

# Adomian decomposition method for solving the population dynamics model of two species

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**Abstract.** Adomian decomposition method has been a powerful method to solve differential equations. In this paper, we propose the method to solve the population dynamics model of two species for mutualism, parasitism, and competition. These three scenarios are considered for the completion of our research. Adomian decomposition method uses initial values of the unknowns and provides series of approximate solutions to the problem. We obtain that the Adomian decomposition method provides fast computation for the solution.

## 1. Introduction

Population dynamics model has been developed in the field of mathematical biology. Populations of species increase or decrease over time depending on a number of factors. A population certainly interacts with other populations, causing a system dynamics [1].

Two populations interact with the properties of mutualism, parasitism, and competition. These interaction can be formed from a general model (system) of differential equations. Interaction type in both populations can vary depending on the value of each given parameter in the general model. Finding the solution to the model using the aid of computer nowadays is often desired [2-5]. This is because fast and accurate results are important to solve real problems.

In this paper, we solve the population dynamics model of two species using the Adomian decomposition method. The method is chosen because it is meshless, is an analytical approach, possess a fast convergence, and can be implemented on a computer easily [6-11]. It is often related to the variational iteration method [12-16].

The paper is organised as follows. Section 2 contains the problem formulation that we want to solve. Section 3 present the Adomian decomposition method for the population dynamics model. Computational results are provided in Section 4. We conclude the paper in Section 5.

## 2. Problem formulation

We consider the nonlinear system of the form [12]

$$\begin{cases} \frac{dx}{dt} = x(a_1 + b_1x + c_1y), \\ \frac{dy}{dt} = y(a_2 + b_2y + c_2x). \end{cases} \quad (1)$$



Here  $x = x(t)$  and  $y = y(t)$  are populations of the first and second species at time  $t$ , respectively. In addition,  $a_1, b_1, c_1, a_2, b_2, c_2$  are constants described as follows:

- $x$  is the population of the first species,
- $a_1$  denotes the growth rate of the population of the first species,
- $b_1$  denotes the carrying capacity of the population of the first species,
- $c_1$  denotes the interacting constant of the population of the first species with the second one,
- $y$  is the population of the second species,
- $a_2$  denotes the growth rate of the population of the second species,
- $b_2$  denotes the carrying capacity of the population of the second species,
- $c_2$  denotes the interacting constant of the population of the second species with the first one.

This model (1) governs mutualism, parasitism, and competition interactions.

### 3. Adomian decomposition method

We derive the Adomian decomposition method to solve the model following Batiha *et al.* [12].

Model (1) can be rewritten as

$$\begin{cases} \frac{dx}{dt} = a_1x + b_1x^2 + c_1xy, \\ \frac{dy}{dt} = a_2y + b_2y^2 + c_2xy. \end{cases} \quad (2)$$

Introducing the derivative operator  $L = \frac{d}{dt}$ , we obtain

$$\begin{cases} Lx = a_1x + b_1x^2 + c_1xy, \\ Ly = a_2y + b_2y^2 + c_2xy. \end{cases} \quad (3)$$

Applying  $L^{-1} = \int_0^t (\cdot) dt$  to both sides of the nonlinear system (3) gives

$$\begin{cases} L^{-1}Lx = L^{-1}a_1x + L^{-1}b_1x^2 + L^{-1}c_1xy, \\ L^{-1}Ly = L^{-1}a_2y + L^{-1}b_2y^2 + L^{-1}c_2xy. \end{cases} \quad (4)$$

The Adomian decomposition method admits the decomposition of  $x$  and  $y$  into an infinite series components

$$x(t) = \sum_{n=0}^{\infty} x_n \quad y(t) = \sum_{n=0}^{\infty} y_n \quad (5)$$

and the nonlinear terms  $x^2, y^2$  and  $xy$  are assumed to be in the following forms:

$$x^2 = \sum_{n=0}^{\infty} A_n, \quad y^2 = \sum_{n=0}^{\infty} B_n, \quad xy = \sum_{n=0}^{\infty} D_n. \quad (6)$$

We define  $A_n, B_n$  and  $D_n$  as

$$A_n = \sum_{k=0}^n x_k x_{n-k}, \quad B_n = \sum_{k=0}^n y_k y_{n-k}, \quad D_n = \sum_{k=0}^n x_k y_{n-k}. \quad (7)$$

