

Atom interferometers' phases at the presence of heavy masses; their use to measure Newtonian gravitational constant; optimization, error model, perspectives

Boris Dubetsky

1849 S Ocean Dr, Apt 207, Hallandale, Florida 33009, USA

E-mail: bdubetsky@gmail.com

Abstract. The contribution to the phase of the atom interferometer caused by the gravity field of a massive test mass is considered. This contribution can be extracted by applying the double difference technique to measure the Newtonian gravitational constant G . Estimates and further calculations showed that after choosing the largest (given the current state of the art) multiphoton wave vector, the time delay between pulses, the mass of the test body and the signal optimization in respect to atomic positions and velocities, one should be able to obtain an accurate G measurement of 200 ppb, which is 2-3 orders of magnitude more accurate than what can currently be obtained. Calculated variations of the phases under the small deviations of atomic variables made it clear that atom clouds' radii and temperatures have to be as small as 100 micron and 100 pK, which has also been achieved already.

1. Introduction

Among the fundamental physical constants c , \hbar , G , the Newtonian gravity constant (CODATA, 2015)

$$G=6.67408(31)\times 10^{-11} \text{ m}^3\text{kg}^{-1} \text{ s}^{-2} \quad (1)$$

is measured at the lowest accuracy of only 46ppm. In this talk, we consider the possibility of increasing this accuracy by using the atom interferometry technique [1]. This technique was first applied [2] by using a test mass moving vertically around the trajectories of atom clouds. For this talk, we assume [3] that the test mass has a cuboid shape with a small cuboid hole for atoms to go through, and that this cuboid shape consists of 2 parts moving horizontally to and from atom clouds (see figure 1). We calculated the phase double difference

$$\Delta^2\phi = \Delta\phi_a - \Delta\phi_b, \Delta\phi = \phi(z_1, v_{1z}) - \phi(z_2, v_{2z}), \quad (2)$$

where $\phi(z,v)$ is a phase of the atom interferometer in which atoms are launched vertically from point z with velocity v . Since both atom clouds are irradiated by the same field and stay on the same platform, the vibration contributes equally to the phases and that contribution is excluded in the 1st order phase difference $\Delta\phi$. When test mass halves are joined or separated (see figure 1a and b), phase differences are equal to $\Delta\phi_{a,b}$. The part of the phase difference caused by Earth's gravitational field is evidently the same



for both differences, and this part is eliminated in the phase double difference. Therefore, $\Delta^2\phi$ depends only on the gravitational field of the test mass, which is linear in G .

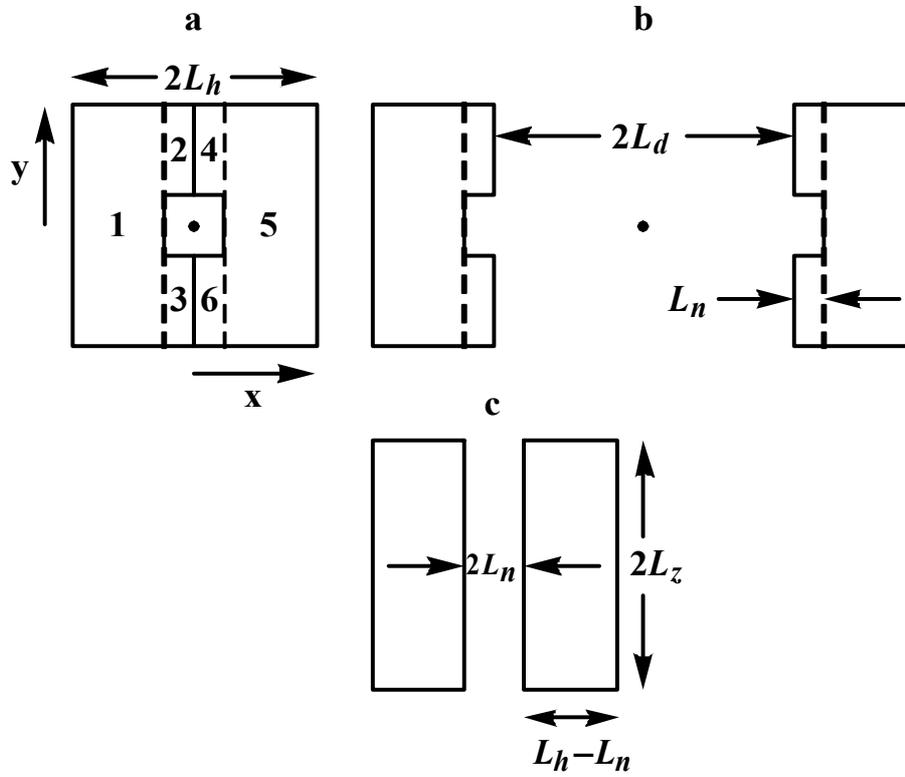


Figure 1. The test mass as a whole is cuboid with a narrow hole for Raman fields and atom trajectories. Atoms are launched from the points z_1 and z_2 with velocities v_{1z} and v_{2z} . Test mass consists of 2 halves. (a) Top view. Joined halves. (b) Top view. Halves separated on the distance $2L_d$. (c) Side view, cross section $x=0$ for joined halves.

