IOP Conf. Series: Journal of Physics: Conf. Series **789** (2017) 012050 doi:10.1088/1742-6596/789/1/012050

Determination of steel bar dispersed mass in electric discharge with alternative electrode

G Yu Shakirova[1], Yu I Shakirov, V I Ilyin, R A Valiev and L N Drogaylova

Kazan Federal University Naberezhnye Chelny Institute, 68/19 Mira Street, Naberezhnye Chelny, 423810, Russian Federation

E-mail: guzya81@mail.ru

Abstract. The mathematical model of plane problem of metal bar dispersion in electric discharge with liquid electrolyte is suggested in this research. The analogy with the plane problem of the theory of jets in an ideal fluid is used to solve the task. It actually means determination of analytic function in the field with one area of unknown boundary. The formula for determination of dispersed metal powder mass assuming the bar axial symmetry has been calculated.

1. Introduction

Powder developers have been more and more successfully used for the production of heavy duty details over recent years. Composite powders are produced by synthesis under the conditions of technological burning followed by pulverization and classification due to the fractions. Along with the traditional methods of powder production, electric discharge dispersion of metals and alloy materials with the usage of liquids is being applied more and more [1-5].

The mathematical model suggested in this work allows to evaluate the influence of the task parameters on the eventual result using at least qualitative and partially quantitative data.

2. Experiment

Figure 1 shows the scheme of a powder developer synthesis unit that consists of a constant power supply, an electrolytic bath with an electrolyte and a steel conductive electrode with different carbon content (0.2 % - 1.0 %).

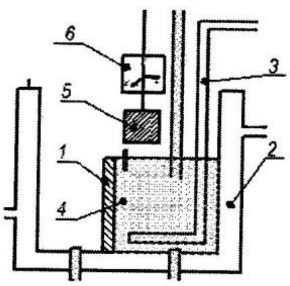

Figure 1. Scheme of a powder developer synthesis unit: 1 - electrolytic bath, 2 - water-cooling jacket, 3 – bubbler, 4 – electrolyte, 5 – steel electrode, 6-co-ordination unit.

[1] To whom any correspondence should be addressed.

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The results of the trials performed have shown big dependency of powder quantity and quality (the size of granules) on many geometrical and physical parameters [1, 2].

3. Results and their discussion

To solve the task first of all it should be mentioned that in the process of powder production the dependency of η current efficiency on anode current density j_a (figure 2) is identical to the analogical dependency $\eta(j_a)$ characteristic of electrolytic processing of metals [6, 7]. Therefore, we consider that the speed of metal removal V_m from the anode surface per unit weight is calculated similarly to Faraday's law $V_m = j_a\eta\varepsilon$, where $\eta = \eta(j_a)$ - current efficiency, equal to the amount of energy spent for the powder production in the electric discharge, j_a- current density, ε- electrochemical equivalent of metal. We consider that the anode surface is moving with the constant speed v and the linear speed of the points on the anode surface is calculated like the following:

$$V_a = v \cdot cos\,\theta, \quad (1)$$

where θ — angle between the vector of anode delivery speed and the unit vector $\overrightarrow{n_a}$ of outward normal to the surface of the anode.

In this case the general scheme of the process is not changing within time and the process can be considered steady or stationary.

Due to formula (1) steady state current density distribution j_a on the stationary anode boundary is determined by the following one:

$$\eta(j_a)j_a = \frac{\rho v}{\varepsilon} cos\,\theta$$

where ρ – anode material density.

