IOP Conf. Series: Journal of Physics: Conf. Series **793** (2017) 012033 doi:10.1088/1742-6596/793/1/012033

Atomic spectroscopy in periodic fields

V I Yudin[1-3], M Yu Basalaev[1,2] and A V Taichenachev[1,2]

[1] Novosibirsk State University, 2 Pirogova str., Novosibirsk 630090, Russia
[2] Institute of Laser Physics SB RAS, 13/3 Acad. Lavrentyev ave., Novosibirsk 630090, Russia
[3] Novosibirsk State Technical University, 20 K. Marx ave., Novosibirsk 630073, Russia

E-mail: mbasalaev@gmail.com

Abstract. Using the density matrix formalism, we prove the existence of the periodic steady-state for an arbitrary periodically driven system described by linear dynamic equations. The presented derivation simultaneously contains a simple and effective computational algorithm, which automatically guarantees a full account of all frequency components.

For the last several decades rapid scientific and technological progress has been substantially connected with an expansion of the lasers and laser technologies at different areas of the science, engineering, and industry. Many impressive successes in these directions are due to the theoretical support, motivation, and interpretation of experimental researches. In this context, of paramount importance is the formulation of mathematical models (equations) and finding of their solutions, which adequately describe the physical picture of investigated problems. During long time, steady states (which arises under the interaction of a quantum system with stationary external fields) play a key role in the theoretical description of the basic problems in laser physics and spectroscopy (for example, see [1–3]). However, in the last few years the devices in which different parameters of electromagnetic fields are periodically modulated have gained a greater importance. First of all, the so-called frequency comb generators use the periodic pulse modulation of a laser field. Such sources of pulse radiation are actively used now in modern atomic clocks for frequency measurements, and they have promising perspectives for direct frequency comb spectroscopy. Also, the phase (frequency) and/or amplitude periodic modulation of the laser field is now widely used for different tasks and applications (including atomic clocks and magnetometers). In all these examples the standard concept of steady state based on the time-independent equation is inapplicable.

We generalize the steady-state concept for an arbitrary quantum system under arbitrary periodic external influence. In this way we prove the following existence theorem: if the coefficients of density matrix dynamic equation

$$\frac{\partial}{\partial t}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H},\hat{\rho}(t)] + \hat{\Gamma}\{\hat{\rho}(t)\}, \qquad \mathrm{Tr}\{\hat{\rho}(t)\} = 1 \tag{1}$$

have the period T, then the periodic solution with the same period T exists always. Acompletely unexpected result is that so universal and fundamental a statement is based only on the normalization condition for the density matrix. Due to the relaxation processes this solution is realized as an asymptotics ($t \to +\infty$) and, therefore, can be characterized as a periodic steady state. The developed simple algorithm allows us to directly construct this solution independently of initial conditions and without the use of either Floquet or Fourier formalisms. Our approach considerably simplifies the analysis regardless of the periodic modulation character: from smoothly harmonic type to ultrashort pulses.

Some authors supposed (without proof) the existence of the periodic steady-state solution for some certain problems. In this case they usually used the Fourier analysis for numerical calculations). However, an intuitive assumption about the periodic steady state is now rigorously substantiated [4]. At the same time, of special interest is the direct and simple method, which allows us to construct the periodic solution without Fourier expansion.

Let us describe one possible numerical algorithm allowing to construct the periodic solution of equation (1). First, rewrite the differential equation (1) for the density matrix in the vector form:

$$\frac{\partial}{\partial t}\vec{\rho}(t) = \hat{L}(t)\vec{\rho}(t)\,, \qquad \mathrm{Tr}\{\hat{\rho}(t)\} = 1\,, \tag{2}$$

where the column vector $\vec{\rho}(t)$ is formed by the matrix elements $\rho_{ab}(t)$ using some definite rule, the linear operator $\hat{L}(t)$ corresponds to the right-hand member of equation (1). In accordance with equation (2), for other instant of time t_2 we can write

$$\vec{\rho}(t_2) = \hat{A}(t_2,t_1)\vec{\rho}(t_1) \tag{3}$$

where the two-time evolution operator $\hat{A}(t_2,t_1)$ is determined by the matrix $\hat{L}(t)$. Consider $\vec{\rho}(t)$ at arbitrary instant of time t. In conformity with equation (3), the vector $\vec{\rho}(t+T)$ is determined as

$$\vec{\rho}(t+T) = \hat{A}(t+T,t)\vec{\rho}(t)\,, \tag{4}$$

where$T$ is time period in the operator $\hat{L}(t)$, i.e. $\hat{L}(t+T) = \hat{L}(t)$. Supposing the existence of the periodic solution $\vec{\rho}(t+T) = \vec{\rho}(t)$, it follows from equation (4) that this solution satisfies the equation

$$\vec{\rho}(t) = \hat{A}(t+T,t)\vec{\rho}(t)\,, \qquad \mathrm{Tr}\{\hat{\rho}(t)\} = 1\,, \tag{5}$$

which always has a nonzero solution as it has been proven in our work [4]. We consider an arbitrary periodic dependence of the operator $\hat{L}(t)$. For instance, under an atom-field interaction such a dependence can be produced by the modulation of the field parameters (amplitude, phase, polarization, etc.).

