The Shannon entropy information for mixed Manning Rosen potential in D-dimensional Schrodinger equation

A Suparmi, C Cari, Beta Nur Pratiwi and Dewanta Arya Nugraha
Physics Department, Graduate Program, Sebelas Maret University, Indonesia

Email: soeparmi@staff.uns.ac.id

Abstract. D dimensional Schrodinger equation for the mixed Manning Rosen potential was investigated using supersymmetric quantum mechanics. We obtained the energy eigenvalues from radial part solution and wavefunctions in radial and angular parts solution. From the lowest radial wavefunctions, we evaluated the Shannon entropy information using Matlab software. Based on the entropy densities demonstrated graphically, we obtained that the wave of position information entropy density moves right when the value of potential parameter q increases, while its wave moves left with the increase of parameter α. The wave of momentum information entropy densities were expressed in graphs. We observe that its amplitude increase with increasing parameter q and α

1. Introduction

The Schrodinger equation or Dirac equation in the case of spin and pseudo spin symmetry which are reduced to Schrodinger like equation for a class of shape invariant potentials are solvable using supersymmetric quantum mechanic (SUSY QM) [1-8], Nikiforov-Uvarov method [9-10], Factorization methods [11-12] and Asymptotic Iteration Method (AIM) [13-17]. SUSY QM can be considered very similar to Factorization method. In addition, the derivation of energy eigenvalue, in AIM and SUSY QM method also very similar. Moreover, some methods are derived from generalized hypergeometric equation.

D-dimensional Schrodinger equation, Klein Gordon equation and Dirac equation in the case of spin and pseudo spin symmetry, which are reduced into one dimensional Schrodinger equation, are recently attractive for some researcher. Even the direction of this problem is not clear enough but it is worthy to be put on consideration. In this paper, we will obtain the energy eigen value and wavefunctions of single particle that is governed by separable D-dimensional mixed Manning Rosen non-central potential which are defined as

$$V(r,\theta_1,\theta_2,...,\theta_{D-1})=\frac{\hbar^2}{2M}\left\{V(r)+\frac{1}{r^2}\sum_{i=1}^{D-3}\left(\frac{V_i(\theta_i)}{\sin^2\theta_{i+1}...\sin^2\theta_{D-1}}\right)+\frac{1}{r^2}\frac{V_{D-2}(\theta_{D-2})}{\sin^2\theta_{D-1}}+V_{D-1}(\theta_{D-1})\right\} \tag{1}$$

where

$$V(r)=\alpha^2\left(\frac{v(v-1}{\sin^2\alpha r}+\frac{\gamma(\gamma-1)}{\cos^2\alpha 2}+2q\tan\alpha r+2p\cot\alpha r\right) \tag{2}$$

$$V_i(\theta_i)=\left(\frac{v_i(v_i-1)}{\sin^2\theta_i}+\frac{\gamma_i(\gamma_i-1)}{\cos^2\theta_i}+2q_i\tan\theta_i+2p_i\cot\theta_i\right) \tag{3}$$

for $i=1,2,3...$ The Manning Rosen potential is used as a mathematical model in the description of diatomic molecular vibrations and it constitutes a convenient model for other physical situations. [18-

21]. Radial wavefunction will be used to investigated the Shannon entropy information. The Shannon entropic plays an important rule in the measure of uncertainly. [22-24]

In this paper, solution of Schrodinger equation in D dimension for mixed Manning- Rosen potential was studied using SUSY QM. The Shannon entropy was calculated for explaining about the uncertainty system. The paper is organized as follows. The basic theory is presented in section 2. The results and discussion are presented in section 3 and a conclusion in section 4.

