

Fast conical surface evaluation via randomized algorithm in the null-screen test

D Aguirre-Aguirre^{1,2}, R Díaz-Uribe¹ and B Villalobos-Mendoza^{2,3}

¹Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma de México, Cd. Universitaria, Apdo. Postal 70-186, C.P. 04510, Cd Mx, México

²Polo Universitario de Tecnología Avanzada, Universidad Nacional Autónoma de México, Vía de la Innovación No. 410, Autopista Monterrey-Aeropuerto Km. 10, PIIT C.P. 66629 Apodaca, N. L.

³Centro de Investigación y de Estudios Avanzados, Unidad Monterrey, Apodaca N.L. 66629, México

E-mail: daniel.aguirre@ccadet.unam.mx, rufino.diaz@ccadet.unam.mx

Abstract. This work shows a method to recover the shape of the surface via randomized algorithms when the null-screen test is used, instead of the integration process that is commonly performed. This, because the majority of the errors are added during the reconstruction of the surface (or the integration process). This kind of large surfaces are widely used in the aerospace sector and industry in general, and a big problem exists when these surfaces have to be tested. The null-screen method is a low-cost test, and a complete surface analysis can be done by using this method. In this paper, we show the simulations done for the analysis of fast conic surfaces, where it was proved that the quality and shape of a surface under study can be recovered with a percentage error < 2.

1. Introduction

In recent years, monitoring systems and real-time inspection have been certainly very attractive tools for the manufacturing industry and industry in general. When these inspection systems are not destructive and have a fast image capture and processing time, besides small dimensions and low weight, their interest increases greatly. Nowadays, the demand for new and more precise non-destructive industrial control techniques that allow automatic or almost automatic evaluation of the object to be inspected has been growing. This requires accurate and fast methods able to perform in situ measurements? i.e. in the production line, in a workshop or in a place where the piece is measured sometimes outside of a laboratory environment.



For this purpose, we decided to use the technique of null-screens [1, 2]. This technique allows us to evaluate specular or partially reflective surfaces without contact, it is very robust and easy to handle, and besides that this method is capable of analyzing very large surfaces with a wide range of shapes [3, 4].

2. Null-screen method

The basic concept of this test is to design null-screens starting from the ideal shape of the surface under study. These null-screens are printed with geometric shapes which can be stripes, dots, drops, among others (Figure 1). This drawing is located on the screen in such way that, when it is reflected by the surface under test, if the surface is ideal or has a high quality, the observed image that also depends on the used geometry, can be stripes or dots perfectly aligned without deformation [1] (Figure 2a).

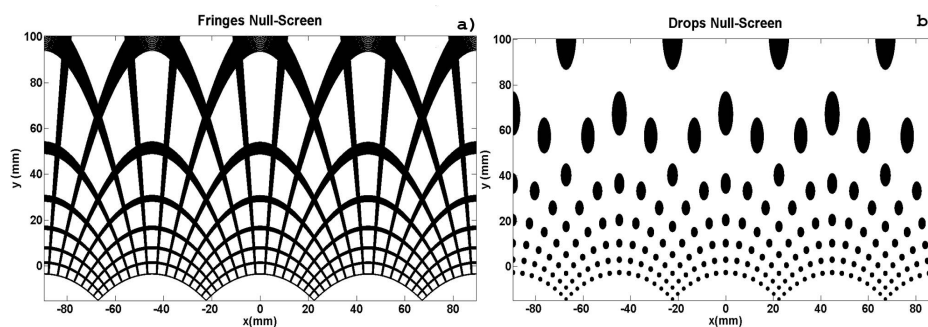


Figure 1. Null-screens a) fringes and b) drops.

However, if the surface is not ideal, or is not the surface for which the screen was designed, the observed image could have stripes or points that do not follow a perfect grid (Figure 2b). From the differences between the perfect grid (theoretical) and the distorted grid (experimental) can be obtained relevant information about the shape of the surface that is being analyzed.

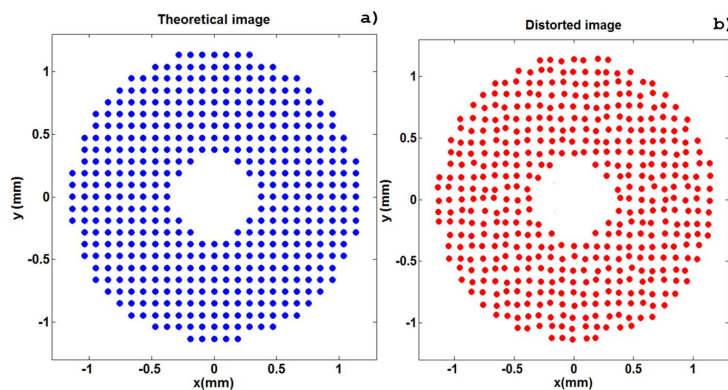


Figure 2. Centroids on the CCD, from the surface a) theoretical and b) distorted.

