Shannon Information Entropy for Modified Anisotropic Non-quadratic Potential in Axial Part for Cylindrical Coordinates System

Miftahul Ma'arif*, Suparmi, Cari
Physics Department, Faculty of Mathematics and Natural Sciences,
Sebelas Maret University
Jl. Ir. Sutami 36A Kentingan Jebres Surakarta 57126, Indonesia

E-mail: miftahulmaarif15@student.uns.ac.id

Abstract. Entropy information in quantum mechanics is important to get the quantify of quantum entanglement and correlations. Modified anisotropic non-quadratic potential in Schrodinger equation which has been solved before, and now the solution of wave function in axial part will be used to get its Shannon information entropy. Numerical results are investigated to get the position S_z and the momentum S_p Shannon information entropy.

1. Introduction

Developing correlation measures is very important things to increase our understanding of correlation effects. Recently, information measures has been considerable interest with the applications in atomic and molecular structures. Entropic measures are used to find quantum correlations and entanglement. The development in quantum information, quantum computation, quantum cryptography and quantum teleportation are interest thins which quantum entanglement plays a central roles in it [1-4]. In 1948, Claude E. Shannon had introduced the entropy to measures the uncertainty and formalized the idea of assigning probabilities to the outcome of uncertain events [5]. In 1975, Beckner, Bialynicki-Birula and Mycieslki presented an entropic uncertainty relation as given by following equation [6],

$$S_x + S_p \geq D\left(1+\ln\pi\right) \tag{1}$$

where D represents the spatial dimension. In one dimensional coordinate system, the position-space (S_x) and momentum-space (S_p) information entropies are defined by

$$S_x = -\int_{-\infty}^{\infty} \left|\psi\left(x\right)\right|^2 \ln\left|\psi\left(x\right)\right|^2 dx \tag{2}$$

$$S_p = -\int_{-\infty}^{\infty} \left|\phi\left(p\right)\right|^2 \ln\left|\phi\left(p\right)\right|^2 dp \tag{3}$$

where $\psi\left(x\right)$ is normalized eigenfunction and $\phi(p)$ is normalized Fourier transform. Interval of integration for the variable x depend on the concrete quantum system [7].

The Shannon entropy information for a few molecular potential has been obtained, i.e., harmonic oscillator [8], Pöschl-Teller [9, 10], Morse [9, 11], Coulomb [12], isospectral to the Pöschl-Teller [13],

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classical orthogonal polynomials [14], infinite circular well [7], and the other potentials and studies[15-17]. In this paper, the modified anisotropic non-quadratic potential which has been got the analytical solution by us before [18]. Now, we will review the solution of this potential before especially in the axial part that has been modified in exponential form and tried to get the Shannon entropy information in position-space, momentum-space, total entropy information and its correlation with uncertainty that presented by BBM.

2. Review to The Solution of Axial Parts

Modified anisotropic non-quadratic potential is expressed as

$$V(\bar{r}) = V(r,\theta,z) = -\frac{V_0}{\sqrt{x^2+y^2}} + \frac{V_1}{x^2} + \frac{V_2 y}{x(x^2+y^2)} + \frac{V_3 e^{-\alpha z}}{(1-e^{-\alpha z})^2} + V_4 \frac{1+e^{-\alpha z}}{1-e^{-\alpha z}} \tag{4}$$

We are only interested in the axial part because it is the modified object. By using cylindrical coordinates system so the axial part in Schrodinger equation can be separated directly, and the exponential form must be simplified by change it in to hyperbolic form and each potential constants has been changed , so the Schrodinger equation for axial part can be written as follows

$$-\frac{\hbar^2}{2m}\frac{\partial^2 Z}{\partial z^2} + \frac{\hbar^2}{2m}\left(\gamma^2 \frac{\nu(\nu-1)}{\sinh^2 \gamma z} - 2q\coth\gamma z\right)\Phi = E'Z \tag{5}$$

The Schrodinger equation that expressed in Equation (5) is solvable by using SUSY QM method and shape invariant approach [19-21], we had got the solutions for Equation (5). The un-normalized ground state wave function for axial part is given by

$$Z_0 = C'\sinh^{\gamma(\nu-1)}\gamma z\, e^{-\frac{q}{\gamma(\nu-1)}z} \tag{6}$$

The un-normalized ground state wave function have to normalized using the following equation

$$\int_{-\infty}^{\infty} Z_0^{*} Z_0 dz = 1 \tag{7}$$

By substitute Equation (6) in to Equation (7) and solve the integration by using integral tables [22] so we get the constant value is given by

$$C' = 2\gamma\sqrt{\nu-1} \tag{8}$$

The study case in this paper is the system with the quantum number $n=0$ and a few quantum number ν which would be the independent variable to get its correlation between information entropy.

