IOP Conf. Series: Journal of Physics: Conf. Series **795** (2017) 012037 doi:10.1088/1742-6596/795/1/012037

Solution of five dimensional Dirac equation with Asymtotic Iteration Method in the case of pseudo spin symmetry

Y.A Kurniawan, A Suparmi and C Cari

Physics Department , Sebelas Maret University,
Jl. Ir. Sutami 36A Kentingan Jebres Surakarta 57126, Indonesia

E-mail: yosuaardi@student.uns.ac.id

Abstract. The relativistic energies of 5 dimensional Dirac equation in the case of pseudo spin symmetry which governed by a sparable non central using the asymtotic iteration method (AIM).The separable five dimensional shape invariant potentials consisted of Hulthen radial potential and Manning-Rosen angular potentials. The relativistic energies were calculated numerically from the relativistic energy equation.

1. Introduction

Exact solutions for relativistic energies can be obtained from the five dimensional Dirac equation. The Dirac equation use to describe a half spin particle such as electrons and positrons. Unlike Schrodinger [1] and Klein Gordon equation [2], this equation add relativistic field and particle spin characteristic to get positive probability in every condition.

Hulthen Potential [3,4] behaves like Coloumb potential in the short range [5]. Hyperbolic Hulthen potential is written as

$$V(r) = -\frac{V_0}{2}\left(\coth\frac{\alpha r}{2} - 1\right) \tag{1}$$

The Hulthen Potential for the Dirac Equation [5, 6] can be solved exactly using Adlrich approximation. Manning-Rosen potential [7, 8] is used in angular section. Since five dimensional Dirac Equation it has 4 angular parts. Here we use trigonometric Manning-Rosen potentials

$$V(\theta_1) = \frac{V_1}{\sin^2\theta_1} - V_2\cot\theta_1 \tag{2}$$

$$V(\theta_2) = \frac{V_3}{\sin^2\theta_2} \tag{3}$$

$$V(\theta_3) = \frac{V_4}{\sin^2\theta_2} - V_5\cot\theta_3 \tag{4}$$

$$V(\theta_4) = -V_6\cot\theta_4 \tag{5}$$

Where V_1, V_2, V_3, V_4, V_5 and V_6 are constant. Manning-Rosen potential is used as a mathematical model in the description of diatomic molecular vibrations and it constitutes a convenient model for other physical situations. It is known that for this potential the Dirac equation can be solved exactly using suitable substitution.

Dirac Equation in the Case of Pseudo Spin Symmetry [9] with combination of Hulthen Potential and Manning-Rosen Potential can be written

$$\left[\begin{array}{l} -\frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right)-\frac{1}{r^2}\left[\begin{array}{l}\frac{1}{\sin^2\theta_2\sin^2\theta_3\sin^2\theta_4...\sin^2\theta_{D-1}}\left(\frac{\partial^2}{\partial\theta_1{}^2}\right)\\ +\frac{1}{\sin^2\theta_3\sin^2\theta_4...\sin^2\theta_{D-1}}\frac{\partial}{\partial\theta_2}\left\{\frac{1}{\sin\theta_2}\left(\sin\theta_2\frac{\partial}{\partial\theta_2}\right)\right\}\\ +\frac{1}{\sin^2\theta_4...\sin^2\theta_{D-1}}\frac{\partial}{\partial\theta_3}\left\{\frac{1}{\sin^2\theta_3}\left(\sin^2\theta_3\frac{\partial}{\partial\theta_3}\right)\right\}\\ +\frac{1}{\sin^2\theta_{D-1}}\frac{\partial}{\partial\theta_{D-1}}\left\{\frac{1}{\sin^{D-2}\theta_{D-1}}\left(\sin^{D-2}\theta_{D-1}\frac{\partial}{\partial\theta_{D-1}}\right)\right\}\end{array}\right]\\ +\frac{h^2}{2m}\left[V(\bar{r})+\frac{1}{r^2}\left(\begin{array}{l}\frac{V(\theta_1)}{\sin^2\theta_2\sin^2\theta_3\sin^2\theta_4...\sin^2\theta_{D-1}}\\ +\frac{V(\theta_2)}{\sin^2\theta_3\sin^2\theta_4...\sin^2\theta_{D-1}}\\ +\frac{V(\theta_3)}{\sin^2\theta_4...\sin^2\theta_{D-1}}+...+V(\theta_{D-1})\end{array}\right)\right](E-M)\end{array}\right]gnk(r) \tag{6}$$
$$=(E^2-M^2)gnk(r)$$

Where gnk is wave function of Dirac equation in case of pseudo spin symmetry. E represent of relativistic energy and M represent of relativistic mass. Pseudo Spin Symmetry happen when difference between scalar potential and vector potential is constant. If we take constant is zero, the value of scalar potential is negative vector potential.

