Visualization of heat transfer in material for varians of boundary value with Relaxation Iteration Gauss-Seidel method

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Abstrack— The research was aimed to know the effect of initial boundary value to the heat propagation rate pattern using iterations over Gauss-Seidel relaxation method and to analyze the exact value of each node descritization profile of test material. This study was an analytical study to fine analytical an numerical solution. Result from this study is that the pattern of variation of the boundary or initial conditions of a material with regard conductivity value remains at steady state the exact value of the smallest are in the same iteration value. The indicates that the value of the thermal equilibrium tend to be at the same iteration. Result from study showed that the pattern of initial boundary values that causes steady state of heat propagation of test material that has smallest exact similar to the iteration value.

1. Introduction

Physics concepts are produced through experiments and theoretical works which are in the form of mathematical equation. Physics is said as the father of technology since the advanced development of physics is in the form of technology, and in turn, physics developed rapidly due to the application of technology in physics, theoretically, physics concepts are described using mathematical modeling in the form of mathematical equation tha can be solved analytically or numerically. Physics is closely related to mathematics for theoretical physics often expressed in mathematical notation and mathematical logic can provide a framework in which the laws of physics can be formulated appropriately [1]. One development of the mathematical methods used in the case of the physical and engineering fields is a numerical method [2]. Numerical methods are techniques to solve problems that are formulated mathematically by means of operating a matter of analytic [3]

Heat transfer is one of the physical phenomena that is described partial differential equations Laplace or Poison [4]. In accordance with the usefulness of mathematics in the fields of physics, the solution partial differential equations can be obtained by several methods. The selection of methods and approaches based on the purpose and complexity of the problems.

Heat transfer is a concept to predict the energy transfer occurs due to a temperature difference between objects or material [5].

The heat energy can not be observed directly but the direction of movement and its effects can be observed and measured, as in the event of conduction heat transfer. Conduction is a process that if two objects or two-part of object is contacted with the other temperature it will pass heat transfer [6].

Heat flows from a higher-temperature object to a lower body temperature. Heat transfer rate that passes through solid objects is proportional to the temperature gradient or temperature difference of unity long [7].

As mentioned before that the rate of heat propagation in a variety of geometric shapes material following the equation of state is presented in the form of mathematical equations in the form of differential equations. Solution method or the propagation model of the flow of heat at a material solved analytically and numerically. The analytically solution by using a systematic calculation and the obtained solution in the form of the exact value [8].

However, some form of differential equations, there is difficulty the applying analytical resolution methods so numerical methods is chosen exam alternative to overcome. Numerical methods are used to solve partial differential equations such as Method Crank- Nicholson, methods Milne, Hamming method and Gauss-Seidel method [9].

In this studies obeys the rate of heat flow that follows the model of partial differential equations are solved using numerical methods of Gauss-Seidel and then subsequently visualized using Mathlab with some variation of the initial boundary value.

Shape of specimens analyzed in this study are presented in Figure 1 with an initial boundary values are follow :

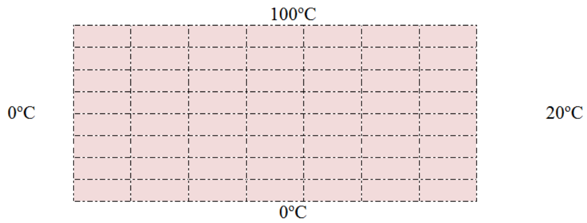

Figure 1. Model workpiece as two-dimensional metal plate with a specific conductivity value in steady state.

Based on the figures, by using Gauss-Seidel methods can obtain the solution of partial differential equations in the hot stream on a metal plate two dimensions in a steady state or heat flow rate system does not change with time (constant), then the temperature at any point does not change.

To apply Gauss-Seidel relaxation method, we set variation value of the initial boundary on the completion of iterations over Gauss Seidel relaxation method to observe how the heat propagation rate pattern seen from the distribution of its exact value on each node discretization test material and visualize the propagation of heat profile.

