

Computer modelling of the Hamamatsu R11410-20 PMT

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Abstract. A computer model of operation of Hamamatsu R11410-20 photomultiplier based on SPICE software package has been developed. The PMT amplification process is simulated with the use of voltage and current controlled sources of current. Boundaries of linear zone were obtained for high anode current (with respect to the base current) operation regime. The results of simulation are in reasonable agreement with the experimentally measured PMT characteristics. The model can be used for simulation of any type of PMT.

1. Introduction

There are various specific photomultiplier tube (PMT) applications when the anode current is relatively high in compare to the base current. The one of those is the use of PMTs for the scintillation and electroluminescent light detection in cryogenic noble gas filled two-phase emission detectors widely used nowadays to search for WIMP particles of dark matter in the Universe [1]. The RED-100 is a two-phase emission detector built for the experiment on coherent elastic neutrino-nucleus scattering (CEvNS) observation [2], another challenging experiment that is carried in a vicinity of a nuclear power plant reactor. There are 38 Hamamatsu R11410-20 3"-diameter PMTs in the RED-100 detector divided into two arrays, the top and bottom ones, with 19 PMTs in each array. The PTS of the RED-100 detector operates in the liquid xenon environment. The PMT bases are directly connected to the photomultiplier pins. The specific of the RED-100 detector operation requires the very wide dynamic range of PMT: from the single photoelectron signals of small scintillation to the very intense electroluminescent signals having several millions of photoelectrons; see for details [3] paper. Moreover, the rate of appearance of scintillation signal is substantially high, and the significant average anode current may result in nonlinearity of the PMT gain. This problem can be solved by the increase of the PMT base current or by designing a special divider circuit having individual power suppliers for the last dynodes. We selected the first option because of the restriction of cabling inside the detector cryostat. On the other hand, the value of the PMT base current must be restricted in order to avoid liquid xenon boiling on the divider circuit resistors.

The main task when designing the PMT divider circuit was to understand the limitation on the average anode current of the PMT when the base current is set to a certain value. The common practice is to do this experimentally. However, computer modeling with the use of standard electrical circuit simulation packages may economize efforts significantly. Surprisingly, we have found only one paper [5] dedicated to computer modeling of a PMT with the use of SPICE package.



2. PMT operation

For principle and theory of PMT operation we refer to the book [4]. The gain g_i at each PMT dynode is defined by the formula:

$$g_i = k_i V_i^\alpha, \quad (1)$$

where k_i and α are coefficients depending on the characteristics of the dynodes, V_i is the voltage applied between i and $i+1$ dynodes. The overall PMT gain G is defined as:

$$G = \prod_{i=1}^N g_i = \prod_{i=1}^N k_i (\varepsilon_i V)^\alpha, \quad (2)$$

where N is the total number of amplification stages and ε_i is a fraction of the overall voltage V defined as:

$$\varepsilon_i = \frac{R_i}{R_{tot}}, \quad (3)$$

where R_i is the resistance between i -th and $i+1$ dynode, R_{tot} is the total resistance of the divider circuit. Thus, the gain versus the photomultiplier voltage is a power-law dependence.

To simulate the PMT operation by a computer model the values ε_i , k_i and α to be known. The ε_i values are defined from the known R_i distribution (see figure 1; $R_{tot} = \sum_{i=1}^N R_i = 20.4 M\Omega$):

$$\varepsilon_1 = \frac{4.4}{20.4}, \quad \varepsilon_2 = \frac{1.55}{20.4}, \quad \varepsilon_{3,12} = \frac{2.2}{20.4}, \quad \varepsilon_{4-11} = \frac{1.1}{20.4}.$$

To simplify the model, we assume that k_i values are equal for each dynode because k_i depends on the dynode material and its configuration that is difficult to account for in the model. Therefore, we have:

$$G = k^{12\alpha} \left(\frac{4.4}{20.4}\right)^\alpha \left(\frac{1.1}{20.4}\right)^{8\alpha} \left(\frac{2.2}{20.4}\right)^{2\alpha} \left(\frac{1.55}{20.4}\right)^\alpha V^{12\alpha} \quad (4)$$

For the known $G = f(V)$ characteristic, it is possible to construct system of equations from which k and α can be obtained. For the typical Hamamatsu R11410-20 characteristic [6]: $k = 0.167$, $\alpha = 0.674$. For the measured characteristic of the PMT with serial #KB0278: $k = 0.0587$, $\alpha = 0.88$.