3. Mathematical basis

To determine the coordinates where each drop will be located on the null-screen, which will generate a perfect grid on the CCD, we make a reversed ray-tracing (figure 3). These rays leave the CCD $P_1(x_1, y_1, z_1)$ passing through the pinhole $P(0, 0, b)$ and intersect the surface at the point $P_2(x_2, y_2, z_2)$.

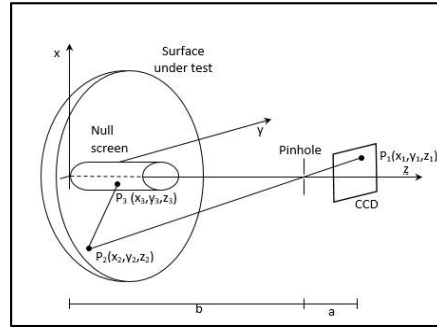


Figure 3. Setup for calculating a null-screen.

From equation of the sagitta which describes this type of surface given by

$$Q(z_2 - z_0)^2 \pm 2r(z_2 - z_0) + (y_2 - y_0)^2 + (x_2 - x_0)^2 = 0,$$

where r is the paraxial radius of curvature, x_0, y_0, z_0 are the decentering of the surface and $Q = (k-1)$, where k is defined by the conic constant.

To obtain the coordinates where the reflected ray intersects the cylinder, we start from the equation of the cylinder

$$(x_3 - x_0)^2 - (y_3 - y_0)^2 = C^2.$$

The parametric equations of the reflected ray are defined by

$$x_3 = x_2 + R_x \lambda$$

$$y_3 = y_2 + R_y \lambda$$

$$z_3 = z_2 + R_z \lambda.$$

where

$$R_x = -x_1 + 2(x_2 - x_0) \left[\frac{(x_2 - x_0)x_1 + (y_2 - y_0)y_1 + a(Q(z_2 - z_0) \pm r)}{(x_2 - x_0)^2 + (y_2 - y_0)^2 + (Q(z_2 - z_0) \pm r)^2} \right]$$

$$R_y = -y_1 + 2(y_2 - y_0) \left[\frac{(x_2 - x_0)x_1 + (y_2 - y_0)y_1 + a(Q(z_2 - z_0) \pm r)}{(x_2 - x_0)^2 + (y_2 - y_0)^2 + (Q(z_2 - z_0) \pm r)^2} \right]$$

$$R_z = -a + 2(z_2 - z_0) \left[\frac{(x_2 - x_0)x_1 + (y_2 - y_0)y_1 + a(Q(z_2 - z_0) \pm r)}{(x_2 - x_0)^2 + (y_2 - y_0)^2 + (Q(z_2 - z_0) \pm r)^2} \right].$$

Substituting the equation into , matching up λ terms we have

$$O\lambda^2 + P\lambda + T = 0,$$

where

$$O = R_x^2 + R_y^2$$

$$P = 2x_2R_x - 2x_0R_x + 2y_2R_y - 2y_0R_y$$

$$T = x_2^2 + x_0^2 - 2x_2x_0 + y_2^2 + y_0^2 - 2y_2y_0 + C^2.$$

Solving with the quadratic formula for λ

$$\lambda = \frac{-P \pm \sqrt{P^2 - 4OT}}{2O}.$$

With this, we obtain the coordinates x_3 , y_3 , and z_3 , which correspond to the positions where the rays reflected by the surface intersect the display.

It should be noted that the diameter of the null-screen in the case of convex surfaces is greater than the diameter of the surface.

4. Surface analysis

In this section, the method used for the analysis of synthetic patterns is described.

4.1 Algorithm used for the analysis

It was decided to use a probabilistic algorithm for the surface analysis. The algorithm has three basic steps: two cycles to found the nearest solution to the problem, and a third cycle was only for the acquisition of the entrance data. The three steps of the algorithm are described next:

Step 1. *Parameter acquisition*

In this step, the theoretical surface data are acquired (i.e., radius of curvature, conic constant, diameter, a and b parameters, the size of the CCD, cylinder diameter, and experimental centroids positions), all this could be written in a .txt or .jpg file.