The mathematical equation that govern the heat propagation rate is given as :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + g = 0$$

With :

T : temperature

x : absis

y : coordinat

g : $\frac{q(\Delta x^2)}{k}$

To solve the above equation, it is used a Taylor series with two independent variables $T\ (x, y)$ is a way to add additional variables so that the Taylor series with two independent variables $T\ (x, y)$ be [10] :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q(\Delta x^2)}{k} = 0$$

To get the second derivative can be done in the following way:

1) If the first derivative of the advanced differential form, then the second derivative settled in the form of differential retreat.

2) If the first derivative of the differential form of retreat, then the second derivative settled in the form of forward differential.

Now we know the second derivative function of T with respect to x and y, then substituted in equation Poisson equation or temperature distribution, in order to obtain:

$$\frac{\left(\frac{T_{i+1,j}-T_{i,j}}{\Delta x}\right)-\left(\frac{T_{i,j}-T_{i-1,j}}{\Delta x}\right)}{\Delta x}+\frac{\left(\frac{T_{i,j+1}-T_{i,j}}{\Delta y}\right)-\left(\frac{T_{i,j}-T_{i,j-1}}{\Delta y}\right)}{\Delta \mathrm{y}}=0$$

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{\Delta x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\Delta x^2}+\frac{q}{k}=0$$

For the size of Δx and Δy are the same, then the above equation simplifies to:

$$T_{i+1,j}-2T_{i,j}+T_{i-1,j}+T_{i,j+1}-2T_{i,j}+T_{i,j-1}+\frac{q(\Delta x^2)}{k}=0$$

or

$$T_{i+1,j}+T_{i-1,j}+T_{i,j+1}+T_{i,j-1}-4T_{i,j}+\frac{q(\Delta x^2)}{k}=0$$

Gauss Seidel equation constructed from the above equation becomes:

$$T_{i,j}=\frac{T_{i+1,j}+T_{i-1,j}+T_{i,j+1}+T_{i,j-1}+\frac{q(\Delta x^2)}{k}}{4}$$

and solving iteratively for i = 1 to n and j = 1 to m. In general, from the heat propagation equation with q is the rate heat transfer, T is the temperature distribution at a distance of x and y, which has a high length L and K. Therefore the value of T at the edge of the plate known temperature (boundary conditions) and at the time before propagation, the value at points it is zero (baseline) settlement is counting equation x and y T in particular.

For partial differential equations

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{q(\Delta x^2)}{k}=0 \qquad 0\le x\le L\ \ dan\ \ 0\le y\le K$$

$$T(0,y)=\ 0$$
$$T(L,y)=\ 0$$
$$T(x,0)=\ 100$$
$$T(x,K)=\ 20$$

known:

Δx = 0,1
Δy = 0,1
q = 10000 Btu/hr ft
k = 40 Btu/hr ft

To find out the solution heat propagation at each point of the Gauss Seidel method, can be done by solving iteratively for i = 0 to n, and j = 1 to m.

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2. Results and Discussion

Completion of the heat propagation equation solved iteratively with over-relaxation equation. Relaxation parameters can be searched using the equation:

$$\omega=\frac{1}{1+\left(\frac{0,1}{0,1}\right)^2}\left[\cos\frac{\pi}{m}+\left(\frac{\Delta x}{\Delta y}\right)^2\cos\frac{\pi}{n}\right]=\frac{1}{1+\left(\frac{0,1}{0,1}\right)^2}\left[\cos\frac{3,14}{50}+\left(\frac{0,1}{0,1}\right)^2\cos\frac{3,14}{50}\right]=0,93$$