We have Adomian polynomials for  $A_n, B_n$  and  $D_n$ :

$$\begin{aligned} A_0 &= x_0 x_0 \\ A_1 &= x_0 x_1 + x_1 x_0 \\ A_2 &= x_0 x_2 + x_1 x_1 + x_2 x_0 \\ A_3 &= x_0 x_3 + x_1 x_2 + x_2 x_1 + x_3 x_0 \\ &\vdots \end{aligned} \quad (8)$$

$$\begin{aligned}
B_0 &= y_0 y_0 \\
B_1 &= y_0 y_1 + y_1 y_0 \\
B_2 &= y_0 y_2 + y_1 y_1 + y_2 y_0 \\
B_3 &= y_0 y_3 + y_1 y_2 + y_2 y_1 + y_3 y_0 \\
&\vdots
\end{aligned} \tag{9}$$

$$\begin{aligned}
D_0 &= x_0 y_0 \\
D_1 &= x_0 y_1 + x_1 y_0 \\
D_2 &= x_0 y_2 + x_1 y_1 + x_2 y_0 \\
D_3 &= x_0 y_3 + x_1 y_2 + x_2 y_1 + x_3 y_0 \\
&\vdots
\end{aligned} \tag{10}$$

The system of nonlinear differential equations from (4) can be expressed as:

$$\begin{cases}
x(t) - x(0) = L^{-1} a_1 \sum_{n=0}^{\infty} x_n + L^{-1} b_1 \sum_{n=0}^{\infty} A_n + L^{-1} c_1 \sum_{n=0}^{\infty} D_n \\
y(t) - y(0) = L^{-1} a_2 \sum_{n=0}^{\infty} y_n + L^{-1} b_2 \sum_{n=0}^{\infty} B_n + L^{-1} c_2 \sum_{n=0}^{\infty} D_n
\end{cases} \tag{11}$$

$$\begin{cases}
\sum_{n=0}^{\infty} x_n = x(0) + L^{-1} a_1 \sum_{n=0}^{\infty} x_n + L^{-1} b_1 \sum_{n=0}^{\infty} A_n + L^{-1} c_1 \sum_{n=0}^{\infty} D_n \\
\sum_{n=0}^{\infty} y_n = y(0) + L^{-1} a_2 \sum_{n=0}^{\infty} y_n + L^{-1} b_2 \sum_{n=0}^{\infty} B_n + L^{-1} c_2 \sum_{n=0}^{\infty} D_n
\end{cases} \tag{12}$$

With initial values  $x(0) = x_0$ ,  $y(0) = y_0$ , we can find the solution of the system. The iterations are determined by the following recursive formulas:

$$x_0 = x(0) = x_0, \tag{13}$$

$$x_{n+1} = L^{-1} a_1 x_n + L^{-1} b_1 A_n + L^{-1} c_1 D_n,$$

$$y_0 = y(0) = y_0,$$

$$y_{n+1} = L^{-1} a_2 y_n + L^{-1} b_2 B_n + L^{-1} c_2 D_n. \tag{14}$$

In this paper, we use 7-term approximations to find the solution to the system. The solution is defined by  $X$  and  $Y$  as

$$X = x_0 + x_1 + x_2 + x_3 + x_4 + x_4 + x_6 + x_7, \tag{15}$$

$$Y = y_0 + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7. \tag{16}$$

#### 4. Numerical results

For computational experiments in this section, we assume to have initial values  $x(0) = x_0 = 4$  and  $y(0) = y_0 = 10$ . We can determine the type of interaction between both populations from the signs of parameter values. We compute components of 7-term approximations using the computer algebra package Maple. The interaction between populations can be mutualism, parasitism, and competition.

##### 4.1. Mutualism interaction

Given that  $a_1 = 0.1$ ;  $a_2 = 0.08$ ;  $b_1 = -0.0014$ ;  $b_2 = -0.001$ ;  $c_1 = 0.0012$ ;  $c_2 = 0.0009$ ; and  $x_0 = 4$  and  $y_0 = 10$ .

