

# Quantum-enhanced protocols with mixed states using cold atoms in dipole traps

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**Abstract.** We discuss the use of cold atoms in dipole traps to demonstrate experimentally a particular class of protocols for computation and metrology based on mixed states. Modelling of the system shows that, for a specific class of problems (tracing, phase estimation), a quantum advantage can be achieved over classical algorithms for very realistic conditions and strong decoherence. We discuss the results of the models and the experimental implementation.

## 1. Introduction

Entanglement is widely recognised as a key resource in quantum technology, however an advantage over classical computing can be achieved without it in the presence of non-classical correlations, quantifiable as *discord* [1]. Experiments using a few photonic qubits have shown that specific computational tasks can be efficiently solved even with no entanglement [2].

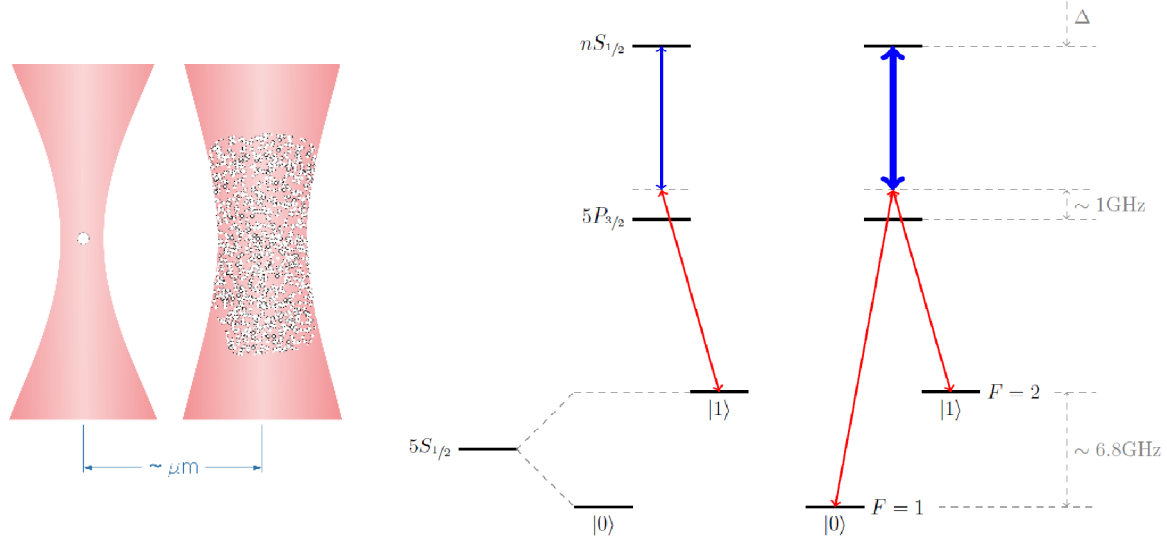
In the past years, there has been outstanding progress in the demonstration of quantum processing based on pure states with a limited number of qubits. However scalability remains an issue, mainly because of decoherence, which scales with the number of qubits. In pure-states quantum computation (QC) this problem can possibly be solved by error correction or using strategies as topologically protecting the qubits. Nevertheless, scaling up to a significant number of qubits and being able to perform a classically intractable calculation has been impossible so far.

As entanglement is very extremely vulnerable to decoherence, the investigation of protocols that are more robust against it is a promising route for progressing the field. One of such protocols is called Deterministic Quantum Computation with 1 clean qubit (or DQC1). This protocol relies on one pure-state control qubit together with a register of completely mixed state qubits, where non-classical correlations are created between the control and the register. DQC1 is a non-universal model of computation (essentially a phase estimation protocol) that can speed up some computational tasks for which no efficient classical algorithms are known. Whilst requiring only a single qubit with coherence, its power scales up with the number of mixed state register qubits. To date, experiments based on photonic implementation of DQC1 have evaluated the normalised trace of a two-by-two unitary matrix [2] and using an NMR implementation, performed the approximation to the Jones polynomial with a system of four qubits, thus experimentally demonstrating the principle of mixed state computation.