2. Asymptotic Iteration Method

AIM [10, 11] solve homogeneous linear second order differential equation to get exact solution of the form

$$y_n''(x)=k_0(x)y_n'(x)+s_0(x)y_n(x) \tag{7}$$

Where $k_0\neq 0$, $s_0(x)$ is coefficient differential equation and n clarifying quantum number. We differentiate Eq (5) with respect to x :

$$y_n'''(x)=k_1(x)y_n'(x)+s_1(x)y_n(x) \tag{8}$$

Where

$$k_1(x)=k_0'(x)+k_0^2(x)+s_0(x)$$
$$s_1(x)=s_0'(x)+s_0k_0 \tag{9}$$

And the second derivative of Eq (5) gives :

$$y''''(x)=k_2(x)y_n'+s_2(x)y_n(x) \tag{10}$$

Where

$$k_2(x)=k_1'(x)+k_1(x)k_0(x)+s_1(x)$$
$$s_1(x)=s_1'(x)+s_0(x)k_1(x) \tag{11}$$

It can iterate up to $(i+1)th$ and $(i+2)th$ derivatives, with $i=1,2,3...$

So we have

$$y_n^{(i+1)}(x)=k_{i-1}(x)y_n'(x)+s_{i-1}(x)y_n(x)$$
$$y_n^{(i+2)}(x)=k_i(x)y_n'(x)+s_i(x)y_n(x) \tag{12}$$

Where

$$k_i(x)=k'_{i-1}(x)+k_{i-1}(x)k_0(x)+s_{i-1}(x)$$
$$s_i(x)=s'_{i-1}(x)+s_0(x)k_{i-1}(x) \tag{13}$$

From Eq (13) we got relation :

$$\frac{y_n^{(i+2)}(x)}{y_n^{(i+1)}(x)}=\frac{k_i\left[y'_n(x)+\frac{s_i}{k_i}y_n(x)\right]}{k_{i-1}\left[y'_n(x)+\frac{s_{i-1}}{k_{i-1}}y_n(x)\right]} \tag{14}$$

Asymptotic aspect of the method makes large $i(>0)$, Eq (14) becomes

$$\frac{s_i}{k_i}=\frac{s_{i-1}}{k_{i-1}}\equiv\Delta \tag{15}$$

For given potential, we convert five dimensional Dirac equation to the form of Eq (5). Than determine k_0 and s_0. Using Eq (11) $s_i(x)$ and $k_i(x)$ are obtained. Energy eigenvalue we get from

$$k_i(x)s_{i-1}(x)-k_{i-1}(x)s_i=\Delta_i=0,\ i=1,2,3... \tag{16}$$

Where k is the iteration number, and radial quantum number n is equal with iteration number k for this case

3. Bound State Solution

We separated Eq (6) in to one dimensional equation of radial, θ_1, θ_2, θ_3, and θ_4.

3.1 Radial Energy Equation

From the separation of Eq (23) one dimensional radial equation written as

$$\frac{1}{R}\left(-r^2\nabla_r{}^2R\right)+r^2V(r)(E-M)-r^2(E^2-M^2)=\lambda_4 \tag{17}$$

Where λ_4 is constant and r is distance a half particle with source of potential field. ∇_r represent of radial laplacian D dimensional is written as

$$\nabla_r{}^2=\frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right) \tag{18}$$

By using $R(r)=\frac{\chi(r)}{r^{(D-1)/2}}$, $\coth\left(\frac{\alpha r}{2}\right)=1-2z$ and $\chi(z)=(z)^{\delta}(1-z)^{\gamma}f_n(z)$ and inserting Eq (1) and Eq (18) to Eq (17) we have

$$z(1-z)f''_n(z)+\left[1+2\delta-(2\delta+2\gamma+2)z\right]f'_n(z)+\left[-(\delta+\gamma)^2-\delta-\gamma+A_s\right]f_n(z)=0 \tag{19}$$

