of the elliptical cross-section of the beam, dependent from the values *u(t)* and *v(t)* in complicated manner [1]. The value C_0 characterizes the mean angular momentum of the particles, moving in magnetic field [1]. The values $\varepsilon_x, \varepsilon_y$ characterizes the partial phase volumes or partial emittances of the beam, corresponding to the coordinate direction *x* and *y* of the transverse plane (the beam profile), κ is the constant of normalization, independent on the time, δ is the delta-function.

Using the model one can estimate the transformation of the phase volumes of the beam in the plane perpendicular the magnetic field lines due to the relation:

$$\varepsilon_{4D} = \int\limits_{I\le 1} d\dot{x}d\dot{y}dxdy = \frac{(\Gamma(\frac{1}{2}))^n}{\Gamma(\frac{n}{2}+1)}, \tag{4}$$

where $\Gamma(x)$ - the gamma-function. In the considered case *n=4* for 4-dimensional phase space or 4-dimensional hyper-ellipsoid, and *n=2* for 2-dimensional phase space or 2-dimensional phase ellipse. Using the well-known properties of the gamma-function, it is easy to obtain the relation between the emittances $\varepsilon_{4D}, \varepsilon_{2D}^{(1,2)}$, which represent the phase volumes in 4-dimensional and 2-dimensional phase spaces respectively:

$$\varepsilon_{4D} = \frac{1}{2}\varepsilon_{2D}^{(1)}\varepsilon_{2D}^{(2)}. \tag{5}$$

This relation determine the property of the magnetic field to transform the energy spread of the particle bunch to the transverse size of the bunch, and vice versa, the longitudinal size of the bunch to the angular divergence of the beam. In the case considered above the maximum effect of the phase characteristics transformation may be estimated as a value equal to the relation $\varepsilon_1 / \varepsilon_2$ during the half of the cyclotron period, i.e. the maximum difference between the initial emittances results in the maximum effect of their transformation.

The model and all the calculations considered here propose that the beam has the small own space charge, but the phase volume transformation doesn't depend on the space charge significantly and only the scale of the effect changes slightly [2].

3. 3D model and the beam characteristics change in magnetic fields

During the interaction of the beam with space plasma the beam current may be interrupted and the linear density of the beam may become modulated. I this case to estimate the phase volume transformation 3D model is more appropriate. It may be built with the same assumptions mentioned in Section 2, but differed in the invariant form because in the case of 3D-bunch one should include the third coordinate:

$$f = \kappa\delta(I_{xy} + I_z^{(1)2} - 1)\delta(I_z^{(2)2}), \tag{6}$$

where I_{xy} - the bilinear invariant, corresponding to the plane, perpendicular to the direction of the bunch motion, $I_z^{(1,2)}$ - the independent linear invariants for the particle motion equation in the direction, corresponding to the third coordinate in the coordinate system, connected with the main axes of the bunch ellipsoid. The invariants I_{xy} and $I_z^{(1,2)}$ may be built in both cases of uniform and nonuniform magnetic field, but in the last case the requirement of the linearity of the field is remained. It means that for the estimation of the effect of the phase characteristics transformation one could consider the magnetic field with dipole and quadrupole components.

In figure 1 the effect of the emittance transformation is shown. The dependence of the coefficient of the emittance transfer is presented against the gradient of the cyclotron frequency which corresponds to the magnetic field with quadrupole type components.

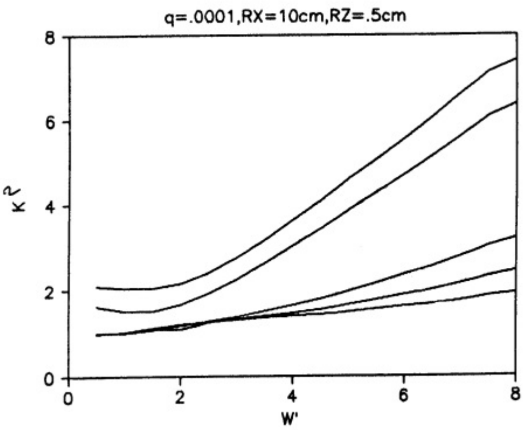

Figure 1. Dependence of the coefficient of the emittance transfer on the gradient of the cyclotron frequency.

It may be seen that the effect of the transformation of the initial characteristics of the beam in terrestrial magnetic field represents itself the transfer of the initial energy spread to the transverse beam size and the initial bunch length to the increase of the transverse particle velocities, that may result in incorrect determination of the beam preliminary "history". Let us estimate roughly this effect for the experimental data from the sattelites Interball-1, Interball-2 and Geotail. According the data of the measurements in the experiments [3,4], the energy spread of non-relativistic light ion beams observed near the boundary of terrestrial magnetosphere was: $\frac{\Delta v_l}{v_l} \sim 0.1 \div 0.8$, where v_l is the value of parallel particle velocity. The corresponding values of the beam energies lye in the range of 20-100 keV. Assuming the simple ellipsoidal geometry of the particle bunches and the maximum transverse particle temperature 0.1 eV one can obtain the maximum coefficient of the emittance transfer not less than 200η, where η is the relation between the length of the particle bunch and the wideness of the ion fluid.

Note here that the influence of the magnetic fields on the phase dynamics of the charged particle beams was first investigated in [5,6] by means of the simple mathematical model of the beam rotation in the dipole bending magnet of the low energy accelerator.

4. Conclusions

In the case of charged particle beam propagation in terrestrial magnetic field the beam behavior may be described in the frame of Vlasov theory. The Vlasov equation requirement of the particle collision absence is fulfilled for the cosmic rays due to the fairly high beam energy and rather low beam intensity. In such approach 2D and 3D self-consistent time-dependent models may be applied, based on the approximation that the particle ensemble is described by the distribution functions dependent on the motion equation invariants. Analytic and numerical solutions of the model equations allow to predict the beam characteristics transformation during the beam propagation in terrestrial magnetic field, in particularly, the beam energy spread transfer into the beam transverse size or the bunch longitudinal size transfer into the angular divergence of the beam, which may be important for the estimate and the interpretation of the measurement results.

Acknowledgements

This work was supported by the Competitiveness Program of NRNU MEPhI.

References

[1] Barminova H Y and Chikhachev A S 1990 *Radiophys Quant Electron* **33**(3) 277
[2] Barminova H Y and Chikhachev A S 1991 *Radiophys Quant Electron* **34**(9) 818
[3] Lutsenko V N, Gavrilova E A and Grechko T V 2008 *Ann Geophys* **26** 2097
[4] Sharma S *et al.* 2008 *Ann Geophys* **26** 955
[5] Kushin V V, Murin B P and Gromov E V 1973 *Proc. RTI AN USSR* **16** 348
[6] Kushin V V 1984 *Sov Phys - Techn Phys* **54**(1) 103