We present theoretical models of a cold-atoms based platform for the benchmarking of the protocol with a large number of qubits, thus extending the implementation to large Hilbert space. In comparison to the photonic or NMR experiments, a cold atoms approach offers scalability. Our scheme is based around a single, pure-state control qubit prepared in a tightly confining microscopic optical dipole trap and a register of  $N$  mixed state qubits prepared in a neighbouring trap with more



relaxed confinement, as shown in figure 1. Conditional operation between the control and register qubits is implemented via the long range interaction between highly excited atomic Rydberg states, which are accessed by laser driven transitions. The Rydberg-Rydberg interactions between the control qubit and register qubits of cold atoms in mixed state can activate the implementation of DQC1. An appropriate sequence of laser pulses individually addressing the two traps will implement an algorithm to calculate the trace of a  $2^N \times 2^N$  matrix.



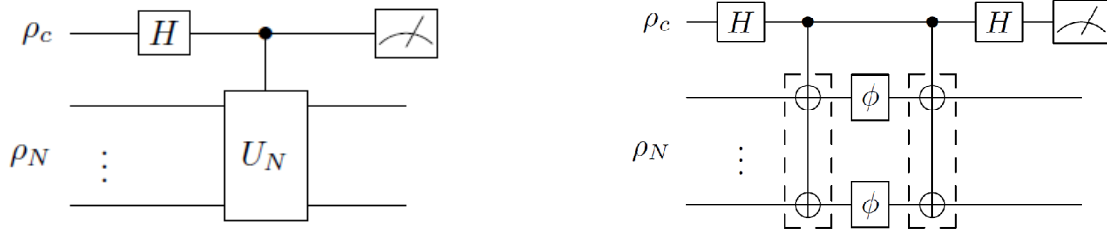
**Figure 1.** *Left panel:* Two dipole traps of different waist can be obtained and filled up with  $N$  atoms or a single atom respectively. If the two traps are positioned at a distance within the range of Rydberg-Rydberg interactions, conditional logic can be performed and a sensitive probe state can be created. *Right panel:* Laser scheme for Rydberg excitation/ Rydberg coupling and conditional logic.

We show that the protocol can be operated with many qubits using a cold atoms setting, and we explore the possibility of tackling non-trivial problems [3], such as many-body physics. The same scheme enables the preparation of quantum enhanced probes for phase estimation and promises high-precision measurement, without relying on quantum entanglement and using highly mixed states [4]. Modelling of this scheme, using cold atoms in dipole traps, demonstrates that the register of partly mixed qubits becomes a powerful resource for phase estimation when supplied with the coherence from the control qubit. A concrete mixed-state model for quantum sensing is also proposed. The scheme can achieve quantum-enhanced precision scaling with the size of the atomic register [5]. We will finally present the full design for this test and our progress in the experimental setup and implementation.

## 2. Computation and metrology with mixed states

DQC1 is a non-universal model of computation based on mixed states. For specific tasks the protocol can offer a quantum advantage that scales with the number of qubits in a maximally mixed state, and can exponentially speed up the processing even with a limited amount of entanglement. DQC1 and DQC1-derived protocols present a remarkable advantage with respect to standard QC protocols, in that they require only a single qubit with non-zero coherence, whilst their power scales up with the number of qubits in mixed state. In sensing applications, DQC1 derived protocols can surpass the standard quantum limit (SQL).

This model of computation was first proposed by Knill and Laflamme [1] and recent work has tried to classify and demonstrate what quantum feature gives rise to the quantum advantage. Although it has been shown that this scheme contains little to no entanglement [9], non-classical correlations are present in the output state of the DQC1 which can be quantified in terms of quantum discord [10].



**Figure 2.** *Left:* Diagram of the DQC1 model of computation. An input single qubit (control qubit) is prepared in a coherent (or partly coherent) state  $\rho_c$  and a register of  $N$  qubits is prepared in the maximally mixed state  $\rho_c = \mathbb{I}_N/2^N$ . The protocol then performs a controlled unitary operation on the register qubits conditional on the state of the control. The normalized trace of the unitary is estimated by measurements on the control qubit only. *Right:* The metrology protocol. The CNOT gate between control and register qubits prepares a probe state that is then let to evolve under interaction with an unknown field. An interferometric scheme then allows to retrieve the phase accumulated during the interaction time. Measurement on the control qubit will retrieve information about the phase and hence recover the field with precision beyond the SQL even in the presence of strong decoherence.