This data are necessary to calculate the theoretical centroids position of every spot in the null-screen. This task is done with the aim of making a comparison between the positions of every spot in the null-screen, with respect to the calculated position regarding the experimental centroids image while the surface form is varying.

Step 2. *“Approximation” cycle*

In this cycle, the conic constant, radius of curvature, surface, and screen decentering (x_0, y_0, z_0) and (x_0', y_0', z_0') , nearest to the real solution are found. This solution is found by modifying a theoretical surface by changing in a random manner the coefficients mentioned before. The found coefficients are introduced in the “limits reduction” cycle which is described next.

Step 3. *“Limits reduction” cycle*

In this cycle, the coefficients found in the approximation cycle are optimized. This process of optimization is done in our case by using the next equation

$$Coef = V - (V * n) + (V * n) 2rand,$$

where V is the value found in the approximation cycle, n is the percent of V where the search was done, and $rand$ is a random value. With this process, the search of the solution is reduced to a $\pm n\%$ of V . At the end of this cycle, the values found for every coefficient are taken as the values that describe the surface under test.

4.2 Decentered surface ($x_0 \neq 0$, $y_0 \neq 0$, $z_0 \neq 0$)

In this subsection, we are going to talk about the decentered surface problem. This is a typical problem in the laboratory when a surface is being tested. The synthetic surface and the decentered coefficients used in this analysis are shown in Table 1.

Table 1. Design parameters of the Synthetic surface.

Surf. Design		Surf. Decentering	
r	8.75 mm	x_0	0.41 mm
k	-3.65	y_0	-0.63 mm
D	35.0 mm	z_0	-0.95 mm

A comparison between the centroids positions without decentering (blue) and the change in the centroids positions when a decentering is introduced in x , y and z (red) are shown in Figure 4a, where the centered and decentered centroids positions can be seen.

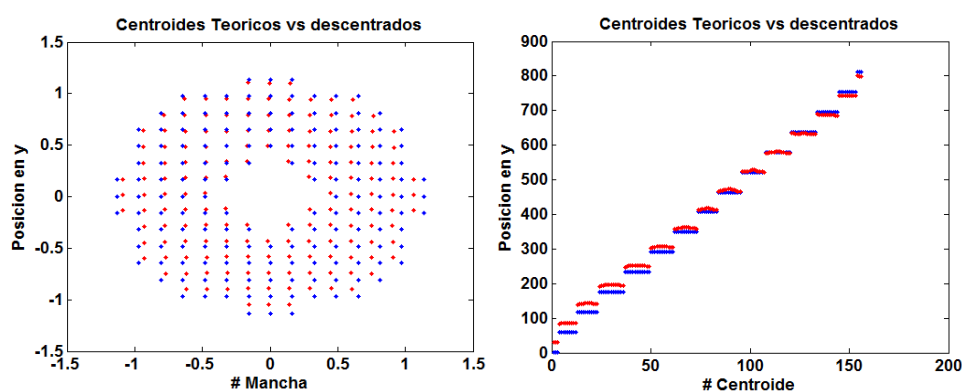


Figure 4. Surface centroids on the CCD, a) comparison between theoretical and decentered centroids position; and b) plot of the y position of each centroid.

In Figure 4b, the y positions of every centroid with respect to its assigned value are shown. This is done to prove that the correspondence between centroids was well done. This correspondence is done as follows: the coordinates in pixels of one centroid is taken from the real image, then the nearest

centroid to this position is searched in the decentered image. To prove that this correspondence was well done, the y positions of each centroid was plotted.