3. Information Entropy

The normalized ground state wave function can be written as follows

$$Z_0 = 2\gamma\sqrt{\nu-1}\sinh^{\gamma(\nu-1)}\gamma z\, e^{-\frac{q}{\gamma(\nu-1)}z} \tag{9}$$

Axial ground state wave function in Equation (9) for $\nu = 1-3$ are displayed in Figure (1). If the quantum number $\nu = 1$ the eigenstate would be zero it can be proven directly using Equation (9) or we can look at the graph in Figure (1). But it would be different if the quantum number $\nu = 2$ it looks like logarithmic graphic but for $\nu = 3$ becomes like exponential graphic.

The position-space and momentum-space information entropies which has been defined by Equation (2) and (3) can be calculated. However, in arbitrary case we should use these following equations

$$S_z = -\int |\psi(z)|^2 \ln|\psi(z)|^2 d^D z \tag{10}$$

$$S_p = -\int |\phi(p)|^2 \ln|\phi(p)|^2 d^D p \tag{11}$$

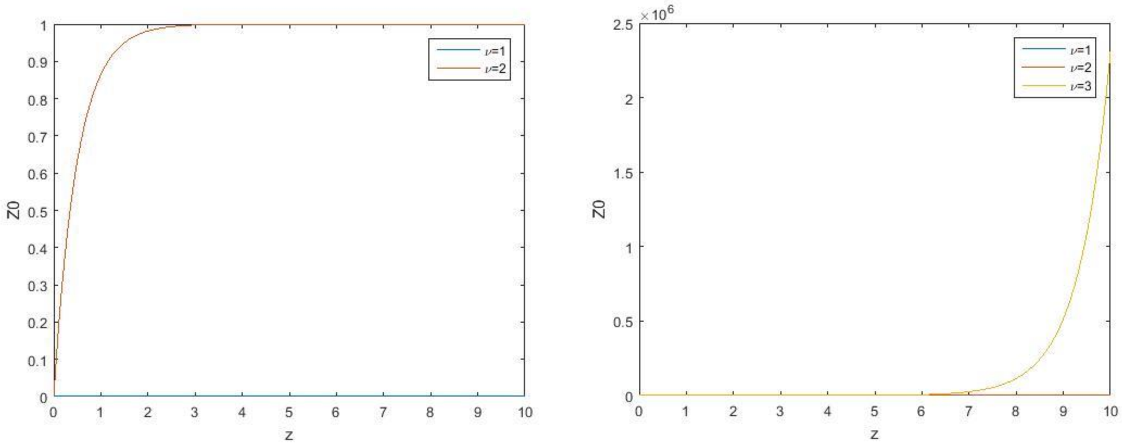

Figure 1. Axial ground state wave function for quantum numbers $\nu = 1-3$.

Before we calculate the momentum-space information entropy, firstly we have to change the the wave function obtained in position-space into momentum-space by using Fourier transform.