2. Basic theory

2.1. *The D-dimensional Schrodinger equation*

D dimensional Schrodinger is separated into 1 one -dimensional radial Schrodinger part, below [25]

$$\left\{ r^2 \frac{1}{r^{D-1}} \frac{\partial}{\partial r}\left(r^{D-1} \frac{\partial}{\partial r} \right) - r^2 \left(V(r) - \frac{2m}{\hbar^2} E \right) \right\} R = \lambda_{D-1} R \tag{4}$$

and (D-1) one- dimensional angular Schrodinger equations as follow

$$\frac{\partial^2 P_1}{\partial \theta_1^2} - V_1(\theta_1) P_1 + \lambda_1 P_1 = 0 \tag{5}$$

$$\left\{ \frac{1}{\sin\theta_2} \frac{\partial}{\partial\theta_2} \left(\sin\theta_2 \frac{\partial P_2}{\partial\theta_2} \right) \right\} - \mathrm{V}_2(\theta_2)\,\mathrm{P}_2 - \frac{\lambda_1 P_2}{\sin^2\theta_2} + \lambda_2 P_2 = 0 \tag{6}$$

$$\left[\frac{1}{\sin^2\theta_3} \frac{\partial}{\partial\theta_3} \left(\sin^2\theta_3 \frac{\partial \mathrm{P}_3(\theta_3)}{\partial\theta_3} \right) \right] - V_3(\theta_3)\mathrm{P}_3(\theta_3) - \frac{\lambda_2}{\sin^2\theta_3} \mathrm{P}_3(\theta_3) + \lambda_3 \mathrm{P}_3(\theta_3) = 0 \tag{7}$$

………

$$\frac{1}{\sin^{D-2}\theta_{D-1}} \frac{\partial}{\partial\theta_{D-1}} \left(\sin^{D-2}\theta_{D-1} \frac{\partial P_{D-1}}{\partial\theta_{D-1}} \right) - V_{D-1}(\theta_{D-1}) P_{D-1} - \frac{\lambda_{D-2} P_{D-1}}{\sin^2\theta_{D-1}} + \lambda_{D-1} P_{D-1} = 0 \tag{8}$$

with $\lambda_1 . \lambda_2,\ \lambda_3, ..., \lambda_{D-1}$ are variable separation constant.

2.2. *Supersymmetric quantum mechanics*

Each of the Schrodinger is solved by using SUSY QM [26] and the properties of shape invariant potential as follows the SUSY partner potentials

$$\begin{aligned} V_-(x) &= w^2(x) - \frac{\hbar}{\sqrt{2M}} w'(x) \\ V_+(x) &= w^2(x) + \frac{\hbar}{\sqrt{2M}} w'(x) \end{aligned} \tag{9}$$

where
$$V(x) = V_-(x) + E_0 \tag{10}$$

and w is super potential.

And by applying the condition of shape invariant potential which is defined as to obtain the mapping parameter:

$$V_+(a_0, x) - V_-(a_1, x) = R(a_1) \tag{11}$$

the energy eigenvalue can be obtained, following equation (12)

$$E_n = \sum_{k=1}^{k=n} R(a_k) + E_0 \tag{12}$$

Then for obtain the wavefunctions we used the lowering and raising operators,

$$A = \frac{\hbar}{\sqrt{2M}}\frac{\partial}{\partial x} + w(a_0, x); \; A^+ = -\frac{\hbar}{\sqrt{2M}}\frac{\partial}{\partial x} + w(a_0, x)$$
$$A\psi_0 = A\psi_0^{(-)} = 0 \; ; \; A^+(a_0, x)\psi_0^{(-)}(a_1, x) = \psi_1^{(-)} \tag{13}$$

2.3. Shanon entropy

Shannon's information entropy which was proposed in the late 1940s has attracted many physicists [22-24] Shannon proposed the idea of probabilities that was used to measure uncertain of events and also proposed the information entropy to measure the uncertainty.

The information entropy consist of position entropy density $\rho(r)$ and momentum entropy density $\rho(p)$ are defined below

$$\rho(r) = |\Psi(r)|^2 \ln|\Psi(r)|^2 \tag{14}$$

$$\rho(p) = |\phi(p)|^2 \ln|\phi(p)|^2 \tag{15}$$

$\Psi(r)$ is the normalized eigenstate of position and $\phi(p)$ is the corresponding Fourier transform, with r is radial radius and p is momentum.