The DQC1 equivalent circuit diagram is shown in figure 2 (left). An input single qubit (control qubit) is prepared in a coherent (or partly coherent) state  $\rho_c$  and a register of  $N$  qubits is prepared in the maximally mixed state  $\rho_c = \mathbb{I}_N/2^N$ . The protocol then performs a controlled unitary operation on the ensemble qubits conditional on the state of the control. The normalized trace of the unitary is estimated by measurements on the control qubit only.

It has been shown theoretically that this model of computation retrieves the trace of  $U_N$  with accuracy scaling with the number  $L$  of measurements on the control qubit, regardless of the number  $N$  of atoms in the mixed state register. In other words, this is a very efficient computational protocol, in that increasing the complexity of the computational task (size of  $U_N$ ) does not increase the overhead in the calculation of its trace. It is remarkable that this protocol brings in a quantum advantage, even if no entanglement is present.

This protocol can be tested and the quantum resources tuned using cold atoms in dipole traps, enabling the conditional logic with Rydberg-Rydberg interactions. This platform has been discussed in [3] and we will review the experimental requirements in the next section.

Furthermore, it has been shown [4] that this protocol can be used effectively for phase estimation. We recently proposed a scheme, where cold atoms in dipole traps can be prepared in a probe state that can reach sensitivities beyond the SQL, even for moderate initial purity of the control and register states [4]. The circuit diagram representing this protocol is shown in figure 2 (right).

The concept that a significant quantum advantage can be achieved even with strongly mixed state qubits is potentially paradigm-shifting for quantum computation and for sensing. If we can understand thoroughly the role of quantum correlations (other than entanglement) in the powering up of quantum protocols, we will be able to bypass the stumbling block of decoherence by using appropriate algorithm (like DQC1 and DQC1-derived).

### 3. A cold atoms platform

We devised a cold-atoms scenario to test and benchmark these protocols. A rubidium magneto-optical trap (MOT) acts as a reservoir to load two optical dipole traps, one to serve as the (single) control qubit and one as a register of  $N$  qubit. In our scheme, the qubits are realised as the hyperfine doublet of the ground state of rubidium atoms. The population of the two traps will be tuned by controlling the trap loading rate (which depends on the number-density of the reservoir, common to both traps) and

by individually controlling the trapping potentials. By these means, the traps will be adjusted so that a different average number of atoms can be achieved in the two traps [11].

The two dipole traps are to be individually laser-addressed so that high-fidelity stimulated Raman rotations [12] can be performed on the control atom and/or on the register atom to prepare a superposition of the ground hyperfine states (thus realising the Hadamard (H) gate in figure 2). It is to be noticed that the level of qubit purity can be tuned both on the control and the register atoms by activating spontaneous scattering on a closed transition (or transitions with different branching ratios), as discussed in [3]. This is a very important feature and allows us to tune the purity of the atomic qubits in each trap, which can range from fully coherent (within experimental limits) to maximally mixed. This tuning of resources is fundamental to understand the role of discord (as opposed to entanglement), the effects of decoherence and ultimately will allow us to explore the boundaries between purely classical to quantum enhanced behavior.

Quantum correlations are built between control and register to either implement a controlled transformation as in figure 2 (left) or a CNOT gate to prepare an  $N+1$  qubit probe state for metrology, as in figure 2 (right). This is done exploiting Rydberg-Rydberg interactions and particularly the phenomenon of *Rydberg blockade* whereby if an atom is excited to Rydberg state, the simultaneous laser excitation of neighbouring atoms is blocked, provided the laser linewidth is small compared to the Rydberg-Rydberg interaction shift. In general for a given Rydberg-Rydberg interaction, a blockade radius is defined. Provided the displacement of all of the register atoms from the control atom fall well within this blockade radius, then a conditional logic scheme based on electromagnetically induced transparency can be used [13, 3].