We obtain:

$$\begin{aligned}
x_1 &= 0.4256000000t, \\
y_1 &= 0.7360000000t, \\
x_2 &= 0.02321664000t^2, \\
y_2 &= 0.02532000000t^2, \\
x_3 &= 0.0008613579093t^3,
\end{aligned} \tag{17}$$

$$\begin{aligned}
y_3 &= 0.0005198410667t^3, \\
x_4 &= 0.00002377241892t^4, \\
y_4 &= 0.00000715508705t^4, \\
x_5 &= 4.762365706 * 10^{(-7)}t^5, \\
y_5 &= 1.122902100 * 10^{(-7)}t^5, \\
x_6 &= 4.92081361 * 10^{(-9)}t^6, \\
y_6 &= 3.924828700 * 10^{(-9)}t^6, \\
x_7 &= -9.99735783 * 10^{(-11)}t^7, \\
y_7 &= 1.355344556 * 10^{(-10)}t^7.
\end{aligned}$$

#### 4.2. Parasitism interaction

Given that  $a_1 = 0.1$ ;  $a_2 = 0.08$ ;  $b_1 = -0.0014$ ;  $b_2 = -0.001$ ;  $c_1 = 0.0012$ ;  $c_2 = -0.0009$ ; and  $x_0 = 4$  and  $y_0 = 10$ .

We obtain:

$$\begin{aligned}
x_1 &= 0.4256000000t, \\
y_1 &= 0.6640000000t, \\
x_2 &= 0.02304384000t^2, \\
y_2 &= 0.01680960000t^2, \\
x_3 &= 0.0008296779093t^3, \\
y_3 &= 0.0000151441067t^3, \\
x_4 &= 0.00002079741808t^4, \\
y_4 &= -0.00001228646758t^4, \\
x_5 &= 2.877838352 * 10^{(-7)}t^5, \\
y_5 &= -4.066091675 * 10^{(-7)}t^5, \\
x_6 &= -3.968167659 * 10^{(-9)}t^6, \\
y_6 &= -5.050637136 * 10^{(-9)}t^6, \\
x_7 &= -4.223308270 * 10^{(-10)}t^7, \\
y_7 &= 8.804344521 * 10^{(-11)}t^7.
\end{aligned} \tag{18}$$

#### 4.3. Competition interaction

Given that  $a_1 = 0.1$ ;  $a_2 = 0.08$ ;  $b_1 = -0.0014$ ;  $b_2 = -0.001$ ;  $c_1 = -0.0012$ ;  $c_2 = -0.0009$ ; and  $x_0 = 4$  and  $y_0 = 10$ .

We obtain:

$$\begin{aligned}
x_1 &= 0.3296000000t, \\
y_1 &= 0.6640000000t, \\
x_2 &= 0.01106304000t^2, \\
y_2 &= 0.01724160000t^2, \\
x_3 &= 0.0001173886293t^3, \\
y_3 &= 0.0000783313067t^3, \\
x_4 &= -0.000004301207450t^4, \\
y_4 &= -0.000007815319423t^4, \\
x_5 &= -1.851824568 * 10^{(-7)}t^5, \\
y_5 &= -2.136855195 * 10^{(-7)}t^5, \\
x_6 &= -1.635576185 * 10^{(-9)}t^6, \\
y_6 &= -7.0058057 * 10^{(-9)}t^6, \\
x_7 &= 8.190722735 * 10^{(-11)}t^7, \\
y_7 &= 1.240343622 * 10^{(-10)}t^7.
\end{aligned} \tag{19}$$

We find the solution to the system for each initials conditions. Approximate solutions for both populations are given by:

$$X = x_0 + x_1 + x_2 + x_3 + x_4 + x_4 + x_6 + x_7 ,$$

$$Y = y_0 + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 .$$

Approximate solutions for mutualism interaction are given by:

$$X1 = 4 + 0.4256000000 * t + 0.02321664000 * t^2 + 0.0008613579093 * t^3$$

$$+ 0.00002377241892 * t^4 + 4.762365706 * 10^{-7} * t^5 + 4.92081361 * 10^{-9} * t^6$$

$$- 9.99735783 * 10^{-11} * t^7,$$

$$Y1 = 10 + 0.7360000000 * t + 0.02532000000 * t^2 + 0.0005198410667 * t^3$$

$$+ 0.00000715508705 * t^4 + 1.122902100 * 10^{-7} * t^5 + 3.924828700 * 10^{-9}$$

$$* t^6 + 1.355344556 * 10^{-10} * t^7.$$

Approximate solutions for parasitism interaction are given by:

$$X2 = 4 + 0.4256000000 * t + 0.02304384000 * t^2 + 0.0008296779093 * t^3$$

$$+ 0.00002079741808 * t^4 + 2.877838352 * 10^{-7} * t^5 - 3.968167659 * 10^{-9}$$