$$\phi_0(p) = \frac{1}{2\pi}\int Z_0(z)e^{-ip.z}dz \tag{12}$$

By substituting Equation (9) into Equation (12) so it becomes

$$\begin{aligned}
\phi_0(p) &= \frac{\gamma\sqrt{\nu-1}}{\pi}\int \sinh^{\gamma(\nu-1)}\gamma z\, e^{-\left(\frac{q}{\gamma(\nu-1)}+ip\right)z}dz \\
&= \frac{\gamma\sqrt{\nu-1}}{\pi 2^{\gamma(\nu-1)}}\int \left(e^{\gamma z}-e^{-\gamma z}\right)^{\gamma(\nu-1)} e^{-\left(\frac{q}{\gamma(\nu-1)}+ip\right)z}dz \\
&\simeq \frac{\gamma\sqrt{\nu-1}}{\pi 2^{\gamma(\nu-1)}}\int \left(1-\gamma\left(\nu-1\right)e^{-2\gamma z}\right)e^{\left(\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip\right)z}dz \\
&\simeq \frac{\gamma\sqrt{\nu-1}}{\pi 2^{\gamma(\nu-1)}}\left[\frac{\left(e^{\left(\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip\right)z}-1\right)}{\gamma^2(\nu-1)-\frac{q}{\gamma\left(\nu-1\right)}-ip}-\frac{\gamma(\nu-1)\left(e^{\left(\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip-2\gamma\right)z}-1\right)}{\gamma^2(\nu-1)-\frac{q}{\gamma\left(\nu-1\right)}-ip-2\gamma}\right]
\end{aligned} \tag{13}$$

The third process to get Equation (13) can be obtained by using binomial rules approximation. The momentum-space is displayed in Figure (2).

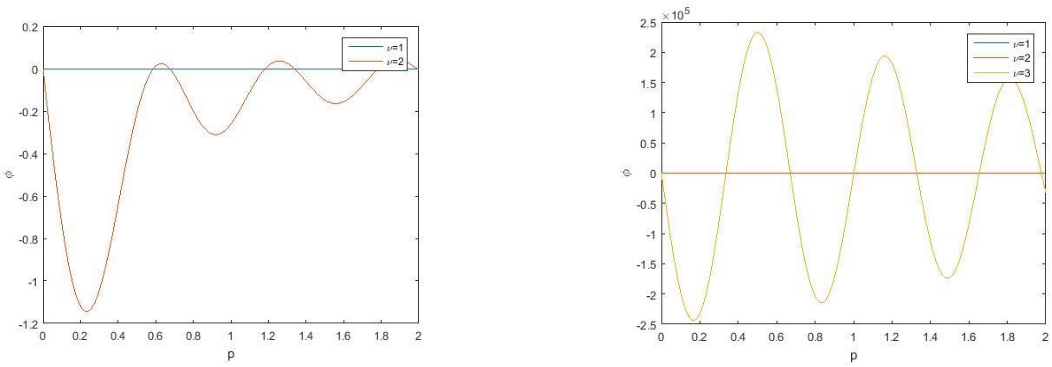

Figure 2. Ground state wave functions in momentum-space for different quantum numbers $\nu = 1-3$.

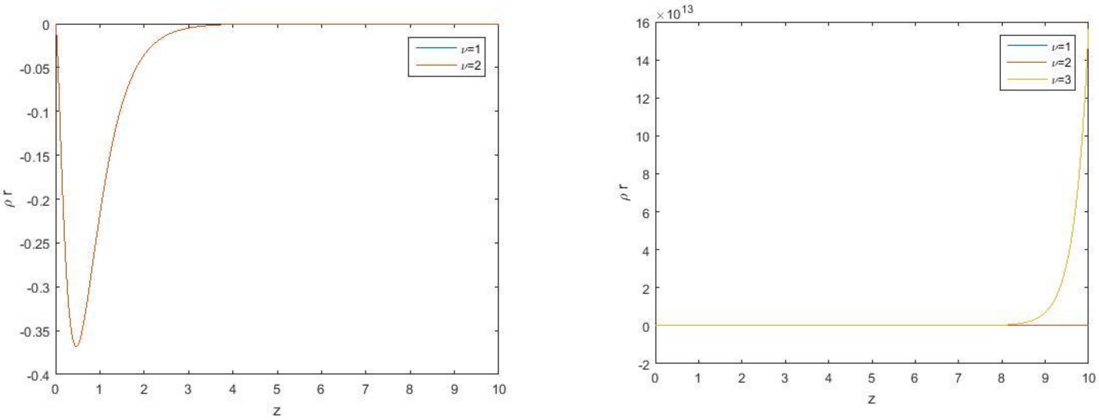

Figure 3. Position entropy densities as a function of the position z .