In figure 1 (right) a level scheme showing the two-photon excitation of the Rydberg state and the EIT-based method of conditional rotation. The qubits are encoded in the hyperfine ground states  $F=1$  (“0”) and  $F=2$  (“1”) of  $^{87}\text{Rb}$ . For the control qubit, state  $F=2$  is coupled to a Rydberg state via 2-photon laser transition. The 0 and 1 states of the  $N$  register qubits are coupled by a 2-photon Raman transition ( $\Lambda$  configuration) scheme following [13], which drives rotations in the computational basis via the  $5P_{3/2}$  intermediate state. Additionally, an intense laser beam coupling the  $5P_{3/2}$  intermediate state to the Rydberg state is added so that the electromagnetically induced transparency condition is fulfilled and the interaction with the rotation beam is inhibited. However, the coupling of the control atom to Rydberg state leads to a strong Rydberg-Rydberg interaction and an additional shift of the Rydberg level that removes the condition for EIT, so that resonant Raman transfer between the 2 qubit states in the register atom is activated.

This scheme is particularly attractive as it allows for global addressing on the register to operate parallel conditional logic on the register qubits. It also maximizes the fidelity of the gates as it is not affected as much as other schemes by the spread in interaction strengths within the register (for a comprehensive discussion Ref. [13]).

We first modeled the performance of the protocol in figure 2 on a realistic cold atoms platform. We considered a register trap containing an average of 100 atoms, over which we perform a controlled Raman rotation. This is a trivial parallel operation on the register qubit, but one that would allow us to benchmark the protocol in high-dimensional Hilbert space. The Rydberg states are chosen so that the register can be fully blockaded by the control qubit. Imperfections in the controlled rotations are taken into account and are mainly due to spontaneous decay from the intermediate level (figure 1, right). For a separation between the traps of  $\sim 2\text{ }\mu\text{m}$  and interaction strength of 15 GHz (achievable at  $2\text{ }\mu\text{m}$  with specific Rydberg states) the trace of  $U_N$  can be retrieved with accuracy of 10% with  $L=400$  measurements on the control qubit. It is remarkable here that the principal source of uncertainty is due to the random loading of the register dipole trap [3], thus it becomes less significant for larger  $N$ .

Upon successful demonstration of the quantum advantage for trivial tasks, this platform offers the ability to implement a range of non-trivial unitaries [3].

Finally we modelled the metrology protocol (figure 2, right) for a particular scenario which will allow the probe state to detect gravity beyond the SQL. The C-NOT gate between control and register qubits prepares a probe state that is then let to evolve under interaction with the unknown field.

We consider  $N=25$  atoms in the register (Poissonian loading), loaded in state-dependent trapping potentials, which allow for the atoms to be shifted vertically conditionally on their internal state. The potentials are initially overlapping to allow for initialization and probe state preparation. Subsequently the two state-dependent traps are adiabatically displaced along the  $z$ -axis so that the two states evolve a differential phase shift due to a variation in the gravitational potential experienced (state-dependent). In the case of Rubidium, with realistic trapping potentials (that we can achieve with the setup described in the next paragraph), and realistic purity levels (i.e. realistic decoherence effects) one can still gain a quantum advantage. With  $N = 25$ , and purity of control and register of 0.95 (well within what can be realistically achieved), the sensitivity of the probe exceeds the classical equivalent by a factor of 3.5. We also confirm that even in this scenario, the sensitivity of the protocol scales with the number of correlated atoms, thus demonstrating the quantum advantage even in the presence of strong decoherence.