Thus, λ can be searched using the following equation:

$$\lambda=\frac{2}{1+\sqrt{1-\omega^2}}=\frac{2}{1+\sqrt{1-0,93^2}}=1,47=15$$

For partial differential equations:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q(\Delta x^2)}{k} = 0 \qquad 0 \le x \le L \; dan \; 0 \le y \le K$$

using boundary conditions and the same coefficient of relaxation. Furthermore solved iteratively for $i = 1$ to n and $j = 1$ to m with over-relaxation following equation:

$$T_{i,j}^{new} = \lambda T_{i,j}^{new} + (1 - \lambda) T_{i,j}^{old}$$

Iteration can be stopped if the relative error has reached the limit. The magnitude of the relative error is defined as:

$$\left|(\varepsilon_a)_{i,j}\right| = \left|\frac{T_{i,j}^{new} + T_{i,j}^{old}}{T_{i,j}^{old}}\right| \times 100\%$$

Analytical solutions obtained are as follows: In the initial condition or Iteration 0, the solution in the first iteration by means of Gauss Seidel, then at the point $T_{i,j}$, :

$$T_{1,1} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} + \frac{q(\Delta x^2)}{k}}{4} = \frac{0 + 100 + 0 + 50 + \frac{10000 \times (0.1)^2}{40}}{4} = 37{,}75$$

of the equation over-relaxation (λ = 1.5) was obtained

$$T_{1,1} = 1.5(37.75) + (1 - 1.5)0 = 56{,}625$$

for $T_{2,1}$:

$$T_{2,1} = \frac{0 + 56.625 + 0 + 50 + 2.5}{4} = 26.906$$

From over-relaxation of the equation is obtained:

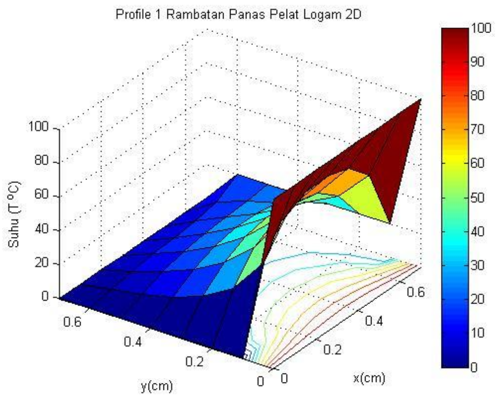
$$T_{2,1} = 1.5(26.906) + (1 - 1.5)0 = 40.359$$

iteration followed by Mathlab until iteration on $T_{7,7}$, point, in order to obtain a value as shown below. From the results of this iteration is then performed visualization of heat propagation using Mathlab [11] with some variation of the initial boundary conditions. And the results are as follows :

a) $T(0,y) = 0; T(L,y) = 20; T(x,0) = 100; T(x,K) = 0$

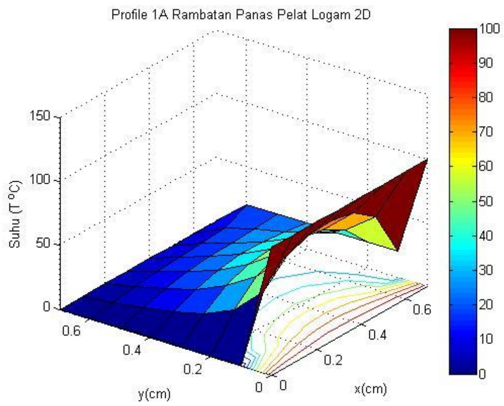
Profile 1
Iteration Matrix
Iteration = 24
Computing times = 0.0042768

100	100	100	100	100	100	100	100
0	481.716	658.982	726.063	738.412	700.802	572.574	20
0	267.883	428.148	506.856	526.784	492.221	389.495	20
0	161.668	278.869	346.431	369.647	351.804	293.184	20
0	99.920	179.230	230.353	253.569	252.165	231.435	20
0	58.781	107.779	142.181	162.109	171.852	180.393	20
0	27.426	50.922	68.484	80.834	92.743	118.284	20
0	0	0	0	0	0	0	0