2. Optimization

In contrast to [3] (where the calculation was made for the parameters' modest values), we calculated the phases for ^{87}Rb and the maximal value of the parameters achieved at the current state of the art in atom interferometry, i.e. for the time delay between pulses $T=1.15$ s [5], the effective wave vector $k=7.248 \times 10^8 \text{ m}^{-1}$, the test mass $M=1080$ kg of Pb [6], and the phase noise $\phi_{err}=10^{-4}$ rad. The chosen value of k vector can be obtained by using multiphoton processes, first considered in [1]. This value is 45 times greater than the wave vector associated with 2-quantum Raman process in ^{87}Rb . It was achieved [7] using a sequential technique.

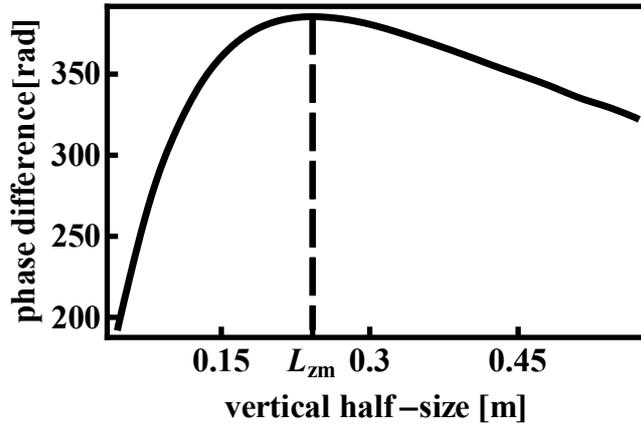


Figure 2. Dependence of the maximum of phase difference on cuboid half-size.

The contribution to the phase caused by the test mass $\delta\phi$ was studied in detail in review [8]. We obtained a response linear in the test mass gravity δg assuming that [9] the magnitude of this field is small,

$$\delta g \ll g, \quad (3)$$

where g is the magnitude of Earth's gravitational field. The ratio $\delta g/g$ is a small parameter of our theory. Evidently, only this part contributes to the double difference, $\Delta^2\phi = \Delta^2\delta\phi$. Using the Wigner representation of the atomic density matrix (first applied for atom interferometry in [10]), we showed that $\delta\phi$ consists of 3 parts, the classical part, the recoil term and the Q-term. The recoil term was obtained without expanding over recoil velocity $\hbar\mathbf{k}/M_a$, M_a is the atomic mass, while for the Q-term, we used perturbation over gravity curvature tensor. Calculations performed in [8] showed that for the chosen value of the wave vector, the recoil term would overcome the classical part, while the Q-term is 2-3 orders of magnitude smaller, and we did not include it in the calculations presented here. From equations (62, 66, 73) in [8], the following expression is obtained for the sum of the classical part and the recoil term:

$$\delta\phi(z, v) = \mathbf{k} \cdot \int_0^T dt \left\{ (T-t) \delta g \left[\mathbf{X}^{(0)}(\mathbf{x}, \mathbf{v}, T+t_1+t) + \frac{\hbar\mathbf{k}}{2M_a}(T+t) \right] + t \delta g \left[\mathbf{X}^{(0)}(\mathbf{x}, \mathbf{v}, t_1+t) + \frac{\hbar\mathbf{k}}{2M_a}t \right] \right\}, \quad (4)$$

where $\mathbf{x}=\{0,0,z\}$ and $\mathbf{v}=\{0,0,v\}$ are the initial atom cloud's position and velocity, $\mathbf{X}^{(0)}(\mathbf{x}, \mathbf{v}, t)$ is the atom trajectory, t_1 is the time delay between the moment of the atoms' launching and 1st Raman pulse. Evidently, $\Delta^2\delta\phi$ achieves the maximum when $\{z_1, v_{1z}\}$ is the point of absolute maximum, and $\{z_2, v_{2z}\}$ is the point of absolute minimum. We found these extrema iteratively using a reasonably wide area and reasonably small steps in z and v . The sizes of the area and steps were restricted by my PC's speed and power. For the given test mass M and density ρ , we found extrema as a function of cuboid vertical halfsize L_z , see figure 2. To get a maximum signal, we recommend choosing L_{zm} shown in figure 2 as vertical halfsize of test mass. Values of the other optimum parameters of the system are presented in table 1.