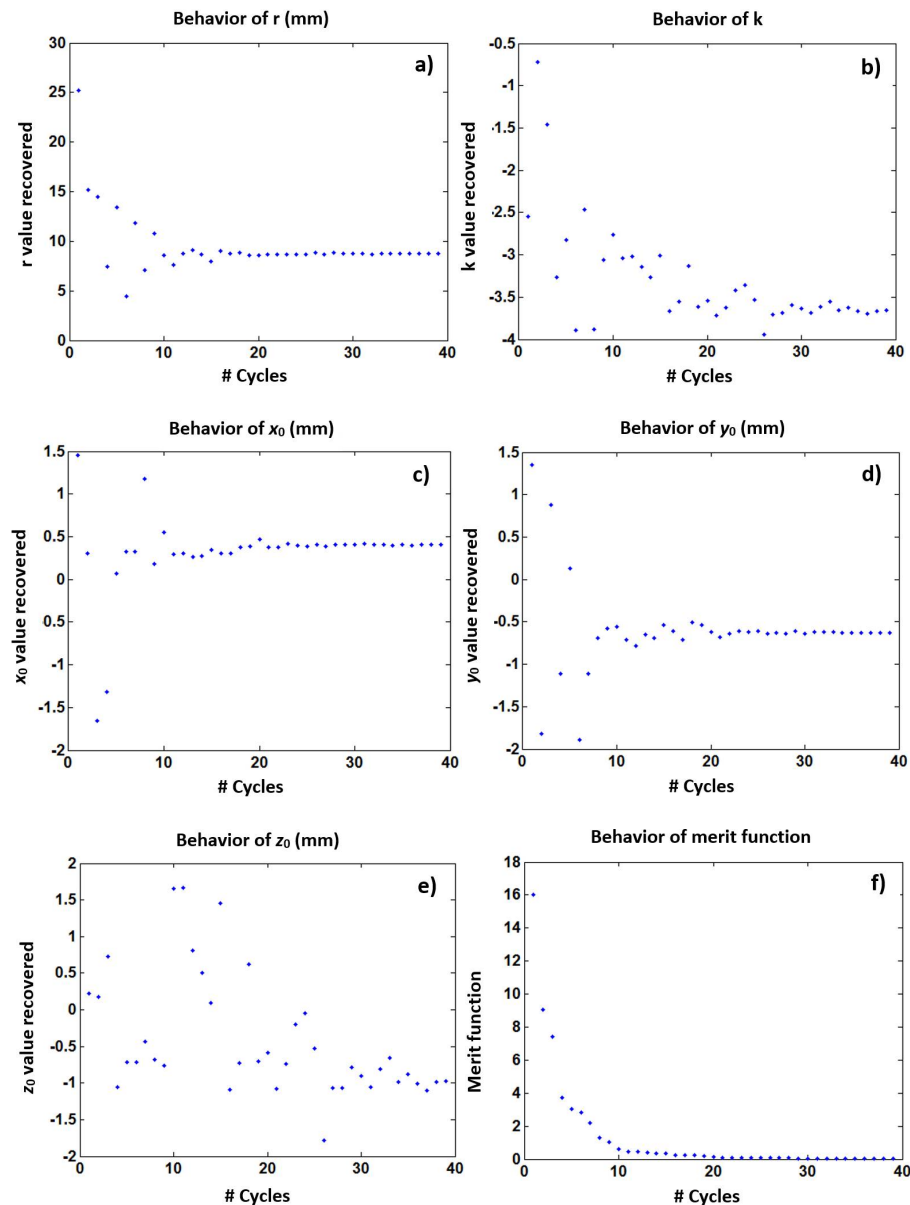


Figure 5. The behavior of, a) radius of curvature, b) conic constant, decentering in c) x , d) y , e) z and f) merit function when the program converged.

The recovered values by our modified algorithm are shown in Table 2. It can be seen that the percentage errors of the decentered coefficient in z and the conic constant was 1.89% and 0.28%, respectively.

Table 2. Comparison between the design and the recovered values.

	Design value	Recovered value	Percentage error
r	8.75 mm	8.7491 mm	0.01
k	-3.65	-3.6605	0.28
x_0	0.41 mm	0.4060 mm	0.97
y_0	-0.63 mm	-0.6274 mm	0.41
z_0	-0.95 mm	-0.9680 mm	1.89

5. Conclusions

In this paper, an algorithm for obtaining the shape of a conical concave or convex synthetic surface was presented. We described the null-screen design procedure and the randomized algorithm used for the surface analysis, where instead of using an integration process as is usually performed, the shape of the surface is found in a direct and in a random way.

We found that the surface decentering coefficients were recovered with a maximum percentage error less than 2%, while for the case of the coefficients which describe the surface shape (r and k), the maximum percentage of error was less than 0.3%. With our method, we eliminated the errors that are introduced during the integration process, therefore, our percentage error is reduced in a great manner.

On the other hand, this algorithm can be modified for the analysis of surfaces of any size and shape due the hardness of our method, where we can modify the parameters of the surface that we want to analyze, i.e. from surfaces of the human eye to the new generation of telescopes surfaces.

6. References

- [1] Díaz-Urbe R and Campos-García M 2000 *Appl. Opt.* **39** 2670-2677
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- [3] Díaz-Urbe R, Campos-García M and Granados-Agustín F S 2002 *Proc. SPIE* **4818** 63
- [4] Moreno-Oliva V, Campos-García M, Román-Hernández E and Santiago-Alvarado A 2014 *Opt. Eng.* **53** 11