3. PMT model

Here we describe the PMT model built with the use of the LTSPICE package including two unsuccessful attempts in order to show the mistakes that can be done by other users of this package.

We started to build the PMT model in analogy with the physical processes that take place in a real photomultiplier, i.e. the interdynode gap can be considered as an ideal current source since it has the

infinite internal resistance. For simulation of the current, which flows between the i -th and $i+1$ -th dynodes, an “arbitrary behavioral dependent current source” was used. The current of this source is set by the formula:

$$i_n = i_{n-1} * k * V_i^\alpha, \quad (5)$$

where i_{n-1} is the current flowing between $i-1$ -th i -th and dynodes, V_i - voltage drop between $i-1$ -th i -th and dynodes. The fragment of the model for two neighboring interdynode gaps is shown in figure 2. The overall voltage V is applied to the resistor chain from the anode side is produces by a “voltage source”. The cathode of the PMT is grounded.

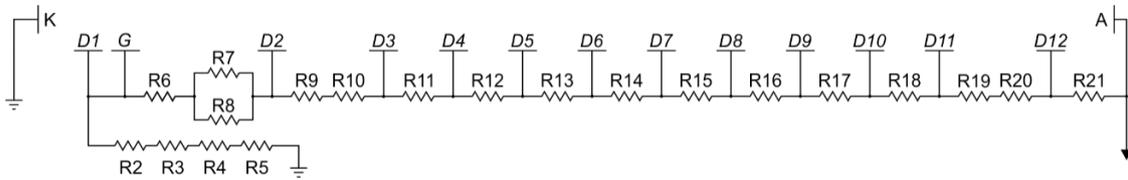


Figure 1. Simplified divider scheme

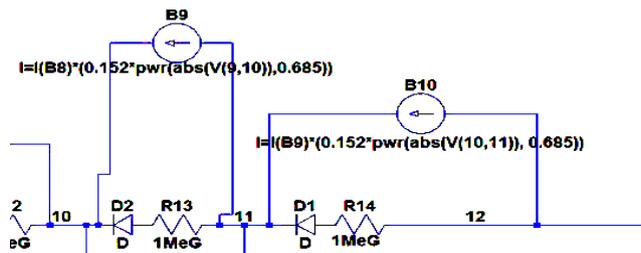


Figure 2. Fragment of model for two dynodes

In a real PMT, the dynode current can flow only in one direction, i.e. from the anode to the cathode. Another words, the current generated by the source B10, for example, cannot split at the point 11, thus the part of it flows from the point 11 to the point 12 and then back to the source through the small loop. But in the LTSPICE model, it does. To avoid this, we tried to introduce the diodes in the resistor chain. However, it has not solved the problem since in the LTSPICE package, a diode is described as an element having voltage dependent resistor, quite small in compare to the R_i value.

The next model version was design in analogy to the model described in [5] in order to avoid small loops. In this model, each dynode (and the anode) is represented by a current source connected to the ground. The current produced by this source is set to be equal to the difference between the incoming and outcoming current as in a real PMT. In this case, the dynode current is given by the formula:

$$i_n = \frac{i_{n-1} * (k * V_i^\alpha - 1) * (k * V_{i-1}^\alpha)}{k * V_{i-1}^\alpha - 1}, \quad (6)$$

the anode current:

$$i_{anode} = \frac{i_{n-1} * (k * V_i^\alpha) * (k * V_{i-1}^\alpha)}{k * V_{i-1}^\alpha - 1}. \quad (7)$$

The overall voltage V is applied the same way by “voltage source”. However, this model suffers with the problem of “wrong current” as well. But now, the current from each source is splitted at a dynide point (a point between the resistor); and part of it goes to the ground in the “wrong direction”,

through the zero internal resistor of the voltage source. This results in incorrect potential distribution between dynodes. Note that this is also valid to the model described in [5].

To avoid this problem, the internal resistance of the overall supplier must be infinite and simultaneously the anode must be kept at the potential equal to V . To arrange this, we replaced the voltage source by a “voltage controlled current source” controlled by potential difference between the last dynode and the anode:

$$I_N = \frac{V - V_{last}}{R_N}, \quad (8)$$

where V_{last} is the potential of the last dynode, R_N is the resistor between the last dynode and the anode. With this source, the potential of the anode is maintained at a constant level V by variation of the current I depending of the potential of the last dynode. In this model, the currents from all sources at each dynode goes to the cathode (ground), i.e. to the “right” direction,

Simplified diagram of the part of the model is shown in figure 3. The fragment of the model illustrating setting of the current sources according to the formulas 7 and 8 is shown in figure 4.