Table 1. Maximal value of the phase double difference and optimal values of the test mass and atom clouds variables.

phase difference	$\Delta^2 \delta\phi = 386.52738$
vertical half-size	$L_{zm} = 0.24219097\text{m}$
horizontal half-size	$L_{hm} = 0.22162188\text{m}$
1st cloud position	$z_{1m} = -6.8517823\text{m}$
1st cloud velocity	$v_{1m} = 11.158930\text{m/s}$
2nd cloud position	$z_{1m} = -6.2983410\text{m}$
2nd cloud velocity	$v_{1m} = 11.172994\text{m/s}$

One can estimate the accuracy of measurement as

$$err \equiv \frac{\delta G}{G} = \frac{\phi_{err} = 10^{-4} \text{ rad}}{387 \text{ rad}} \approx 200 \text{ ppb}. \quad (5)$$

This accuracy is more than 200 times better than that claimed in CODATA 2015.

3. Error model

To achieve highly precise measurements of the interferometers' phases, one has to prepare both the atomic and proof mass system with great accuracy. The most challenging part is precisely positioning the atom clouds [3]. The extrema of the clouds positions and velocities are, obviously, preferable here. That is why the extrema (found above) in $\{z, v\}$ space allow one not only to maximize the response, but also to make less stringent the requirements for atom clouds' position, velocity, temperature and size because the response becomes quadratic on variations of these variables near the extrema.

Let us now allow small variations of the atom clouds' initial positions, velocities (atomic variables) and small displacements of the test body halves (see figure 3). In this talk we calculated linear and quadratic dependences on atomic variables only. Their relative weights are presented in table 2.

One sees that in spite of using extremum points $\{z_i, v_i\}$ linear terms are not equal to 0. It is because extrema $\{z_i, v_i\}$ have been found in section 2 approximately. One can find that coefficients in the linear dependences small enough so that, for allowed variations of position and velocity (see below table 3), the quadratic dependences are not overcome by linear contributions.

One can use nonlinear terms to estimate atom clouds' radii and temperatures. Consider for example relative contribution

$$\delta\phi_z = \alpha z_i^2 \quad (6)$$

If Raman fields are sufficiently flat to neglect ac-Stark shift variation across the atom cloud, and if Raman pulses are sufficiently short to neglect the Doppler broadening of the Raman transition, then one needs only to average (6) over atoms' spatial distribution.

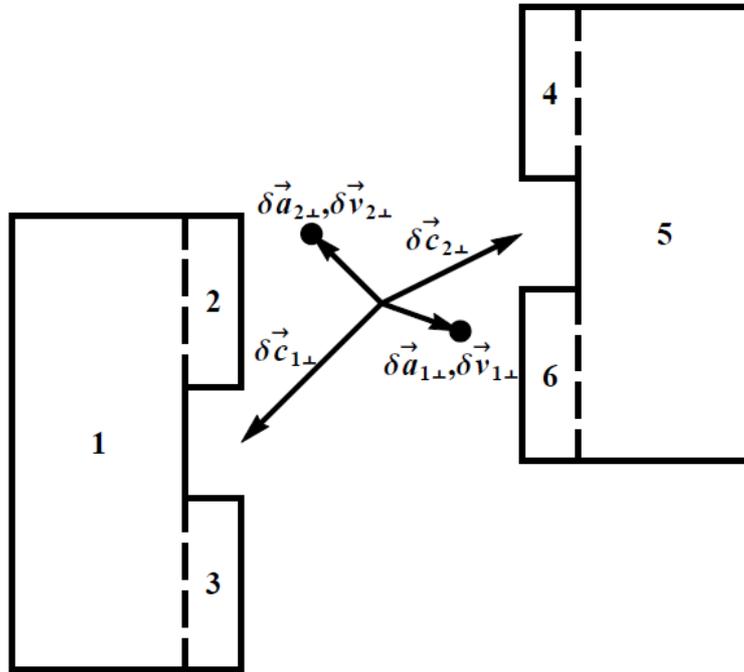


Figure 3. Top view. Small variations of the atomic and proof mass variables.

Table 2. Error model for 1080 kg test mass, atomic variables.