As it is seen in figure 2, dependency of $\eta(j)_a$ current efficiency can be approximated by the hyperbolic equitation and it can be considered that the following relation is fulfilled on the anode boundary [8]

$$j_a = \frac{\rho v}{\varepsilon}(a_0 + b_0 \cos\theta) \quad (2)$$

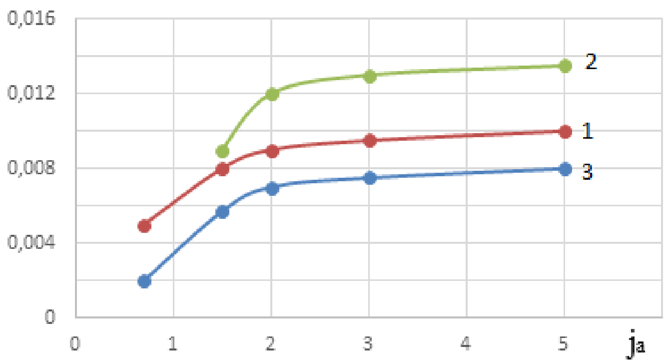

Figure 2. Dependency of current efficiency on anode current density while producing Fe_3O_4 powder of У10 steel under different current intensity.

Let's take a closer look at the two-dimensional model of the process (figure 3). We introduce the Cartesian coordinate systemx_1, y_1 connected with the anode moving towards the coordinate axis.

Ignoring the electrode phenomena, we consider that there exists electric field potential ψ_1 in the interelectrode space (IES) consistent with Laplace's equation

$$\Delta\psi_1 = 0 \quad (3)$$

and the condition of potentials' constancy $\psi_{1a} = u_a$, $\psi_{1k} = u_k$ is fulfilled on the electrodes' boundaries. Due to equation (3), there exists function $\varphi_1$the harmonicψ_1 conjugate and it is possible to allow the complex potential of electrostatic field W (x_1,y_1) = $\varphi_1(x_1,y_1)$ + $i\psi_1(x_1,y_1)$ that is the analytical function in the field z_1=x_1+iy_1.

Let's allow typical values of current density $j_a = \frac{\rho v}{\varepsilon}$, length $H = \kappa(u_a - u_k)/j_0$ (κ – electrical conductivity of the circumference) and move on to the dimensionless variables

$$x = \frac{x_1}{H}, \qquad y = \frac{y_1}{H}, \qquad z = x + iy, \qquad W = \varphi + i\psi = W_1 - \frac{iu_k}{u_a - u_k}$$

Then, taking into account (2) that function ψ in the interelectrode field is consistent with Laplace's equation and the boundary conditions on the electrodes' surface

$$\psi_a = 1, \qquad \psi_k = 0, \qquad \frac{\partial \psi_a}{\partial n} = \frac{j_a}{j_0} = a_0 + b_0 \cos\theta$$

where $a_0, b_0 -$ constants sensitive to the dependency of current efficiency on its density.

Therefore the initial task is modeled by potential plane-parallel flow of ideal incompressible liquid. Moreover, the analogue of $\vec{\mathrm{E}}$ electric field intensity is the speed of dummy flow $\vec{V}$ while vectors $\vec{\mathrm{E}}$ and $\vec{V}$ are orthogonal. The following equation $\partial\psi/\partial n$= V, V $=|\vec{V}|$ is fulfilled along the line ψ =const. V $=a_0 + b_0 \cos\theta$ on the anode surface.

A two-dimensional problem is studied in this research. The IES field in plane$z = x + i$, determined by symmetry lines A_1C and B_1D is represented in figure 3, where CD – cathode boundary, A_1ABB_1 – anode boundary, A_1B_1, - points at infinity. The anode boundary is divided into two fields: straight-line segmentsA_1A, B_1B, where V_a=0 and segment AB where condition (2) is fulfilled. The origin of coordinates is C.

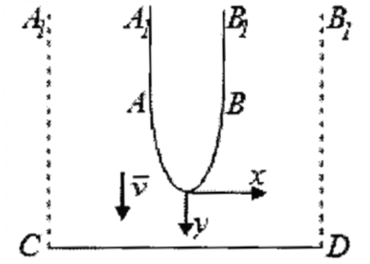

Figure 3. IES field.

