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A simple model for determining the atmospheric thermal conductivity

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Abstract. This paper presents a simple method based on Finite-Time Thermodynamics (FTT) to determine the thermal conductivity of atmospheric air. The method considers an atmospheric heat engine in which the air moves convectively on the border day-night driven by the Sun's energy. The numerical value obtained for the thermal conductivity reasonably accords with the reported value by Van Ness by the number of Rayleigh.

1. Introduction

A finite-time thermodynamics has been developed four decades ago, from the pioneering work of Curzon and Ahlborn [1]. As it is well known, Carnot's theorem states that for thermal cycles operating between two heat reservoirs at absolute temperatures T_1 and T_2 ($T_1 > T_2$), the cycle of maximum efficiency that can work between they temperatures is the Carnot cycle, consisting of two isotherms and two adiabat that take place reversibly. This fact implies that to the Carnot cycle has zero power output, since a reversible cycle consisting of quasistatic process require an infinite time to complete. The Curzon and Ahlborn (CA) engine (see figure 1) consists of two parts, a reversible one (endoreversible) which is an internal Carnot engine operating between T_{1w} and T_{2w}, and another part formed by the two couplings between T_{1w} and T_1 and between T_{2w} and T_2 respectively, which constitute the irreversible part of the model. In addition, the heat transfer along the mentioned couplings is given by a linear heat transfer law of the Newtonian type, although other heat transfer laws can be used to model heat fluxes, so as Dulong-Petit law and Stefan-Boltzmann law among others. The endoreversible CA engine was a first step towards irreversible engine models by incorporating irreversible elements as thermal resistances at the couplings between the internal reversible cycle and the external heat reservoirs. The CA engine under maximum power conditions leads to the famous CA–efficiency given by $\eta_{CA} = 1 - \sqrt{\tau}$, being $\tau = T_2/T_1$. The present paper is organized as follows: In Section 2, we define the atmospheric winds; in Section 3 we maximize the power output; in Section 4 we analyze the tidal winds as heat engine driven by the Sun; finally in Section 5 we present some concluding remarks.

2. Atmospheric winds

The existence of two heat reservoirs at different temperatures makes possible the production of work. In fact, the planet's atmosphere is a working fluid illuminated by the Sun, thus the atmospheric gases tend to increase its temperature and they expand. Therefore, the illuminated gas by the Sun rises to the upper atmosphere. When moving to less enlightened places it is cooled, and the more density tends to

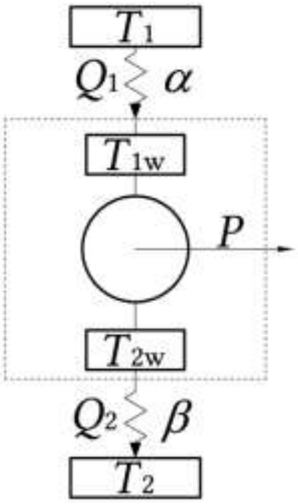

Figure 1. Curzon-Ahlborn endoreversible engine model.

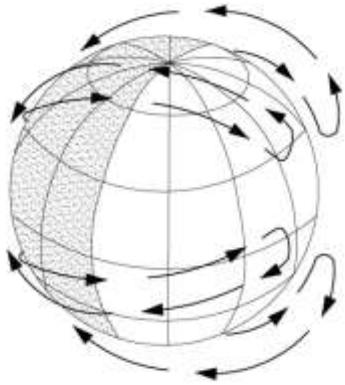

Figure 2. Model of tidal winds or thermal tides [2].

sink back to the planet's surface. That is, the movement of gases is promoted by the differences in longitudinal temperature (geographical coordinates): the day lengths (floodlit) are warmer than the night lengths (unlit). Such macroscopic cycles are one of the sources of the winds. These kinds of winds are called tidal winds or thermal tides, because air travel is much like the movement of tidal water from the sea [2], see figure 2.

Taking the model of CA as a basis for modelling tidal winds, it follows that the temperature of the hot reservoir is the temperature of the Sun ($T_1 = 5778K$) and the temperature of the cold reservoir is the temperature of the surface of the Earth at night ($T_2 = 287.15K$). On the other hand, thermal conductance α (see figure 1), is proportional to the Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$, that is

$$\alpha = f\sigma, \tag{1}$$

being f the dilution factor, defined by $f = R_s^2/r^2$ with R_s the Sun's radius and $r = \sqrt{bc}$, with b the half major axis and c the half minor axis of the elliptical orbit of a planet around the Sun [3]. Similarly, the thermal conductance β (see figure 1) is attenuated by the absorption coefficient a [4], thus

$$\beta = ak, \tag{2}$$

where k is the atmospheric thermal conductance, $[k] = W m^{-2} K^{-1}$.

2.1. Power output

Figure 1 shows the CA model, which consists of two heat reservoirs (T_1 and T_2) connected by two irreversible components (α and β associated to thermal conductances) to two isotherms of the working fluid (T_{1w} and T_{2w}) with an internal reversible Carnot cycle. Thus, the endoreversibility hypothesis [3] is

$$\frac{Q_1}{T_{1w}} = \frac{Q_2}{T_{2w}}. \tag{3}$$

being Q_1 and Q_2 heat fluxes (heat transfer per unit time). From first law of thermodynamics, we have

$$Q_1 = P - Q_2, \tag{4}$$

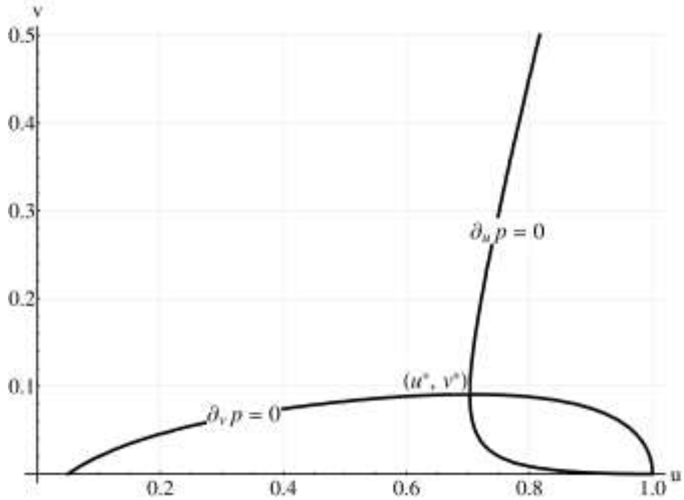

Figure 3. Overlap of implicit equations (10) and (11).