Profile 1A
Iteration matrix with Q = 10000; k = 40
Iteration = 24
Computing times = 0.0076659

100	100	100	100	100	100	100	100
0	481.979	659.382	726.524	738.873	701.203	572.837	20
0	268.284	428.777	507.589	527.517	492.851	389.896	20
0	162.129	279.602	347.289	370.505	352.537	293.645	20
0	100.381	179.963	231.211	254.426	252.898	231.897	20
0	59.182	108.408	142.914	162.842	172.482	180.794	20
0	27.689	51.323	68.946	81.295	93.143	118.547	20
0	0	0	0	0	0	0	0



The number of iterations resulting from this limit value is 24 times. Profile distribution of heat to show the temperature decreases in the same area or exactly on point $T_{7,1}$.

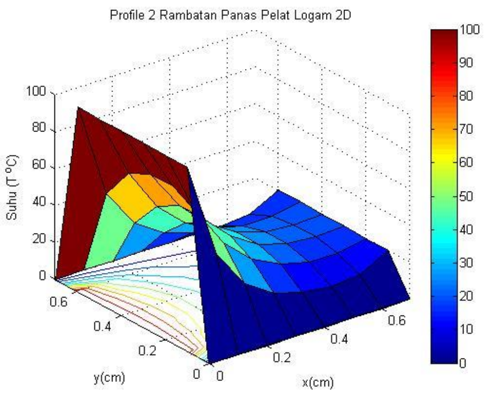
b) $T(0,y) = 100; T(L,y) = 20; T(x,0) = 0; T(x,K) = 0$

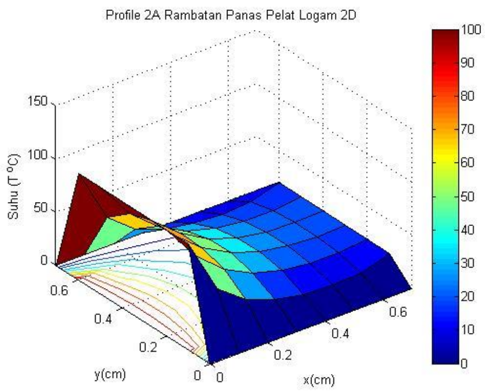
Profile 2
Iteration Matrix
Iteration = 24
Computing times = 0.0099215

0	0	0	0	0	0	0	0
100	481.716	269.848	169.527	120.128	102.566	118.284	20
100	657.017	428.148	288.131	208.420	171.852	170.569	20
100	718.204	497.594	346.431	253.569	205.854	192.141	20
100	718.204	497.594	346.431	253.569	205.854	192.141	20
100	657.017	428.148	288.132	208.420	171.852	170.569	20
100	481.716	269.848	169.527	120.128	102.566	118.284	20
0	0	0	0	0	0	0	0

Profile 2A
Iteration Matrix with Q = 10000; k = 40
Iteration = 24
Computing times = 0.012379

0	0	0	0	0	0	0	0
100	481.979	270.249	169.988	120.589	102.967	118.547	20
100	657.418	428.777	288.864	209.153	172.482	170.970	20
100	718.665	498.327	347.289	254.426	206.587	192.602	20
100	718.665	498.327	347.289	254.426	206.587	192.602	20
100	657.418	428.777	288.864	209.153	172.482	170.970	20
100	481.979	270.248	169.988	120.590	102.967	118.547	20
0	0	0	0	0	0	0	0





The resulting number of iterations 24 times. Second heat distribution profile to show the temperature decreases in the same area or exactly at a node $T_{5,1}$ dan $T_{5,6}$.
When observed more than a second variation of the above limits, namely: 0; 20; 100; 0 and 10; 20; 0; 0 produce the same number of iterations equilibrium which is 24 times. And when the second iteration matrix deductible would generate diagonal matrix with value 0.