Where

$$\delta=\sqrt{E_s-B_s},\ \gamma=\sqrt{B_s+E_S} \tag{20}$$

$$A_s=2-\lambda_4,\ B_s=\frac{V_0(E-M)}{2\alpha^2},\ E_s=\frac{V_0}{2\alpha^2}(E+M)+\frac{(E^2-M^2)}{\alpha^2} \tag{21}$$

s_0 and k_0 are obtained from Eq (26), so we can easy to find Δ_n, written as

$$(\delta+\gamma+n)(\delta+\gamma+(n+1))-A_{sn}=\Delta_0=0 \tag{22}$$

By inserting Eq (20) and (21) to equation (22) we have

$$\left[\sqrt{2-\lambda_4+\frac{1}{4}}-\left(n+\frac{1}{2}\right)\right]^2+\frac{4\left[\frac{V_0(E-M)}{2\alpha^2}\right]^2}{\left[\sqrt{2-\lambda_4+\frac{1}{4}}-\left(n+\frac{1}{2}\right)\right]^2}=4\left[\frac{V_0}{2\alpha^2}(E-M)+\frac{(E^2-M^2)}{\alpha^2}\right] \tag{23}$$

and the wave function for $n=1$ is

$$\chi_{r1}(r)=\left(\frac{1-\coth(\alpha r/2)}{2}\right)^{\delta}\left(1-\frac{1-\coth(\alpha r/2)}{2}\right)^{\gamma}\left[-C_2(2\delta+1)\left(1+\frac{(-2\delta-2\gamma-2)}{2\delta+1}\left(\frac{1-\coth(\alpha r/2)}{2}\right)\right)\right] \tag{24}$$

3.2 θ_1 Equation

The result of separation Eq (6) in variable θ_1 is written as

$$\frac{1}{P_1}\left(\frac{d^2P_1}{d\theta_1^{\,2}}\right)-V(\theta_1)(E-M)=\lambda_1 \tag{25}$$

Where λ_1 is constant and P_1 is an equation respect to θ_1. Eq (25) is reduced to homogeneous linear second order differential equation by inserting Eq (2) and taking $\cot\theta_1=(1-2z_1)i$, $P_1=z_1^{\ \delta}(1-z_1)^{\gamma}f_n(z_1)$, than solve second differential of P_1 and we have

$$z_1(1-z_1)f_n^{''}+[1+2\delta-z_1(2\delta+2\gamma+2)]f_n^{'}-\left[(\delta+\gamma)^2+\delta+\gamma+V_1(E-M)\right]f_n=0 \tag{26}$$

We can determine s_0 and k_0 from Eq (26). From Eq (19) we find Δ_n

$$(\delta+\gamma+n)(\delta+\gamma+n+1)+V_1(E-M)=\Delta_3=0 \tag{27}$$

Can be rewrite as

$$(\delta+\gamma+n+1)^2-(\delta+\gamma+n+1)+V_1(E-M)=\Delta_n=0 \tag{28}$$

Where

$$\delta=\sqrt{\frac{V_2i(E-M)+\lambda_1}{4}},\ \gamma=\sqrt{\frac{-V_2i(E-M)+\lambda_1}{4}} \tag{29}$$

By inserting Eq (29) to Eq (28) we have

$$\lambda_1=\frac{[V_2i(E-M)]^2}{4\left[\sqrt{-V_1(E-M)+\frac{1}{4}}-\left(n+\frac{1}{2}\right)\right]^2}+\left[\sqrt{-V_1(E-M)+\frac{1}{4}}-\left(n+\frac{1}{2}\right)\right]^2 \tag{30}$$

the wave function for θ_1 section for $n=1$ is

$$P_{11}=\left(\frac{1}{2}+\frac{i\cot\theta_1}{2}\right)^{\delta}\left(\frac{1}{2}-\frac{i\cot\theta_1}{2}\right)^{\gamma}\left[-C_2(2\delta+2)\left(1-\frac{2\delta+2\gamma+2}{2\delta+1}z_1\right)\right] \tag{31}$$

3.3 θ_2 Equation

Variable θ_2 from separation of Eq (6) written as

$$\frac{\lambda_1}{\sin^2\theta_2}+\frac{1}{P_2}\left(\frac{1}{\sin\theta_2}\frac{d}{d\theta_2}\left(\sin\theta_2\frac{dP_2}{d\theta_2}\right)\right)-V(\theta_2)(E-M)=\lambda_2 \tag{32}$$

By using $P_2=\dfrac{Q_2(\theta_2)}{\sin^{1/2}\theta_2}$, $\cot\theta_2=i(1-2z_2)$, and $Q_2(z_2)=z_2^{\ \alpha}(1-z_2)^{\beta}f_n(z_2)$ Eq (32) become

$$z_2(1-z_2)f_s^{''}(z_2)=-[1+2\alpha-z_2(2+2\alpha+2\beta)]f_s^{'}(z_2)+\left\{(\alpha+\beta)^2+\alpha+\beta+\lambda_1-V'(E-M)+\frac{1}{4}\right\}f_s(z_2) \tag{33}$$