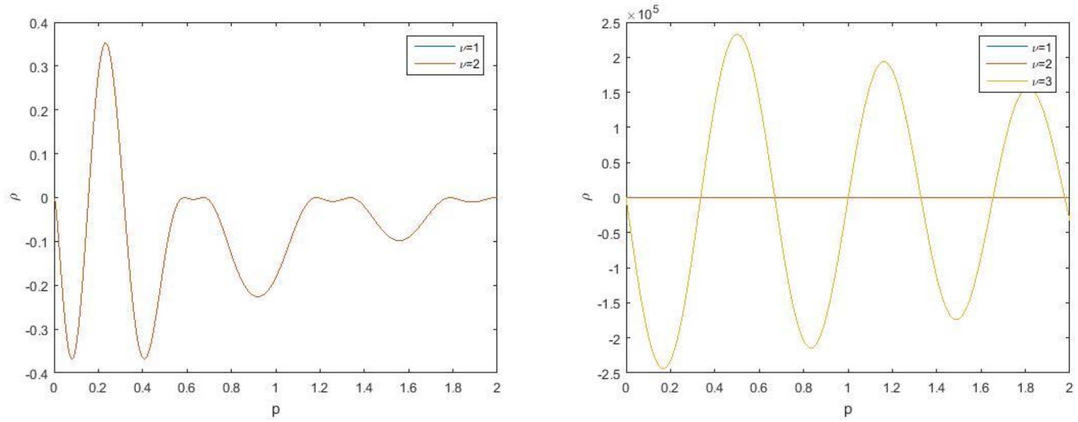

Figure 4. Momentum entropy densities as a function of the momentum p .

Characteristics features of position ρ_z and momentum ρ_p entropy densities are displayed in Figure (3) and (4). To calculate these entropy densities, we used the following equations bellow,

$$\rho_z = |Z_0|^2 \ln |Z_0|^2 \tag{14}$$

$$\rho_p = |\phi_0|^2 \ln |\phi_0|^2 \tag{15}$$

By substituting Equations (9) and (13) into Equations (14) and (15), so we get position and momentum entropy densities are given by

$$\rho_z = 4\gamma^2(\nu-1)\sinh^{2\gamma(\nu-1)}\gamma z\, e^{-\frac{2q}{\gamma(\nu-1)}z} \ln 4\gamma^2(\nu-1)\sinh^{2\gamma(\nu-1)}\gamma z\, e^{-\frac{2q}{\gamma(\nu-1)}z} \tag{16}$$

$$\rho_p = \frac{\gamma^2(\nu-1)}{\pi^2 2^{2\gamma(\nu-1)}}\left[\frac{\left(e^{\left(\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip\right)z}-1\right)}{\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip}-\frac{\gamma(\nu-1)\left(e^{\left(\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip-2\gamma\right)z}-1\right)}{\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip-2\gamma}\right]^2 \ln\frac{\gamma^2(\nu-1)}{\pi^2 2^{2\gamma(\nu-1)}}\left[\frac{\left(e^{\left(\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip\right)z}-1\right)}{\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip}-\frac{\gamma(\nu-1)\left(e^{\left(\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip-2\gamma\right)z}-1\right)}{\gamma^2(\nu-1)-\frac{q}{\gamma(\nu-1)}-ip-2\gamma}\right]^2 \tag{17}$$

In Table (1) we present position-space, momentum-space and total Shannon information entropies, it is obtained by using numerical analysis using trapezoidal integration method with 100 times iterations. We just obtained the Shannon information entropies for quantum number $\nu = 2-6$. The higher

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quantum number ν for S_p couldn't be obtained because the output value is too high and time-consuming in the calculations.

Table 1. Information entropies for $n = 0$ states with respect to different quantum number ν .

ν	S_z	S_p	$S_z + S_p$	$D(1+\ln\pi)$
2	-0.1038	0.0300	-0.0738	4.28946
3	241.9233	1.6344e+10	1.63E+10	4.28946
4	1.00E+04	3.3904e+19	3.39E+19	4.28946
5	2.48E+05	7.4903e+27	7.49E+27	4.28946
6	4.96E+06	2.9664e+35	2.97E+39	4.28946

4. Conclusions

The position S_z and the momentum S_p Shannon information entropies for modified anisotropic nonquadratic potential in axial parts has been investigated. We have taken the ground state wave function with some different value of quantum number ν . The interesting features like position and momentum entropy densities for different quantum numbers $\nu = 1-3$ has been displayed and it has different properties. Next study of Shannon information entropy for this case is needed with the higher quantum number n .

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