0	389134	556536	618284	598236	454290
-389134	0	218725	318364	320369	218926
-556536	-218725	0	116078	145950	101043
-618284	-318364	-116078	0	46311	39294
-598236	-320369	-145951	-46311	0	9824
-454290	-218926	-101043	-39294	-9823	0

0	389.133	556.536	618.284	598.236	454.290
-389.134	0	218.725	318.364	320.369	218.926
-556.536	-218.725	0	116.079	145.950	101.043
-618.284	-318.364	-116.078	0	46.311	39.295
-598.236	-320.369	-145.950	-46.311	0	9.824
-454.290	-218.925	-101.042	-39.295	-9.824	0

Both matrices show has a positive value on the triangle top and negative on the lower triangle. And both showed the smallest iteration value in the same area.

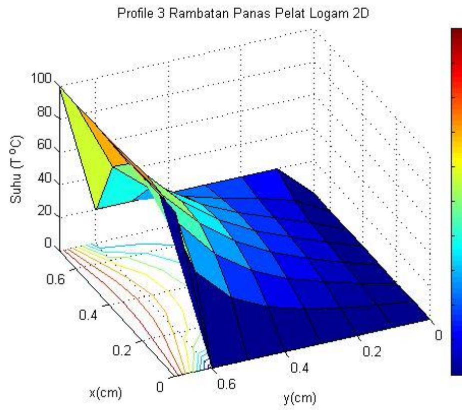
c) $T(0,y) = 100; T(L,y) = 0; T(x,0) = 20; T(x,K) = 0$

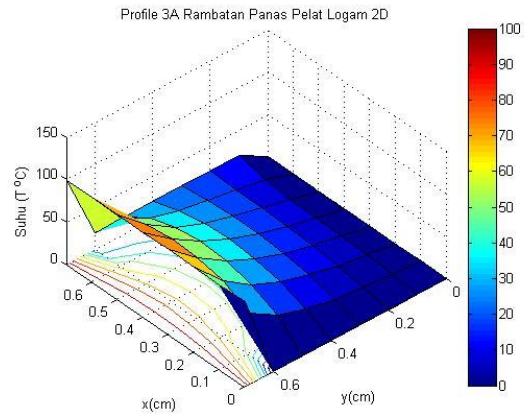
Profile 3
Iteration matrix
Iteration = 28
Computing times = = 0.14655

0	0	0	0	0	0	0	0
0	27.426	50.922	68.484	80.834	92.743	118.284	20
0	58.781	107.779	142.181	162.109	171.852	180.393	20
0	99.920	179.230	230.353	253.569	252.165	231.435	20
0	161.668	278.869	346.431	369.647	351.804	293.184	20
0	267.883	428.148	506.856	526.784	492.221	389.495	20
0	481.716	658.982	726.063	738.412	700.802	572.574	20
100	100	100	100	100	100	100	100

Profile 3A
Iteration Matrix with Q = 10000; k = 40
Iteration = 28
Computing times = 0.010536

0	0	0	0	0	0	0	0
0	27.689	51.323	68.946	81.295	93.143	118.547	20
0	59.182	108.408	142.914	162.842	172.482	180.794	20
0	100.381	179.963	231.211	254.426	252.898	231.897	20
0	162.129	279.602	347.289	370.505	352.537	293.645	20
0	268.284	428.777	507.589	527.517	492.851	389.896	20
0	481.979	659.382	726.524	738.873	701.203	572.837	20
100	100	100	100	100	100	100	100





The number of iterations resulting from this limit value is 28 times. Profile distribution of heat to show the temperature decreases in the same area or exactly on point $T_{1,1}$

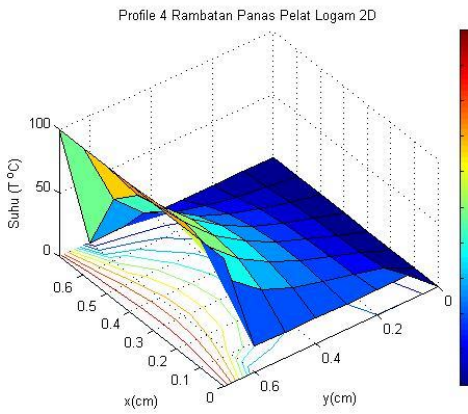
d) $T(0,y) = 100; T(L,y) = 20; T(x,0) = 0; T(x,K) = 0$