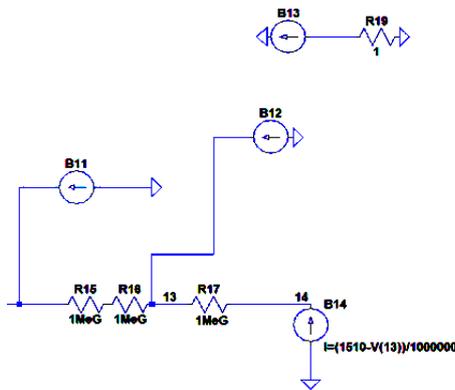


Figure 3. Fragment of the model final stages.

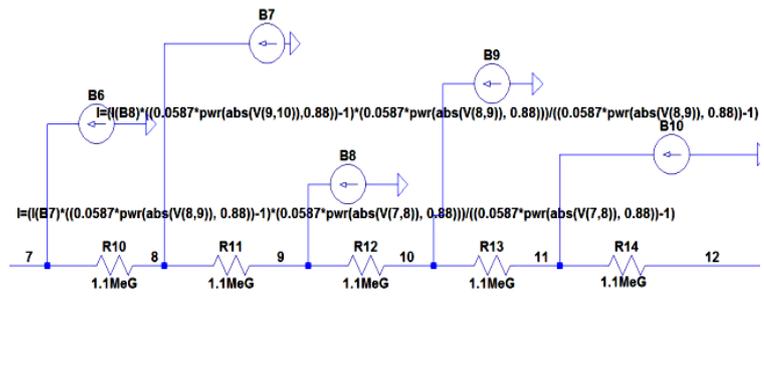


Figure 4. Fragment of the model with formulas.

4. Results

To verify the model, PMT gain dependences versus applied voltage was obtained at the small anode current ($\ll 1\%$ of the base current) for the measured and typical characteristic (figure 5, a and b correspondingly). Excellent coincidence of the curves both in figure 5 and in figure 6 proves the correctness of the model.

Finally, the model was used for understanding of the limitation on the average PMT anode current caused by the gain nonlinearity at different values of the base current. The overall voltage was set to $V = 1510$ V, and $R_{tot} = 27.75, 20.35, 18.5$ and 9.25 M Ω (corresponds to the base current 54.41, 74.2, 81.62 and 163.24 μ A). The results are shown in figure 6 (curves 2 – 5). The curve 1 was obtained experimentally for the PMT with serial # KB0278 for $V = 1510$ V and $R_{tot} = 20.4$ M Ω . The curve 2 was obtained by model for the same set of V and R_{tot} . One can see the qualitative agreement between the curves 1 and 2. The discrepancy between them may be explained by the difference in the coefficients of k_i for various dynodes in the real PMT (as mentioned above, in the model, they are all assumed to be equal k). The curves 1 and 3 are shown in larger scale in the insert of figure 6. One can conclude that the model predicts quite correctly the anode current when the gain nonlinearity starts to exceed the 1% value. It is approximately 1.3 μ A, while the experimental value corresponding to the 1% of nonlinearity is ~ 0.6 μ A (1.7% and 0.8% of the base current respectively). One can see also the

qualitatively right behavior the dependences 2 – 5: at smaller R_{tot} values, i.e. at higher base currents, the curves shift to the right.