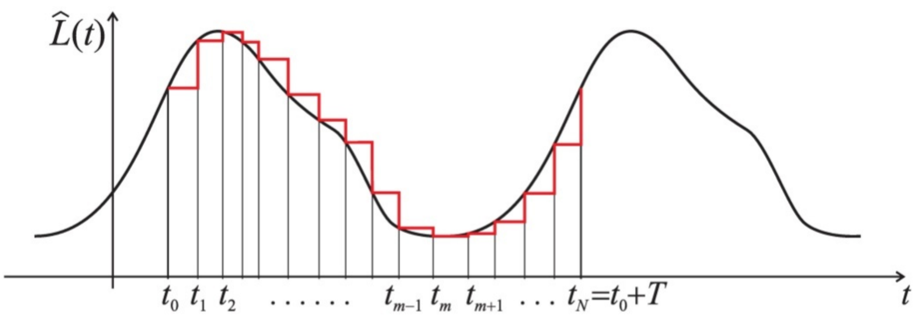

Figure 1. Partition of the time interval [t_0,t_0 + T] at N subintervalsand symbolic approximation of the dependence $\hat{L}(t)$ by the step function (red step line).

The selected time interval [t_0, t_0 + T] is divided into N small subintervals, where t_N = t_0 + T. The character of partition (uniform or nonuniform discrete mesh) and number of subintervals are determined in conformity with the studied problem. The dependence $\hat{L}(t)$ we will approximate by step function where the matrix $\hat{L}(t)$ has the constant value $\hat{L}(t_{m-1})$ inside of subinterval (t_{m-1}, t_m]. In this case the vector $\vec{\rho}(t_0)$ in initial point $t_0$is determined by equation (5), where the evolution operator $\hat{A}(t_0+T,t_0)$ has the form of a chronologically ordered product of the matrix exponents:

$$\hat{A}(t_0+T,t_0) \approx \prod_{m=1}^{m=N} \mathrm{e}^{(t_m-t_{m-1})\hat{L}(t_{m-1})} = \mathrm{e}^{(t_N-t_{N-1})\hat{L}(t_{N-1})} \times \ldots \times \mathrm{e}^{(t_1-t_0)\hat{L}(t_0)} \tag{6}$$

The vectors $\vec{\rho}(t_m)$ in other points of the interval [t_0, $t_0 + T$] are determined by the recurrence relation

$$\vec{\rho}(t_m) = \mathrm{e}^{(t_m-t_{m-1})\hat{L}(t_{m-1})}\, \vec{\rho}(t_{m-1}) \tag{7}$$

In summary, in the framework of density matrix formalism we have rigorously proven the existence theorem of the periodic steady state for an arbitrary periodically driven system. Due to the relaxation processes this state is realized as an asymptotic ($t \to +\infty$) independently of initial conditions, i.e., periodicity is the main attribute of steady state. The proof simultaneously contains a computational algorithm, which uses neither Floquet nor Fourier theories. Our method radically simplifies the calculations for arbitrary types of periodic modulation (including the ultrashort pulses) and opens up great possibilities for analysis and development of new methods in laser physics, nonlinear optics, and spectroscopy.As animportant case, the developed method can be applied for calculation of the line shape and field-induced shift of the coherent population trapping resonances, which are widely used in atomic clocks.

Acknowledgments
The work was supported by the Russian Science Foundation (project no. 16-12-10147). The calculations for direct frequency comb spectroscopy of two-level atom were supported by the Russian Foundation for Basic Research (project no. 16-32-60050 mol_a_dk).

References
[1] Demtroder W 2003 *Laser Spectroscopy* (Berlin: Springer-Verlag) p 986
[2] Letokhov V S and Chebotaev V P 1977 *Nonlinear Laser Spectroscopy* (Berlin: Springer-Verlag) p 466
[3] Rautian S G and Shalagin A M 1991 *Kinetic Problems of Nonlinear Spectroscopy* (Amsterdam: North-Holland) p 454
[4] Yudin V I, Taichenachev A V and Basalaev M Yu 2016 *Phys. Rev.* A **93** 013820