Where

$$\alpha = \sqrt{\left(\frac{1}{4}-\lambda_2\right)\Big/4}\,,\quad \beta = \sqrt{\left(\frac{1}{4}-\lambda_2\right)\Big/4} \tag{34}$$

We can find Δ_n from Eq (33). We have

$$\Delta_n = (\alpha+\beta+n+1)^2-(\alpha+\beta+n+1)+\lambda_1-V'(E-M)+\frac{1}{4} \tag{35}$$

So we have eigenvalue from θ_2 equation

$$\lambda_2 = \frac{1}{4}-\left[\sqrt{-\lambda_1+V'(E-M)}-\left(n+\frac{1}{2}\right)\right]^2 \tag{36}$$

the wave function for θ_2 section for $n=1$ is

$$P_{21}(z_2)=\left(\frac{1}{2}+\frac{i\cot\theta_2}{2}\right)^a\left(\frac{1}{2}-\frac{i\cot\theta_2}{2}\right)^b\left[-C_2(2a+2)\left(1+\frac{-(2a+2b+2)}{2a+1}\left(\frac{1}{2}+\frac{i\cot\theta_2}{2}\right)\right)\right] \tag{35}$$

3.4 θ_3 Equation

Separation variable of Eq (6) for θ_3 is

$$\frac{\lambda_2}{\sin^2\theta_3}+\frac{1}{P_3}\left(\frac{1}{\sin^2\theta_3}\frac{d}{d\theta_3}\left(\sin^2\theta_3\frac{dP_3}{d\theta_3}\right)\right)-V(\theta_3)(E-M)=\lambda_3 \tag{35}$$

Using AIM, consider $P_3=\dfrac{Q_3(\theta_3)}{\sin^{2/2}\theta_3}$, $\cot\theta_3=i(1-2z_3)$, $Q_3={z_3}^{\alpha}(1-z_3)^{\beta}f_n(z_3)$ we reduce Eq (35) to be homogeneous linear second order differential equation

$$z_3(1-z_3)f_n''+[2\alpha+1-z_3(2+2\alpha+2\beta)]f_n'-\left\{(\alpha+\beta)^2+\alpha+\beta-4V_1''(E-M)+4\lambda_2\right\}f_n=0 \tag{36}$$

From Eq (36), we have

$$\Delta_n=(\alpha+\beta+n+1)^2-(\alpha+\beta+n+1)-4V_1''(E-M)+4\lambda_2 \tag{37}$$

Where

$$\alpha=\sqrt{V_2''i(E-M)-(\lambda_3+1)},\ \beta=\sqrt{-V_2''i(E-M)-(\lambda_3+1)} \tag{38}$$

By inserting Eq (38) to Eq (37) we obtain

$$\lambda_3=-\frac{[V_2''i(E-M)]^2}{\left[\sqrt{4V_1''(E-M)-4\lambda_2+\frac{1}{4}}-\left(n+\frac{1}{2}\right)\right]^2}-\frac{\left[\sqrt{4V_1''(E-M)-4\lambda_2+\frac{1}{4}}-\left(n+\frac{1}{2}\right)\right]^2}{4}-1 \tag{39}$$

the wave function for θ_3 section for $n=1$ is

$$P_{31}(z_3)=\left(\frac{1}{2}+\frac{i\cot\theta_3}{2}\right)^a\left(\frac{1}{2}-\frac{i\cot\theta_3}{2}\right)^b\left[-C_2(2\alpha+2)\left(1+\frac{-(2\alpha+2\beta+2)}{2\alpha+1}z_3\right)\right] \tag{40}$$

3.5 θ_4 Equation

The last equation from separating Eq (6) is

$$\frac{1}{\sin^3\theta_4}\frac{d}{d\theta_4}\left(\sin^3\theta_4\frac{dP_4}{d\theta_4}\right)-\left(V(\theta_4)(E-M)-\lambda_4+\frac{\lambda_3}{\sin^2\theta_4}\right)P_4=0 \tag{41}$$

With $P_4=\dfrac{Q_4}{\sin^{3/2}\theta_4}$, $\cot\theta_4=i-2iz_4$, and $Q_4={z_4}^{\delta}(1-z_4)^{\gamma}f_n(z_4)$ we have