3. Solution and discussion

3.1. Solution of radial part

By using equation (2) and equation (4), the general solution for four dimensional Schrodinger equation is given as

$$\frac{1}{r^3}\frac{\partial}{\partial r}\left(r^3\frac{\partial R}{\partial r}\right) - \alpha^2\left(\frac{v(v-)}{\sin^2\alpha r} + \frac{\gamma(\gamma-1)}{\cos^2\alpha r} + 2q\tan\alpha r + 2p\cot\alpha r\right)R + \frac{2M}{\hbar^2}ER - \frac{\lambda_3}{r^2}R = 0 \tag{16}$$

By setting $R = \dfrac{u}{r^{3/2}}$ in equation (16) we have

$$u'' - \frac{3/4 + \lambda_3}{r^2}u - \alpha^2\left(\frac{v(v-1)}{\sin^2\alpha r} + \frac{\gamma(\gamma-1)}{\cos^2\alpha r} + 2q\tan\alpha r + 2p\tan\alpha r\right)u = -\frac{2ME}{\hbar^2}u \tag{17}$$

By applying centrifugal term approximation defined as $\dfrac{1}{r^2} \cong \dfrac{\alpha^2}{\sin^2\alpha r}$, then equation (17) becomes

$$u'' - \alpha^2\left(\frac{v(v-1) + 3/4 + \lambda_3}{\sin^2\alpha r} + \frac{\gamma(\gamma-1)}{\cos^2\alpha r} + 2q\tan\alpha r + 2p\tan\alpha r\right)u = -\frac{2ME}{\hbar^2}u \tag{18}$$

By using equations (9-13) we get the super potential, the super partner potential V_+ and V_- mapping para meter, raising and lowering operators as

$$w(r) = \gamma\alpha\tan\alpha r - v\alpha\cot\alpha r + \alpha\frac{q}{\gamma} \tag{19}$$

$$V_-(r) = \frac{\hbar^2\alpha^2}{2M}\left(\frac{v'(v'-1)}{\sin^2\alpha r} + \frac{\gamma(\gamma-1)}{\cos^2\alpha r} + 2q\tan\alpha r + 2p\cot\alpha r\right) - \frac{\hbar^2\alpha^2}{2M}(v+\gamma)^2 + \frac{\hbar^2\alpha^2}{2M}\frac{q^2}{\gamma^2} \tag{20}$$

$$V_+(r) = \frac{\hbar^2\alpha^2}{2M}\left(\frac{v'(v'+1)}{\sin^2\alpha r} + \frac{\gamma(\gamma+1)}{\cos^2\alpha r} + 2q\tan\alpha r + 2p\cot\alpha r\right) - \frac{\hbar^2\alpha^2}{2M}(v+\gamma)^2 + \frac{\hbar^2\alpha^2}{2M}\frac{q^2}{\gamma^2} \tag{21}$$

where
$$v' = \frac{1}{2} + \sqrt{v(v-1) + 1 + \lambda_3} \tag{22}$$

so we get the mapping parameters given as

$$a_0 = v', a_1 = v'+1, ..., a_n = v'+n; \quad b_0 = \gamma, b_1 = \gamma+1, ..., b_n = \gamma+n \tag{23}$$

The lowering and rising operator is determined by,

$$A = \frac{\hbar}{\sqrt{2M}}\frac{\partial}{\partial r} + \frac{\hbar}{\sqrt{2M}}\left(\gamma\alpha\tan\alpha r - v'\alpha\cot\alpha r + \alpha\frac{q}{\gamma}\right);$$
$$A^{+} = -\frac{\hbar}{\sqrt{2M}}\frac{\partial}{\partial r} + \frac{\hbar}{\sqrt{2M}}\left(\gamma\alpha\tan\alpha r - v'\alpha\cot\alpha r + \alpha\frac{q}{\gamma}\right) \tag{24}$$

By using SUSY, we obtained

$$E_0 = \frac{\hbar^2\alpha^2}{2M}\left(v'+\gamma\right)^2 - \frac{\hbar^2\alpha^2}{2M}\frac{q^2}{\gamma^2} \tag{25}$$

So the energy eigenvalue was obtained from equations (12, 23 and 25) ,

$$E_n = \frac{\hbar^2\alpha^2}{2M}\left(v'+\gamma+2n\right)^2 - \frac{\hbar^2\alpha^2}{2M}\frac{q^2}{(\gamma+n)^2} \tag{26}$$

From equation (26,) it is shown that the energy eigenvalue depends on the parameters of all components of the composed potential and also depend on the quantum number *n*.