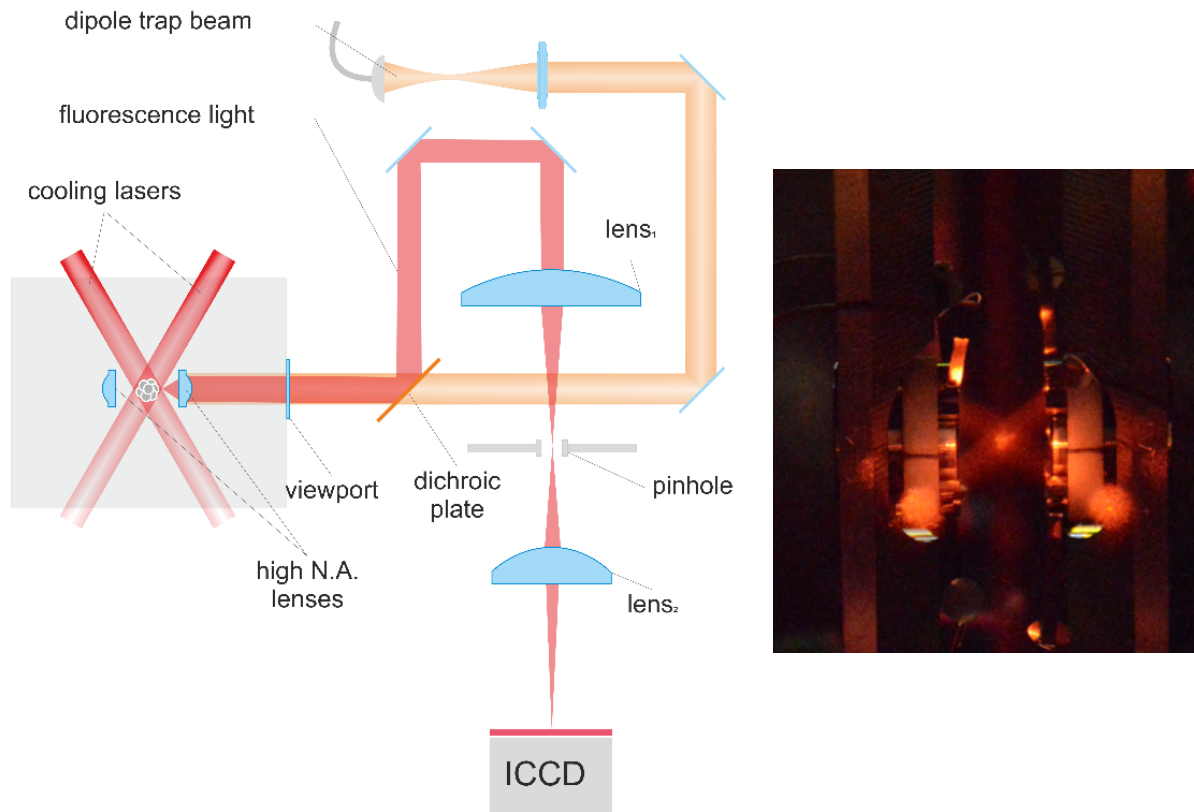
#### 4. The experiment: preparing the control and the register atoms

A magneto-optical trap (MOT) is formed in the area between two identical high N.A. lenses (see figure 3), which are ITO coated to ensure electrical conductivity. Electrodes are also placed around the body of the lens to control electric fields in the trapping region. Each lens has an effective focal distance of 7.03 mm so the distance between the two front faces of the lenses is 14.06 mm. Because of the restricted optical access, two of the MOT beams are crossed at a small angle of  $40^\circ$  (with the third one perpendicular to the plane of the first two) and have a diameter of 2.5 mm. The total MOT beam power is 8 mW and we choose a detuning of  $1.1 \Gamma = 2\pi \times 6.7$  MHz. With these parameters, typical atomic densities are  $10^8$  atoms/cm<sup>3</sup> with a temperature of 300  $\mu$ K (which can be reduced by increasing the laser detuning and simultaneously switching the quadrupole field off). A fluorescence image of the magneto-optical trap formed between the lenses is shown in figure 3 (right). In addition to the lenses are two sets of electric field plates, each set consisting of four electrodes. By applying voltages to these field plates, we are able to apply precise electric field gradients (including spatially constant fields) which allow fine tuning of the Rydberg-Rydberg interactions.

The MOT acts as a reservoir to load two dipole traps, one containing one atom and one containing a register of  $N$  atoms. As mentioned above, to control the populations of the traps we can adjust the reservoir number density to influence the trap loading rate, and individually, the trap depth and geometry [11]. The dipole traps are realised by imaging 852 nm laser beams at the focal plane of one of the high N.A. lenses. We have verified experimentally that the lenses allow us to obtain spot sizes of below 1  $\mu$ m (measured across the  $1/e^2$  points of the beam) over a large area of the focal plane. The same lens is part of an imaging system which observes the atoms which fluoresce at 780 nm. We separate the 780 nm fluorescence from stray 852 nm trap light using a dichroic beam splitter and a narrow band interference filter, and image the trapped atom on an intensified CCD camera.