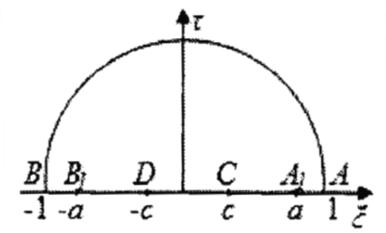

Figure 4. D_u field.

The task at issue in dimensionless form reduces itself to the search of boundary AB on the anode surface in the following task. Function ψ is consistent with Laplace's equation $\Delta\psi$ = 0, on the electrodes' boundaries the following conditions are fulfilled: $\psi|_{CD} = 0, \psi|_{A_1BBB_1} = 1$, $\frac{\partial\psi}{\partial n|_{AB}} = a + b\cos\theta$, on symmetry lines A_1C *and* B_1D – conditions $\frac{\partial\psi}{\partial n|_{A_1C}} = 0, \frac{\partial\psi}{\partial n|_{B_1D}} = 0$.

For the task at issue the flow of dummy stream is created by the system of continuously distributed sources and sinks: sources – along lineA_1C, sinks – along lineB_1D. Condition $V = a_0 + b_0\cos\theta$ is fulfilled on the unknown boundary AB.

Let's suppose that in the plane of additional complex variable $u = \xi + i\tau$ flow range D_z is equivalent to (figure 4)

$$D_u = \{|u| \le 1, \tau \ge 0\}$$

and function z (u) conformally reflects field D_u to field D_z c with the points' correspondence as shown in figures 3,4.

Let's calculate two functions: complex potential of dummy flow [3] W (u) = $\varphi(u)$ + $i\psi(u)$ and Joukowski function [9]

$$x(u) = ln\left(\frac{V_0 dz}{dW}\right) = r(u) + i\theta(u) \tag{4}$$

where $r(u) = \ln V_0 / V$, V- dummy flow speed module, Θ - inclination angle of speed vector to axis x, $V_0 = a_0$- speed value at point A.

Complex potential W (u) is consistent with the boundary conditions

$$\psi(u) = \begin{cases} 0, u = \xi, |\xi| \leq c \\ 1, u = \xi, d \leq |\xi| \leq 1; u = e^{i\sigma}, 0 \leq \sigma \leq \pi \end{cases}$$

$$\varphi(u) = \begin{cases} 0, u = \xi, c \leq \xi \leq d \\ \varphi_0, u = \xi, d \leq |\xi| \leq 1, -d \leq \xi \leq -c \end{cases}$$

W (u) function range – rectangular (figure 5)

$$D_w = \{\varphi + i\psi | 0 \leq \varphi \leq \varphi_0, \quad 0 \leq \psi \leq 1\}$$

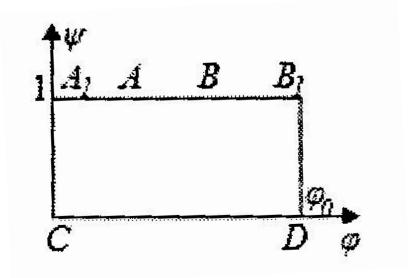

Figure 2. D_w field.

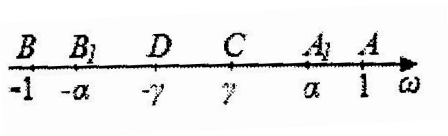

Figure 3. D_w field.

To determine derivative *dW/du* of complex potential we reflect D_u field to upper semiplane D_a with the points' correspondence as shown in figures 5, 6, using the transformation

$$x\omega(u) = \frac{2u}{1+u^2} \tag{4}$$

then, in accordance with Schwarz-Christoffel formula [7] we find the function reflecting D_w field to the *W* (u) function range:

$$W(\omega) = N \int_\gamma^\omega \frac{d\omega}{\sqrt{(\omega^2-\alpha^2)(\omega^2-\gamma^2)}},$$