$$* t^6 - 4.223308270 * 10^{-10} * t^7,$$

$$Y2 = 10 + 0.6640000000 * t + 0.01680960000 * t^2 + 0.0000151441067 * t^3$$

$$- 0.00001228646758 * t^4 - 4.066091675 * 10^{-7} * t^5 - 5.050637136 * 10^{-9}$$

$$* t^6 + 8.804344521 * 10^{-11} * t^7.$$

Approximate solutions for competition interaction are given by:

$$X3 = 4 + 0.3296000000 * t + 0.01106304000 * t^2 + 0.0001173886293 * t^3$$

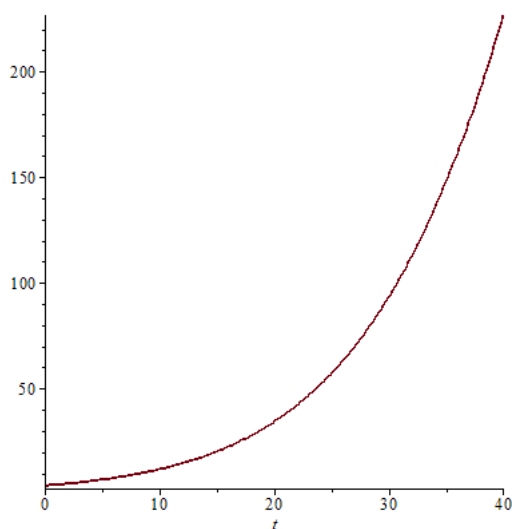
$$- 0.000004301207450 * t^4 - 1.851824568 * 10^{-7} * t^5 - 1.635576185 * 10^{-9}$$

$$* t^6 + 8.190722735 * 10^{-11} * t^7,$$

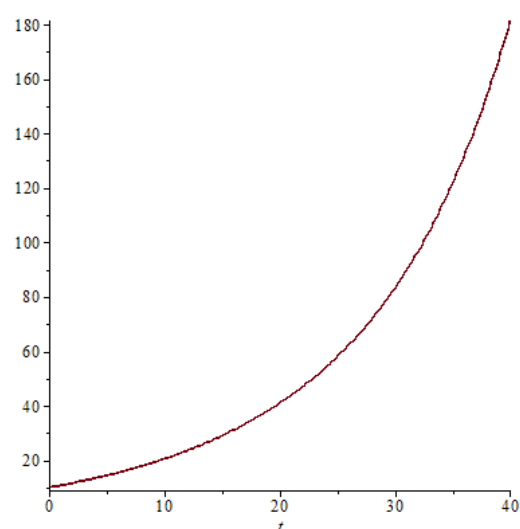
$$Y3 = 10 + 0.6640000000 * t + 0.01724160000 * t^2 + 0.0000783313067 * t^3$$

$$- 0.000007815319423 * t^4 - 2.136855195 * 10^{-7} * t^5 - 7.0058057 * 10^{-11}$$