IOP Conf. Series: Journal of Physics: Conf. Series **795** (2017) 012037 doi:10.1088/1742-6596/795/1/012037

$$z_4(1-z_4)f_n''(z_4)+[1+2\delta-z_4(2+2\delta+2\gamma)]f_n'(z_4)+\left\{-(\delta+\gamma)^2-\gamma-\delta+\lambda_3+\frac{3}{4}\right\}f_n(z_4)=0 \qquad (42)$$

Than we find Δ_n

$$(\delta+\gamma+n+1)^2-(\delta+\gamma+n+1)-\lambda_3-\frac{3}{4}=\Delta_n=0 \qquad (43)$$

Where

$$\delta=\sqrt{\frac{V_3''l(E-M)+\left(\frac{9}{4}+\lambda_4\right)}{4}},\ \gamma=\sqrt{\frac{-V_3''l(E-M)+\left(\frac{9}{4}+\lambda_4\right)}{4}} \qquad (44)$$

By inserting Eq (44) to Eq (43) the eigenvalue represent as

$$\left[\sqrt{\lambda_3+1}-\left(n+\frac{1}{2}\right)\right]^2+\frac{\frac{1}{4}\left(V_3''l(E-M)\right)^2}{\left[\sqrt{\lambda_3+1}-\left(n+\frac{1}{2}\right)\right]^2}-\frac{9}{4}=\lambda_4 \qquad (45)$$

the wave function for θ_2 section for $n=1$ is

$$P_{41}=\left(\frac{1}{2}-\frac{\cot\theta_4}{2i}\right)^p\left(1-\frac{1}{2}+\frac{\cot\theta_4}{2i}\right)^q\left(-C_2(2p+2)\left(1-\frac{2p+2q+2}{2p+1}z_4\right)\right) \qquad (46)$$

4. Result and Discussion

Energy bound state obtained by calculate mathematically the Eq (23), and substituting λ_4 in Eq (39)

Table 4.1 Eigenvalue of 5 dimensional Dirac equation with Hulthen potential and Manning-Rosen potential in the case of pseudo spin symmetry, with variation n_r and α

n_r	$E_{n_r}\left(fm^{-1}\right)$		
	$\alpha=0.25\,fm^{-1}$	$\alpha=0.5\,fm^{-1}$	$\alpha=0.75\,fm^{-1}$
1	4.998354	4.993412	4.985164
2	4.993412	4.973592	4.940374
3	4.985164	4.940374	4.864756
4	4.973592	4.893468	4.756757
5	4.958673	4.832453	4.613969

Where n_r is the orbit of electron, and α is screening parameter of Hulthen potential that work on radial section. Table 4.1 shows positive value of energy, that mean particles feel a repulsive force. The increasing of n_r has effect of decreasing of bound state energy. Higher the orbit of electron the bound state become lower. The reason is the higher orbit of electron, the distance electron from nucleus become longer. The increasing of screening parameter doesn't affect with increasing the number of n_r. It shows that decreasing energies analogously with increasing of n_r and wider range of energies.

Eigen function for radial section shown in Eq (24). Eigen function θ_1 shown in Eq (31). Eigen function for section θ_2 shown in Eq (35), for section θ_3 and section θ_4 shown in Eq (40) and Eq (46).

IOP Conf. Series: Journal of Physics: Conf. Series **795** (2017) 012037 doi:10.1088/1742-6596/795/1/012037

References

[1] Saregar A. 2013. *International Journal of Engінerring And Science. ISSN: 2278-4721, Vol. 2, Issue 3.*
[2] Das T. 2014. *arXiv:1409.1457v1 [quant-ph].*
[3] Agboola D 2009 *arXiv:0811.4441v2 [math-ph].*
[4] Bhagwati N and Singh N N 2014 *Indian J Phys DOI 10.1007/s12648-014-0509-3.*
[5] Falaye B J and Oyewumi K J 2011 *The African Review of Physics 6:0025.*
[6] Hall R L and Zorin P 2016 *CUQM 03.65.Pm, 03.65.Ge, 36.20.Kd.*
[7] Bahar M K and Yasuk F 2012 *DOI 10.1007/s00601-012-1461-8.*
[8] Ikhdiar S M and Sever R 2013 *arXiv:0807.2085v1 [quant-ph].*
[9] Soylu A, Bayrak O, Boztosun I 2007 *Journal of Mathemathical Physics 48, 082302.*
[10] E. Ateser, H. Cifti, and M. Ugurlu, Chinese Journal Of Physics Vol. 45, No. 3.
[11] O. Bayarak, I. Boztosun, H. Ciftci 2006 *Wiley InterScience DOI 10.1002/qua.21141.*