$$N = \left(\frac{1}{\gamma} K\left(\frac{\alpha}{\gamma}\right) - \frac{1}{\alpha} K\left(\frac{\gamma}{\alpha}\right)\right)^{-1}, \ \varphi_0 = \frac{2K(\gamma/\alpha)}{\gamma K\left(\frac{\gamma}{\alpha}\right) - \alpha K(\alpha/y)},$$

$$K(k) = \int_0^1 \frac{d\omega}{\sqrt{(\omega^2 - 1)(k^2\omega^2 - 1)}}$$

where $K(k)$ – complete elliptic integral of the first kind [10]. Differentiating *W* (ω) on variable ω and taking into account the dependency (5), we see:

$$\frac{dW}{du} = \frac{2N(1-u^2)}{\sqrt{4u^2-a^2(1+u^2)^2}\sqrt{4u^2-c^2(1+u^2)^2}}$$

Let's take a closer look at Joukowski function for the dummy flow. On the cathode boundary imaginary part $\lambda(u)$ is equal to zero. The stationary condition of forming (2) is fulfilled on the anode boundary AB. There are the following boundary conditions for Joukowski function $\lambda(u)$

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$$\begin{cases} \frac{\pi}{2}, u = \xi, -1 \le \xi \le -\alpha \\ 0, u = \xi, |\xi| < \alpha \\ -\frac{\pi}{2}, u = \xi, \alpha \le \xi \le 1 \end{cases}$$

$$r(u) = -\ln\left(1 + \frac{b_0}{a_0} cos\theta(u)\right), u = e^{i\sigma}, 0 \le \sigma \le \pi.$$

We are going to find Joukowski function $x(u)$ in the following way

$$x(u)x = x_0(u) + f(u),$$

where function $x_0(u) = r_0(u) + i\theta_0(u)$ is consistent with the boundary conditions

$$\theta_0(u)=\theta_0(u), u = \xi, |\xi| \le 1,$$

$$r_0(u) = 0, u = e^{i\sigma}, 0 \le \sigma \le \pi$$

and has the same peculiarities in field D_u that $\lambda(u)$has, function $\lambda(u)$- analytical in D_u and continuous in $\overline{D_u}$.

The boundary conditions for $\lambda_0(u)$ allow to construct function $d\lambda_0/du$, which is real when $u = \xi, |\xi| \le 1$ and has a simple pole with residue ½ at points $A_1(u = a), B_1 = (u = -\alpha)$. Let's extend function $\frac{\lambda(u)}{du}$ to the whole plane in accordance with the principle of symmetry and construct it using the singularity method [9]. Then, integrating we find out:

$$\lambda_0(u) = \frac{1}{2}\ln\left(\frac{u^2 - a^2}{1 - a^2u^2}\right) + \mathrm{A}_0 u + B_0$$

Using the boundary conditions at points C and D, we find constants $A_0 = 0, B_0 = 0$.

Comparing the boundary conditions for functions $x(u)$ and $x_0(u)$, we get the boundary conditions for the unknown function $f(u) = \lambda(u) + i\mu(u)$:

$$\mu(u) = 0, u = \xi, |\xi| \le 1,$$

$$\lambda(u) = -\ln\left(1 + \frac{b_0}{a_0} \cos\theta\,(u)\right), u = e^{i\sigma}, 0 \le \sigma \le \pi.$$

Taking into account the boundary conditions (6), (7) and the symmetry of the dummy flow, function $f(u)$ can be analytically extended to the whole circle and presented in the form of Laurent expansion:

$$f(u) = \sum_{n=0}^{\infty} c_{2n} u^{2n},$$

where c_{2n} – actual coefficients.