Term	Relative weight
Linear in position	$0.0012117481 \delta z_1$
	$0.0011188391 \delta z_1$
Linear in velocity	$-2.5456033 \times 10^{-6} \delta v_{1z}$
	$-2.3703001 \times 10^{-6} \delta v_{2z}$
Nonlinear in position	$6.6840356 (\delta x_1^2 + \delta y_1^2)$
	$-13.368071 \delta z_1^2$
	$1.4586375 (\delta x_2^2 + \delta y_2^2)$
Nonlinear in velocity	$-2.9172749 \delta z_2^2$
	$9.0832760 (\delta v_{1x}^2 + \delta v_{1y}^2)$
	$-18.166552 \delta v_{1z}^2$
	$1.9881322 (\delta v_{2x}^2 + \delta v_{2y}^2)$
Position-velocity cross-term	$-3.9762644 \delta v_{2z}^2$
	$15.578255 (\delta x_1 \delta v_{1x} + \delta y_1 \delta v_{1y})$
	$-31.156509 \delta z_1 \delta v_{1z}$
	$3.4018485 (\delta x_2 \delta v_{2x} + \delta y_2 \delta v_{2y})$
	$-6.8036970 \delta z_2 \delta v_{2z}$

For Gaussian distribution,

$$\frac{\exp\left(-\delta z_i^2 / \delta z_{i0}^2\right)}{\sqrt{\pi} \delta z_{i0}},$$

after averaging one gets

$$\langle \delta \phi_z \rangle = \frac{\alpha}{2} \delta z_{i0}^2. \quad (7)$$

Requiring it to be equal to the expected relative error of phase measurement, err ; one finds for atom cloud radius

$$\delta z_{i0} = \sqrt{\frac{2err}{\alpha}}. \quad (8)$$

In the same manner, we determine atom clouds velocities' variations and temperatures. These quantities are presented in table 3 for relative error value (5). Even though it is challenging to cool to those temperatures and focus on the radii, the temperatures and radii are higher than those achieved in article [12].

Table 3. Atom interferometers' parameters one should hold to achieve measurements of 200 ppb.

1st cloud vertical radius [m]	0.00017297989
1st cloud vertical velocity [m/s]	0.00014838628
1st cloud vertical temperature [K]	$1.1519682 \times 10^{-10}$
1st cloud horizontal radius [m]	0.00024463051
1st cloud horizontal velocity [m/s]	0.00020984989
1st cloud horizontal temperature [K]	$2.3039364 \times 10^{-10}$
2nd cloud vertical radius [m]	0.00037028943
2nd cloud vertical velocity [m/s]	0.00031717019
2nd cloud vertical temperature [K]	$5.2630531 \times 10^{-10}$
2nd cloud horizontal radius [m]	0.00052366833
2nd cloud horizontal velocity [m/s]	0.00044854639
2nd cloud horizontal temperature [K]	1.0526106×10^{-9}

4. Conclusion

We showed that using the atom interferometry technique, for the chosen geometry of the test body, at the positions and velocities of the atom clouds determined by the optimization, can give the double difference of the atomic interferometers phases as large as 387 rad at the phase noise level 10^{-4} rad. This should allow one to measure the Newtonian gravitational constant G with an accuracy of 200 ppb, which is 2 to 3 orders of magnitude better than that currently achieved using conventional methods. To achieve this result one has to realize **SIMULTANEOUSLY** (a) sequential technique to increase the effective wave vector, (b) small radii and (c) low temperatures of the atom clouds, determined by using the built error model. Each of the parameters is within the current state of the art in atomic interferometry.

References

- [1] Dubetsky B Ya, Kazantsev A P, Chebotaev V P, Yakovlev V P 1984 *JETP Lett.* **39** 651
- [2] Fixler J B, Foster G T, McGuirk J M, Kasevich M A 2007 *Science* **315** 74
- [3] Dubetsky B 2014 Optimization and error model for atom interferometry technique to measure Newtonian gravitational constant *Preprint* Physics/1407.7287
- [4] Snadden M J, McGuirk J M, Bouyer P, Haritos K G and Kasevich M A 1998 *Phys. Rev. Lett.* **81** 971
- [5] Dickerson S M, Hogan J M, Sugarbaker A, Johnson D M S and Kasevich M A 2013 *Phys. Rev. Lett.* **111** 083001
- [6] Biedermann G W, Wu X, Deslauriers L, Roy S, Mahadeswaraswamy C and Kasevich M A 2015 *Phys. Rev. A* **91** 033629
- [7] Kovachy T, Asenbaum P, Overstreet C, Donnelly C A, Dickerson S M, Sugarbaker A, Hogan J M and Kasevich M A 2015 *Nature* **528** 530
- [8] Dubetsky B, Libby S B and Berman P 2016 *Atoms* **4** 14
- [9] Dubetsky B 2008 unpublished work.
- [10] Dubetsky B, Kasevich M A 2006 *Phys. Rev. A* **74** 023615
- [11] Rosi G, Sorrentino F, Cacciapuoti L, Prevedelli M and Tino G M 2014 *Nature* **510** 518
- [12] Kovachy T, Hogan J M, Sugarbaker A, Dickerson S M, Donnelly C A, Overstreet C and Kasevich M A 2015 *Phys. Rev. Lett.* **114** 143004