And then from equations (19, 24) we get the ground state wave function as

$$u_0 = C_0\left(\sin\alpha r\right)^{v'}\left(\cos\alpha r\right)^{\gamma} e^{-\frac{\alpha q}{\gamma}r} \tag{27}$$

and by using raising operator, we get the first exited wave function

$$u_1 = -\frac{\hbar\alpha}{\sqrt{2M}}C_1\left\{\begin{array}{l}\left(2v'+1\right)\left(\sin\alpha r\right)^{v'}\left(\cos\alpha r\right)^{\gamma+2} - \left(2\gamma+1\right)\left(\sin\alpha r\right)^{v'+2}\left(\cos\alpha r\right)^{\gamma} \\ -\left(\frac{q\alpha}{\gamma+1}+\frac{q}{\gamma}\right)\left(\sin\alpha r\right)^{v'+1}\left(\cos\alpha r\right)^{\gamma+1}\end{array}\right\} e^{-\frac{\alpha q}{\gamma+1}r} \tag{28}$$

3.2. Solution of angular part

From equation (3) and equations (5-7), we rewrote the angular part equations below,

$$\frac{\partial^2 P_1}{\partial\theta_1^2} - \left\{\frac{v_1(v_1-1)}{\sin^2\theta_1} + \frac{\gamma_1(\gamma_1-1)}{\cos^2\theta_1} + 2q_1\tan\theta_1 + 2p_1\cot\theta_1\right\}P_1 + \lambda_1 P_1 = 0 \tag{29}$$

$$\left\{\frac{1}{\sin\theta_2}\frac{\partial}{\partial\theta_2}\left(\sin\theta_2\frac{\partial P_2}{\partial\theta_2}\right)\right\} - \left\{\frac{v_2(v_2-1)}{\sin^2\theta_2} + \frac{\gamma_2(\gamma_2-1)}{\cos^2\theta_2} + 2q_2\tan\theta_2 + 2p_2\cot\theta_2\right\}\mathrm{P}_2$$
$$-\frac{\lambda_1 P_2}{\sin^2\theta_2} + \lambda_2 P_2 = 0 \tag{30}$$

$$\left[\frac{1}{\sin^2\theta_3}\frac{\partial}{\partial\theta_3}\left(\sin^2\theta_3\frac{\partial \mathrm{P}_3}{\partial\theta_3}\right)\right] - \left\{\frac{v_3(v_3-1)}{\sin^2\theta_3} + \frac{\gamma_3(\gamma_3-1)}{\cos^2\theta_3} + 2q_3\tan\theta_3 + 2p_3\cot\theta_3\right\}\mathrm{P}_3$$
$$-\frac{\lambda_2}{\sin^2\theta_3}\mathrm{P}_3 + \lambda_3\mathrm{P}_3 = 0 \tag{31}$$

and by repeating the steps used to solve radial Schrodinger equation we get the angular wave functions and the constant of variable separation as follows. By solving equation (29) using SUSY QM we obtain the ground state wave function P_{10} and the constant of variable separation as

$$P_{10}(\theta_1) = C_{10}\left(\sin\theta_1\right)^{v_1}\left(\cos\theta_1\right)^{\gamma_1} e^{-\frac{q_1}{\gamma_1}\theta_1}; \lambda_1 = \left(\gamma_1 + v_1 + 2n_1\right)^2 - \frac{q_1^2}{\left(\gamma_1 + n_1\right)^2} \tag{32}$$