Based on the condition $f(1) = 0$ we find $c_0 = -\sum_{n=0}^{\infty} c_{2n}$,

Condition (7), as function $f(u)$ has been presented in the form of the expansion, looks like

$$c_0 + \sum_{n=1}^{\infty} c_{2n} \cos(2n\sigma) = -\ln(1 + \frac{b_0}{a_0} \cos\theta(\sigma))$$

Multiplying both parts of equation (8) by$\cos(2n\sigma)$, integrating over σ within $0, \pi/2$, we get infinite system of equations for defining coefficients c_{2n}:

$$c_{2n} = -\frac{4}{\pi}\int_0^{\frac{\pi}{2}} \ln(1 + \frac{b_0}{a_0} cos\theta_0(\sigma) + \mu(\sigma)) \cos(2n\sigma) d\sigma, n = \overline{1,\infty}.$$

We calculate the dimensionless coordinates of AB anode boundary points based on (4) using the formula

$$z(u) = \frac{1}{a_0}\int_i^u \frac{dW}{du}\exp\big(x(u)\big)\,du = e^{i\sigma}, 0 \le \sigma \le \pi$$

To solve the task it is necessary to find mathematical parameters c and a. We can do it specifying intervals |CD| and $|A_1B_1|$:

$$z(-c)-z(c)=|CD|,\ \ z(-a)-z(a)=|A_1B_1|$$

It is possible to determine the mass of dispersed powder considering the bar axially symmetric and using the formula for calculation of square of the rotary surface.

$$m = 4\pi\alpha_0\rho H^2 \int_0^{\pi/2} x(\sigma)\exp(-r(\sigma))|\frac{dz}{d\sigma}|d\sigma$$

where ρ – metal density.

The results of calculations of the boundary shape of anode surface for different special cases are shown in figures 7, 8.

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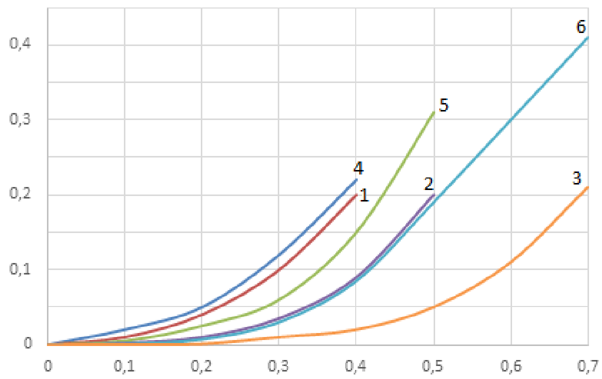

Figure 7. The results are calculated for α=0.1; b=0.9; |CD|=2.0 and different $|A_1B_1|$. Curves 1, 4 – $|A_1B_1|$=0.6; 2, 5 – $|A_1B_1|$=1.0; 3, 6 – $|A_1B_1||$=1.4. Curves 4, 5, 6 – unlimited cathode.

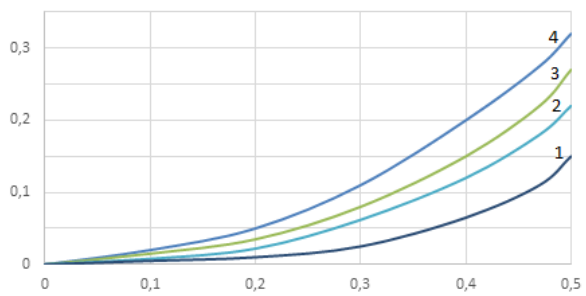

Figure 8. The results are calculated for α=0.1; b=0.9; $|A_1B_1|$=1.0 and different |CD|. Curves 1 – |CD|=1.5; 2 – |CD|=2.0; 3 – |CD||=3.0; 4 – unlimited cathode.

Curves 4, 5, 6 in figure 7 and also curve 4 in figure 8 fit the case when the cathode width tends to infinity in figure 3 |CD|→ ∞, what corresponds with c→a in figure 4.

4. Conclusion

The mathematical model qualitatively enough describes the removal of metal from the anode surface process, and also allows you to determine the approximate weight of the dispersed powder of a steel electrode in the discharge of liquid electrodes.

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