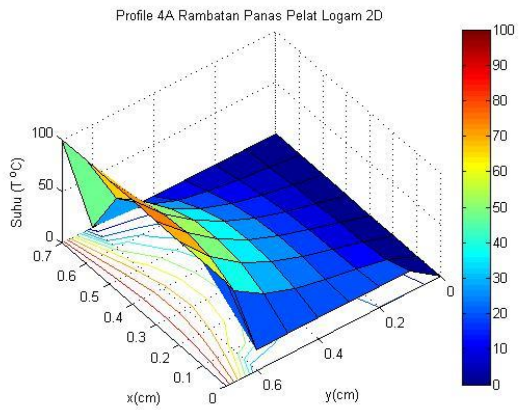
Profile 4
Iteration Matrix
Iteration = 28
Computing times = 0.0096928

0	0	0	0	0	0	0	0
20	118.284	92.743	80.834	68.484	50.922	27.426	0
20	180.393	171.852	162.109	142.181	107.779	58.781	0
20	231.435	252.165	253.569	230.353	179.230	99.920	0
20	293.184	351.804	369.647	346.431	278.869	161.668	0
20	389.495	492.221	526.784	506.856	428.148	267.883	0
20	572.574	700.802	738.412	726.063	658.982	481.716	0
100	100	100	100	100	100	100	100

Profile 4A
Matrix iteration with Q = 10000; k = 40
Iteration = 28
Computing times = 0. 010513

0	0	0	0	0	0	0	0
20	118.547	93.143	81.295	68.946	51.323	27.689	0
20	180.794	172.482	162.842	142.914	108.408	59.182	0
20	231.897	252.898	254.426	231.211	179.963	100.381	0
20	293.645	352.537	370.505	347.289	279.602	162.129	0
20	389.896	492.851	527.517	507.589	428.777	268.284	0
20	572.837	701.203	738.873	726.524	659.382	481.979	0
100	100	100	100	100	100	100	100





The resulting number of iterations 28 times. Second heat distribution profile to show the temperature decreases in the same area or exactly at a node $T_{6,1}$.
When observed more than a second variation of the above limits, namely: 0; 20; 100; 0 and 20; 0; 0; 100 produce the same number of iterations equilibrium which is 28 times. And when the second iteration matrix deductible would generate diagonal matrix with value 0.

-90.858	-41.821	-12.350	12.350	41.821	90.858
-121.612	-64.073	-19.928	19.928	64.073	121.612
-131.515	-72.935	-23.216	23.216	72.935	131.515
-131.516	-72.935	-23.216	23.216	72.935	131.516
-121.612	-64.073	-19.928	19.928	64.073	121.612
-90.858	-41.820	-12.349	12.349	41.820	90.858

-90.858	-41.820	-12.349	12.349	41.820	90.858
-121.612	-64.074	-19.928	19.928	64.074	121.612
-131.516	-72.935	-23.215	23.215	72.935	131.516
-131.516	-72.935	-23.216	23.216	72.935	131.516
-121.612	-64.074	-19.928	19.928	64.074	121.612
-90.858	-41.821	-12.349	12.349	41.821	90.858

Both matrices show has a positive value on right side and negative on left side. And both showed the smallest iteration value in the same area.

Results from this study is that the pattern of variation of the boundary or initial conditions of a material with regard conductivity value remains at steady state the exact value of the smallest are in the same iteration value. This indicates that the value of the thermal equilibrium tend to be at the same iteration.
A recommendation for further research is by varying the initial conditions more and include different conductivity.

3. Refference

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