By setting $P_2 = \frac{Q_2}{\sqrt{\sin\theta_2}}$ and $P_3 = \frac{Q_3}{\sin\theta_3}$ in equations (30) and (31) the both equations becomes

$$Q_2'' - \left\{\frac{v_2(v_2-1)+\lambda_1 - 1/4}{\sin^2\theta_2} + \frac{\gamma_2(\gamma_2-1)}{\cos^2\theta_2} + 2q_2\tan\theta_2 + 2p_2\cot\theta_2\right\}Q_2 + \left(\lambda_2 + 1/4\right)Q_2 = 0 \tag{33}$$

$$Q_3'' - \left\{\frac{v_3(v_3-1)+\lambda_2}{\sin^2\theta_3} + \frac{\gamma_3(\gamma_3-1)}{\cos^2\theta_3} + 2q_3\tan\theta_3 + 2p_3\cot\theta_3\right\}Q_3 + \left(\lambda_3 + 1\right)Q_3 = 0 \tag{34}$$

By setting $v'_2(v'_2-1) = v_2(v_2-1)+\lambda_1 - 1/4$ or $v'_2 = 1/2 + \sqrt{v_2(v_2-1)+\lambda_1}$ and $v'_3 = 1/2 + \sqrt{(v_3-1/2)^2 + \lambda_2}$, these two equations are solved using SUSY QM and we get the lowest wave functions and the variable separation constants as

$$Q_{20} = C_{20}\left(\sin\theta_2\right)^{v'_2}\left(\cos\theta_2\right)^{\gamma_2} e^{-\frac{q_2}{\gamma_2}\theta_2}; \lambda_2 = \left(v'_2 + \gamma_2 + 2n_2\right)^2 - \frac{q_2^2}{\left(\gamma_2 + n_2\right)^2} \tag{35}$$

$$Q_{30} = C_{30}\left(\sin\theta_3\right)^{v'_3}\left(\cos\theta_3\right)^{\gamma_3} e^{-\frac{q_3}{\gamma_2}\theta_3}; \lambda_3 = \left(v'_3 + \gamma_3 + 2n_3\right)^2 - \frac{q_3^2}{\left(\gamma_3 + n_3\right)^2} \tag{36}$$

The lowest total wavefunction from equation (27), equation (32) and equations (35-36) is given as

$$\begin{aligned}\psi(r,\theta_1,\theta_2,\theta_3) = C\alpha^{3/2}\frac{\left(\sin\alpha r\right)^{v'}\left(\cos\alpha r\right)^{\gamma} e^{-\frac{\alpha q}{\gamma}r}}{\left(\sin\alpha r\right)^{3/2}}\left(\sin\theta_1\right)^{v_1}\left(\cos\theta_1\right)^{\gamma_1} e^{-\frac{q_1}{\gamma_1}\theta_1} \\ \left(\sin\theta_2\right)^{v'_2-1/2}\left(\cos\theta_2\right)^{\gamma_2} e^{-\frac{q_2}{\gamma_2}\theta_2}\left(\sin\theta_3\right)^{v'_3-1}\left(\cos\theta_3\right)^{\gamma_3} e^{-\frac{q_3}{\gamma_2}\theta_3}\end{aligned} \tag{37}$$

with $C = C_0C_{10}C_{20}C_{30}$ which is normalization factor of the total lowest wave function.

3.3. The Shannon information entropy of one-dimensional Manning Rosen potential

For the case of $\lambda = 2$ and $v' - 3/2 = 2$, we obtain the radial wavefunction given as

$$\psi_0(x) = C_0\left(\sin\alpha r\right)^2\left(\cos\alpha r\right)^2 e^{-\frac{\alpha q}{2}r} \tag{38}$$