$$* t^6 + 1.240343622 * 10^{-10} * t^7.$$

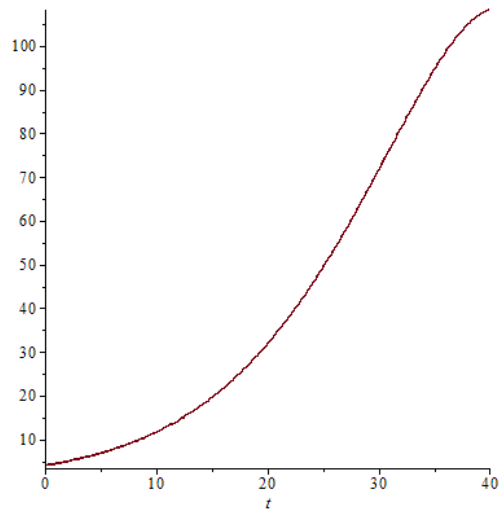
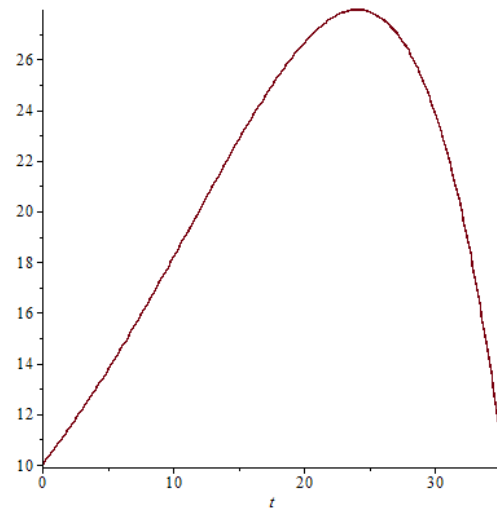
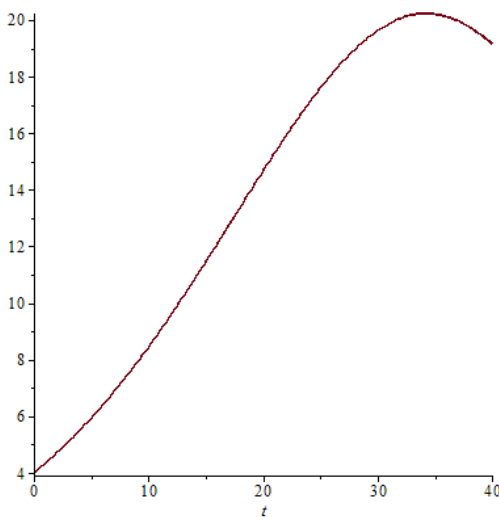
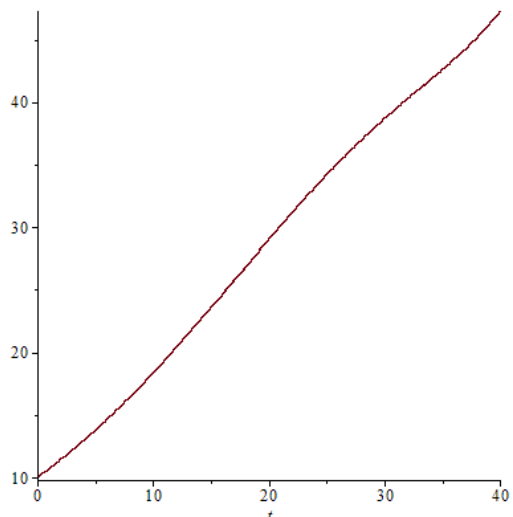


(a). Solution for the first population X1.



(b). Solution for the second population Y1.

**Figure 1.** Solutions for both populations with mutualism interaction.

(a). Solution for the first population  $X_2$ .(b). Solution for the second population  $Y_2$ .**Figure 2.** Solutions for both populations with parasitism interaction.(a). Solution for the first population  $X_3$ .(b). Solution for the second population  $Y_3$ .**Figure 3.** Solutions for both populations with competition interaction.

**Table 1.** Population dynamics for both species obtained using the Adomian decomposition method.

$t$	Mutualism interaction		Parasitism interaction		Competition interaction	
	X1	Y1	X2	Y2	X3	Y3
0	4.000000	10.00000	4.000000	10.00000	4.000000	10.00000
0.1	4.042793	10.07385	4.042791	10.06657	4.033071	10.06657
0.2	4.086056	10.14822	4.086048	10.13347	4.066363	10.13349
0.3	4.129793	10.22309	4.129777	10.20071	4.099879	10.20075
0.4	4.174010	10.29848	4.173981	10.26829	4.133617	10.26836
0.5	4.218713	10.37440	4.218666	10.33620	4.167580	10.33632
0.6	4.263907	10.45083	4.263838	10.40445	4.201767	10.40462
0.7	4.309597	10.52779	4.309501	10.47304	4.236180	10.47327
0.8	4.355790	10.60527	4.355661	10.54196	4.270819	10.54227
0.9	4.402489	10.68329	4.402324	10.61122	4.305684	10.61162
1	4.449702	10.76185	4.449495	10.68081	4.340776	10.68131

Illustration of the solutions for all three cases are shown in Figure 1, Figure 2, and Figure 3. Numerical results are representatively given in Table 1. From these results, the Adomian decomposition method is successful in solving the population dynamics model. These results compared with the Runge-Kutta numerical solutions lead to the discrepancy of order lower than  $10^{-6}$ . This means that solutions obtained using the Adomian decomposition method are very accurate.

## 5. Conclusion

We have solved the population dynamics model and for three different sets of parameters for mutualism, parasitism, and competition. The Adomian decomposition method is meshless, so we can obtain approximate solution simply from the explicit solutions. It gives approximate solutions at every time value without any discretisation of the time domain. This is an advantage of using the Adomian decomposition method to solve the problem.

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