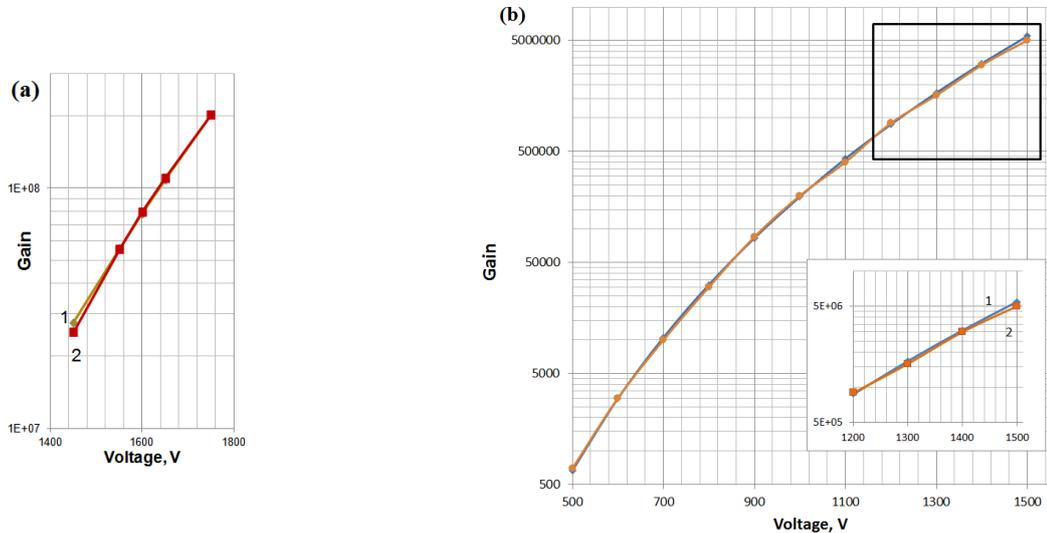


Figure 5. Gain dependence versus supply voltage; (a) – obtained by model (1) and experimental (2) characteristics for PMT #KB0278, (b) – obtained by model (1) and typical gain (2) characteristics for Hamamatsu R11410-20.

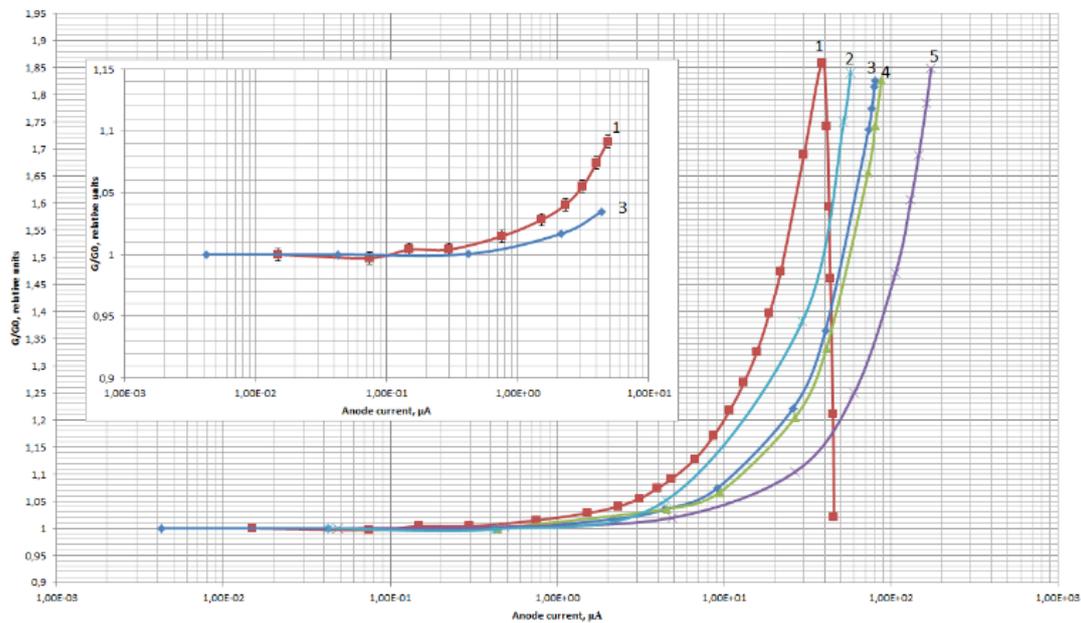


Figure 6. Gain dependence on anode current for PMT #KB0278: 1 – experimental, 2, 3, 4, 5 – simulated for $R_{tot} = 27.75, 20.35, 18.5$ and 9.25 M Ω ; on the insert, curves 1 and 3 in larger scale.

5. Conclusion

Computer model of the Hamamatsu R11410-20 PMT has been built on the basis of the LTSPICE electrical circuit simulation package. The result of simulation of the gain versus anode current dependence is in reasonable agreement with the experimental data. The model is also applicable to other types of PMTs with corresponding tuning of the model parameters.

Acknowledgments

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