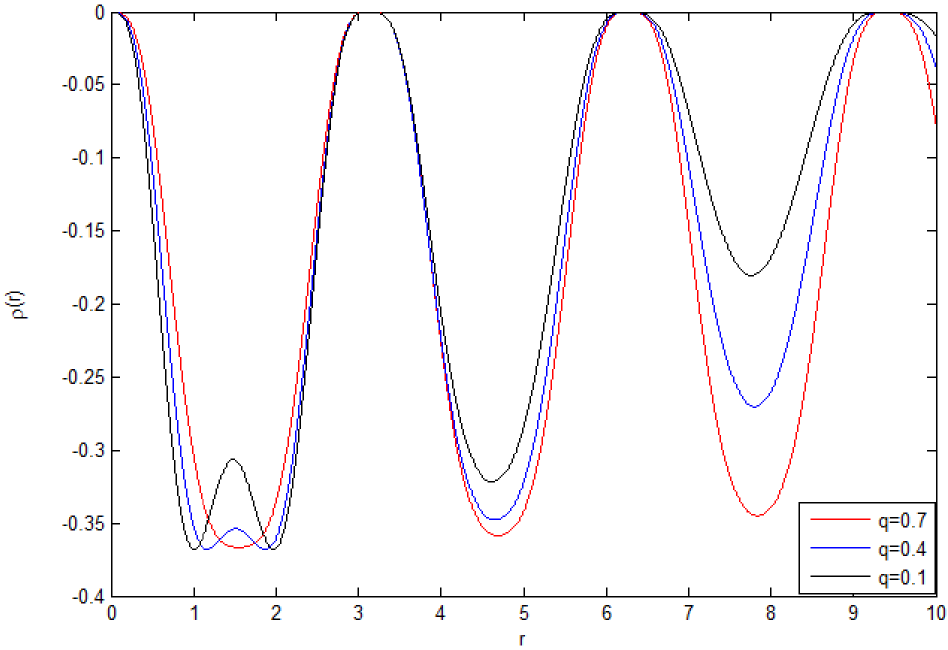
and the momentum eigenstate is obtained by using Fourier transform on position eigenstate as follows

$$\phi_0(p) = \int e^{-ipx}C_0(\sin\alpha x)^2(\cos\alpha x)^2 e^{-\frac{\alpha q}{2}x}dx \tag{39}$$

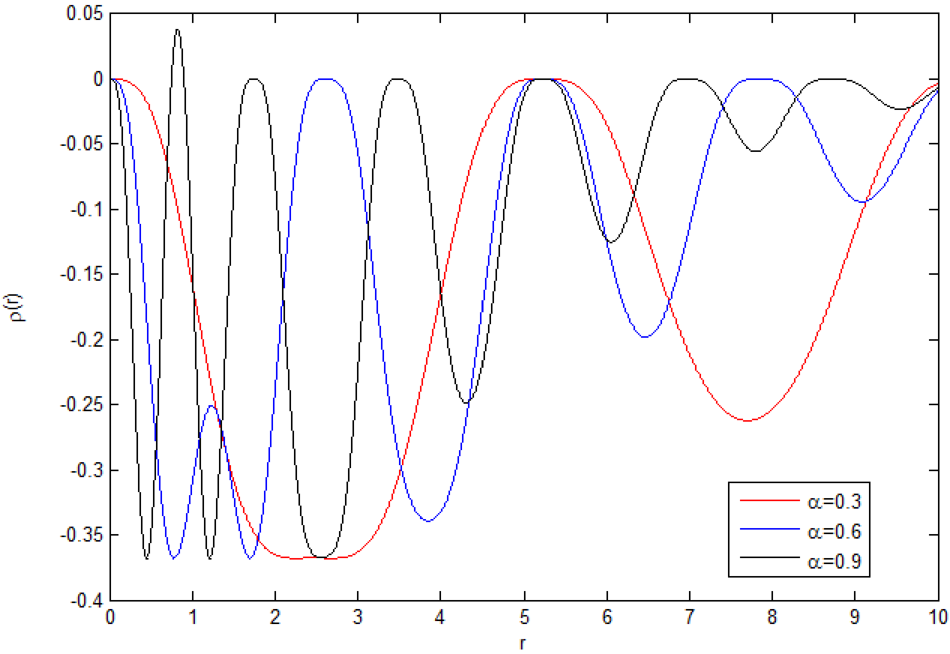
so we obtained

$$\phi_0\left(p\right) = \frac{1}{4}C_0\left\{-\frac{-ip+\frac{\alpha q}{2}}{p^2+\left(\frac{\alpha q}{2}\right)^2} + \left(\frac{\left(ip+\alpha q/2\right)\cos 4\alpha x + 4\alpha\sin 4\alpha x}{-p^2+\alpha^2q^2/4+16\alpha^2+ip\alpha q}\right)\right\}\left(e^{-x\left(ip+\alpha q/2\right)}\right) \tag{40}$$

The Shannon position information entropy density was obtained numerically from equation (14) and equations (38) using Matlab software, which can be shown in Figure 1.