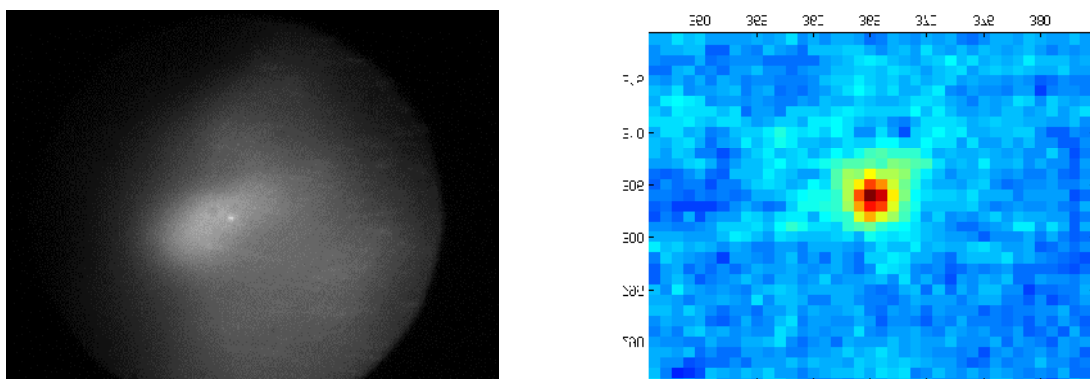
##### 4.1. Register

We prepare the register atoms by loading atoms from the background MOT into a tweezer obtained by focusing an 852 nm laser light down to a 2-4  $\mu$ m waist. Two images of the dipole traps atoms are then taken, using a short 50  $\mu$ s fluorescence pulse – one image with the dipole trapping beam on and one immediately after the dipole trapping beam has been switched off. The short pulse length ensures that the motion of the dense dipole trapped atoms is negligible during the imaging time, and that the expanding atomic density distribution matches the trapped atomic distribution. By comparing the atoms' fluorescence rate in the presence of resonant light it is possible to recover both the number of atoms loaded in the trap and the average light shift induced by the intense laser beam, which enables us to characterise the trap potential. An image of the dipole trap is shown in figure 4.



**Figure 3.** Experimental apparatus for tight dipole trapping of Rubidium atoms. *Left panel:* high numerical aperture lenses are used to produce a very tight ( $\sim \mu\text{m}$  waist) optical tweezer to load  $N$  atoms from a laser-cooled reservoir. Fluorescence light emitted from the atoms is also collected from the same lens and used to image the atoms. *Right panel:* image of the lenses inside the vacuum chamber.

With a trapping laser power of 110 mW we infer that the atoms in the dipole trap experience an average light shift of  $2\pi \times 80$  MHz, which is indeed compatible with a waist between 2 and 4  $\mu\text{m}$ . We also determine the average number of atoms in the trap is  $N = 20$ .