with P the power output. We propose a law of heat transfer for each of the couplings between the working fluid and heat reservoirs, of the type

$$Q_1 = \alpha(T_1^4 - T_{1w}^4), \quad Q_2 = \beta(T_{2w} - T_2). \tag{5}$$

Q_1 corresponds to a radiative transfer and Q_2 to a convective transfer in the day-night boundary [2]. Thus, the power output is given by [5,6]

$$P = \frac{\alpha\beta y[T_1 - T_2 - x - y][T_1^4 - (T_1 - x)^4]}{\alpha[T_1^4 - (T_1 - x)^4][T_2 + y] + \beta y(T_1 - x)}, \tag{6}$$

where $x = T_1 - T_{1w}$, $y = T_{2w} - T_2$ [1]. We define the dimensionless variables, $N = \alpha T_1^3/\beta$, $u = 1 - x/T_1$, $v = y/T_1$ and $\tau = T_2/T_1$. Under these parameters we rewrite equation (6) to obtain

$$P = \alpha T_1^4 \frac{v(1-u^4)(u-v-\tau)}{N(1-u^4)(v+\tau)+uv}. \tag{7}$$

3. Maximization of the normalized power output

Given the above expression we proceed to maximize the power output by using the variables u and v, taking τ and N as system parameters. To simplify the calculations, we normalize the power output in the following manner, $P/\alpha T_1^4$. Then equation (7) becomes,

$$p(u, v, \tau, N) = \frac{v(1-u^4)(u-v-\tau)}{N(1+u\text{^}4)(v+\backslash tau)+uv}. \tag{8}$$

This expression is analogous to the conversion efficiency of solar energy ω, defined by De Vos [3]. To maximize $p(u, v, \tau, N)$given by equation (8) we calculate

$$\frac{\partial p(u,v)}{\partial u} = 0, \quad \frac{\partial p(u,v)}{\partial v} = 0. \tag{9}$$

The following pair of transcendental equations is obtained

$$[v + \tau][u^4(3v - 2N) + (1 + u^8)N + v)] - 4u^5 v = 0, \tag{10}$$

$$N[1 - u^4][(v + \tau)^2 - u\tau] + uv^2 = 0. \tag{11}$$

Figure 3 shows the intersection point (u_0, v_0), of the implicit equations (10) and (11), for N and τ constants. For a different value of N, it can be seen that within this set up is critical point (u_0, v_0) [7].

3.1. Tidal winds as heat engine driven by the Sun

Taking the temperature of the Sun ($5788K$) and the temperature of the surface of the Earth at night ($287.15K$), the parameter $\tau = T_2/T_1 \approx 1/20$, the coefficient of thermal conductances N, determine the appropriate configuration of the heat engine, such that the power output or the efficiency of solar energy conversion, equation (8), are maximal. Thus, for a value of $N = 0.48$ and $\tau = 1/20$. The maximum values of u and v are 0.7044 and 0.1148 respectively. By using equation (8), $p_{max} = 17.3\%$, for the efficiency of solar energy conversion. De Vos reported a value for the conversion efficiency of solar energy, of the tidal winds, of the 12.79%, [3] and Barranco-Jiménez report a value of 17.33% [9], therefore the value of N it is appropriate for the configuration of the tidal winds, such that the power output is maximum. On the other hand, from the definition of N, it follows that

$$N = \frac{\alpha T_1^3}{\beta} = \frac{f\sigma T_1^3}{ak}, \tag{13}$$

where we have used the equations (1) and (2). Then, it is obtained that the atmospheric thermal conductance is given by

$$k = \frac{f}{a}\frac{\sigma T_1^3}{N}. \tag{14}$$

The dilution factor for the Earth is given by $f = 2.16 \times 10^{-5}$ [3]. On the other hand, we know that the absorption coefficient of the atmosphere is $a = 14\%$ [4], then we finally obtain

$$k = 3.5156 W m^{-2} K^{-1}. \tag{15}$$

Which is a value reasonably close to that reported by Van Ness [8], which has a value of $3.3185 W m^{-2} K^{-1}$, for the thermal conductance of air at ambient temperature.

4. Concluding remarks

In this paper we have calculated the approximate value of the thermal conductance of air through a very simple model of heat engine operating at finite time. As is well known, although the atmosphere has a temperature distribution with a different for each one of the atmospheric layers (troposphere, the tropopause, stratosphere, etc.), the behavior of the winds can be simulated, in first approximation, by an atmospheric heat engine operating between two extreme temperatures, with it has is troposphere value [2,9,10,11]. Among these models is that of De Vos and Flater [2] which focuses on the horizontal winds at night-day border. Based on this model we have proposed a simple method for calculating the thermal conductance of air, that gives a very close to that reported by Van Ness [8] by using the Rayleigh number value; a dimensionless number used in calculations of heat transfer by natural convection in fluid mechanics. As can be seen in the present work based on the general methods of thermodynamics, they can be used to calculate some thermal properties of materials (air in our case).

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