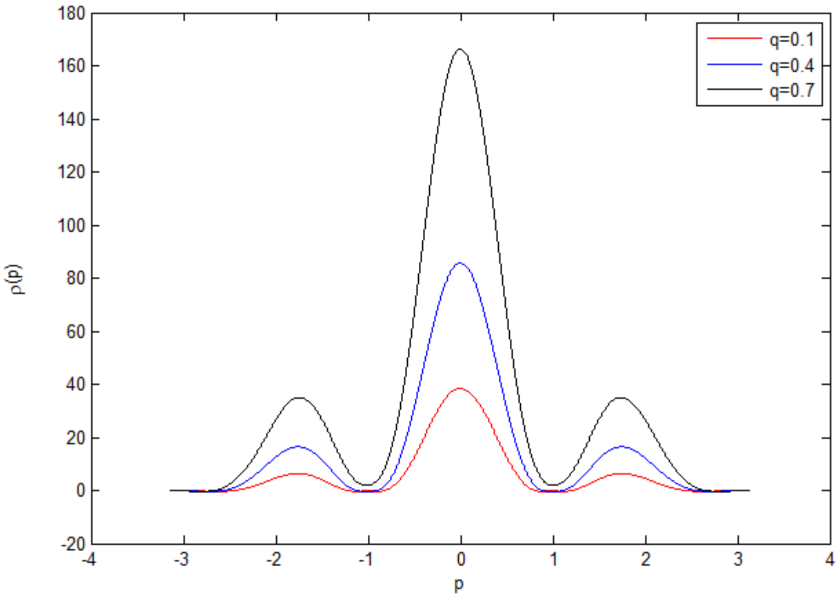
(a)



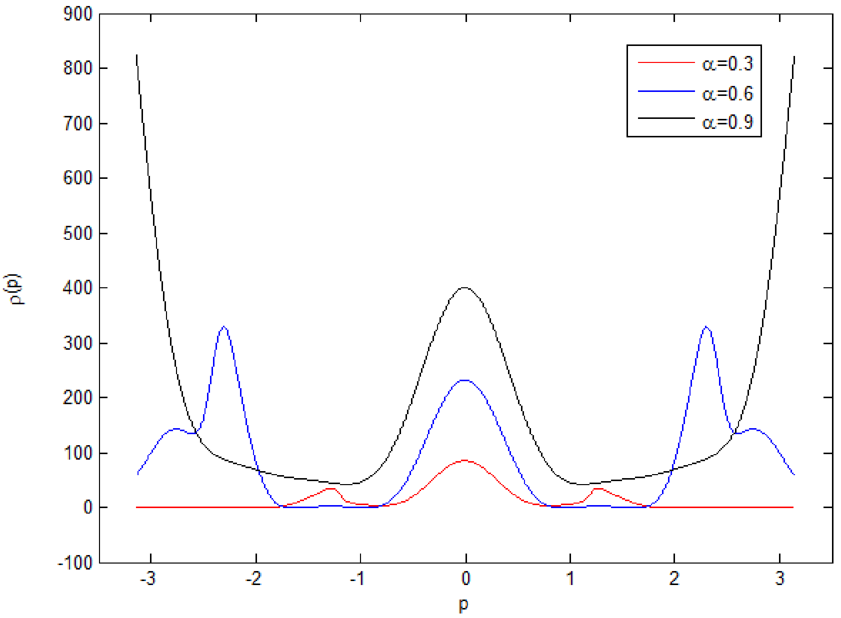
(b)

Figure 1. Plot of the position space entropy density $\rho(r)$ of the Manning Rosen potential for (a) variation of q (b) variation of α