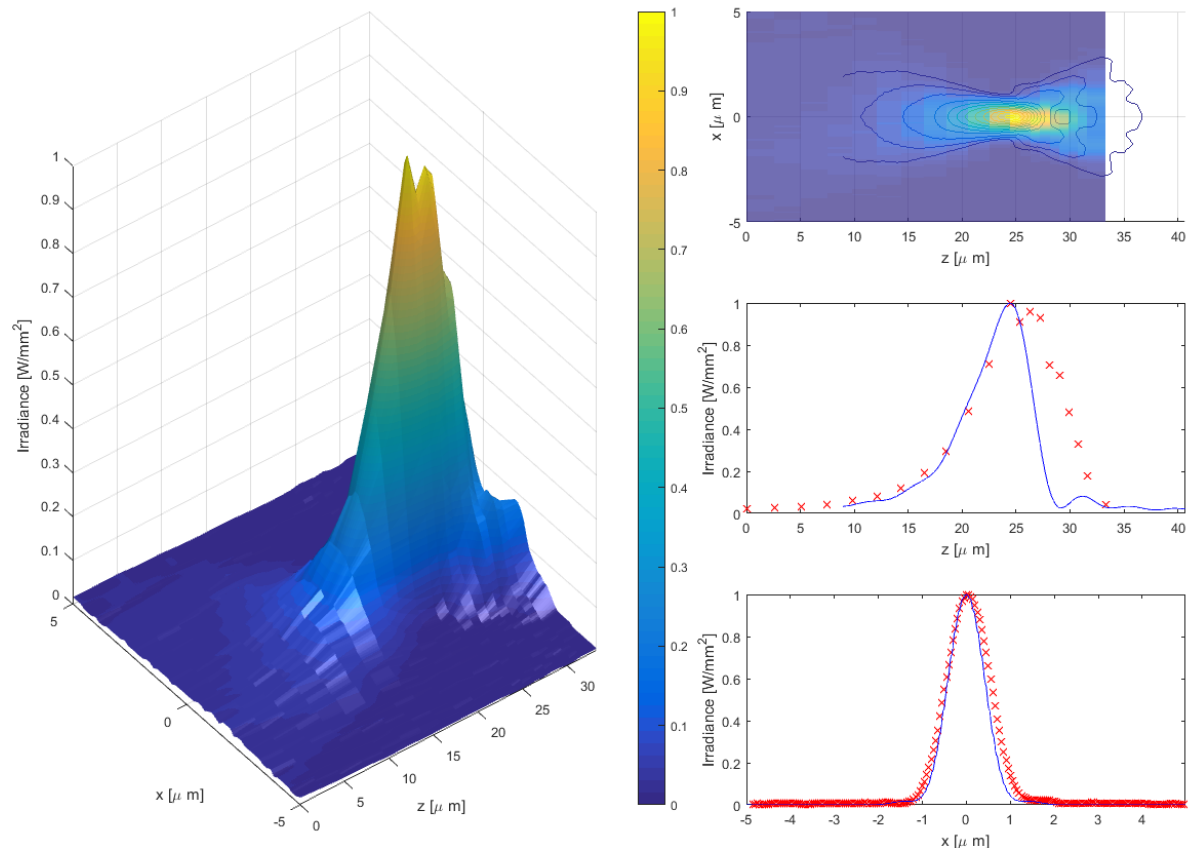
**Figure 4.** Dipole trap loading 20 atoms. *Left panel:* raw fluoresce image of the dipole trap (bright spot at the center) and the MOT reservoir *Right panel:* background subtracted image of a dipole trap containing 20 atoms. The magnification is such that 1 pixel on the ICCD corresponds to 1  $\mu\text{m}$  at the lens focal plane.

#### 4.2. Control

To isolate a single control atom, we need to create a much steeper potential nearby the register trap, so that inelastic laser-induced two body collisions ensure the filling of the trap with either 1 or 0 atoms with sub-poissonian statistics [14].

By changing the input diameter of the incoming beam on the high N.A. lens, we can reduce the transverse size of the beam (and by consequence the Rayleigh length that scales with the square of the waist). By changing the waist by a factor of  $\sim 4$  we would so reduce the trapping volume, by a factor of  $1/4^4$ , thus allowing the trap to enter the regime of *collisional blockade* required for single atom trapping, as discussed in [14].

We have setup a second dipole trap potential and performed measurements of the irradiance spatial profile. In figure 5 we compare the measured profile to the theoretical models and we observe that the measured performance of the optical setup well in agreement with the expectations given by the models. We verify this way the much tighter potential for the trapped atoms that will allow us to achieve simultaneous loading of an ensemble cloud and single control qubit to perform mixed-states protocols.



**Figure 5.** Measured irradiance profile for a  $\sim 1$   $\mu\text{m}$  waist trap and comparison with theoretical models

#### 4.3. Excitation and conditional logic lasers

The excitation of Rydberg states and the EIT scheme for conditional rotation are achieved by the 2-photon laser schemes shown in figure 1. A commercial frequency doubled 960 nm laser (Toptica SHG) produces 270 mW of 480 nm light which we lock to a specific Rydberg state by extracting an error signal from an EIT based spectrometer [16]. The scheme has already been implemented and used to measure the transition strengths of the 480 nm 5P to nL Rydberg states via the Autler Townes effect [15]. We have excited  $n$  in the range 19 – 52 and the total orbital angular momentum quantum number



L can be either S or D. The 780 nm beam is locked by a modulation transfer scheme [17], and is used both for driving the 5S to 5P transition in the dipole trapped atoms and for locking the 480 nm laser.

## 5. Conclusions

We reported on progress on the design and implementation of a cold-atoms based experiment to test the quantum advantage to be gained in the presence of mixed states. At the present time we are able to study registers containing 20  $^{87}\text{Rb}$  atoms. In the future, we aim to increase the number of atoms in the register by tuning the MOT loading rate and the trap geometry. This method gives us the ability to tune the amount of quantum resources and decoherence present in the system and to study the boundaries between classical computing and quantum enhanced computing or sensing.

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