From Figure (1), we observe that the wave of position information entropy density moves right when the value of potential parameter q increases (Figure 1.a), while its wave moves left with the increase of parameter α (Figure 1.b). And then by using equation (15) and equation (40), we found the graphically result in Figure (2) for the momentum information entropy density. It is shown that the wave amplitude increase with increasing parameter q and α



(a)



(b)

Figure 2. Plot of the momentum entropy density $\rho(p)$ of the Manning Rosen potential for (a) variation of q (b) variation of α

4. Conclusion

In this paper, we have presented the solution of D-dimensional Schrodinger for the mixed Manning Rosen equation using supersymmetric quantum mechanics. We obtained the energy eigenvalues from radial part solution, which the energy eigenvalue depends on the parameters of all components of the composed potential and also depend on the quantum number n. The radial wavefunctions and angular wavefunctions was obtained using lowering and raising operator. We presented the Shannon entropy information graphically using Matlab software. We obtained that the wave of position information entropy density moves right when the value of potential parameter q increases, while its wave moves left with the increase of parameter α. While for the wave amplitude increase with incresing parameter *q* and α

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References

[1] Wu Q S, Wen H B, Yun W D and Yu G Z 2002 *Comm. Theor. Phys*.(China) **38** 139
[2] Sheng J C, Ying Z, Lin Z X and Tian S L 2001 *Commun .Theor. Phys*. **36** 641-646
[3] Suparmi and Cari 2014 *J. Math. Fund. Sci.* **46** 3 205-223
[4] Cari C and Suparmi A 2014 *AIP Conference Proceedings* **1615** 101
[5] Alemgadmi K I K, Suparmi, Cari, and Deta U A 2015 *AIP Conference Proceedings* **1677** 030004
[6] Dutt R, Khare A and Sukhatme U P 1988 *Am. J. Phys*. **56** 163
[7] Suparmi A, Cari C and Pratiwi B N 2016 *Journal of Physics: Conference Series* **710** 012026
[8] Gangopadhyaya A, Mallow J V, Rasinariu C and Sukhatme U P 2001 *Chin. J. Phys.* **39**
[9] Nikiforov A E and Uvarov V B 1988 *Special Function in Mathematical Physics* (Birkhausa: Basel)
[10] Falaye B J, Oyewumi K J and Abbas M 2013 arXiv:1109.3894v4 [quant-ph]
[11] Sadeghi J 2007 *Act.Phys.Polonica A* **112** (1) 23
[12] Sadeghi J and Pourhassan B 2008 EJTP **5** (17) 193
[13] Ciftci H, Hall R L and Saad N 2003 *Journal of Physics A: Mathematical and General* **36** 11807
[14] Ikhdair S M, Falaye B J and Adepoju A G 2013 arXiv:1308.0155v1 [quant-ph]
[15] Oyewumi K J, Falaye B J, Onate C A, Oluwadare O J and Yahya W A 2014 arXiv:1402.2181v1 [quant-ph]
[16] Rostami A and Motavali H 2008 *Progress in Electromagnetic Research B* 4 171
[17] Soylu A, Bayrak O and Boztosun I 2008 *Journal of Physics A: Mathematical and Theoretical* **41** 065308
[18] Leroy R J and Bernstein R B 1970 *The Journal of Chemical Physics* **52**, 3869
[19] Cai J M, Cai P Y and Inomata A 1986 *Physical Review A* **34**, 4621
[20] Ikhdair S M 2011 *ISRN Mathematical Physics* **2012** 201525
[21] Chen Z Y, Li M and Jia C S 2009 *Mod. Phys. Lett. A* **24**, 1863
[22] Sun G H, Aoki M V and Dong S H 2013 *Chin. Phys. B* **22** 050302
[23] Shannon C E 1948 *Bell Syst. Tech. J.* **27** 623
[24] Shannon C E 1993 *Collected Papers* (New York :IEEE Press)
[25] Shi-Hai Dong 2011 Waves Equation in Higher Dimension (Springer Science: Business Media) pp 41-43, pp 101-102, p 97
[26] Witten E 1981 *Nucl. Phys. B* **188** 513