The effect of the unpaired nucleons on the β-decay properties of the neutron-rich nuclei

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Abstract. Starting from the T45 Skyrme interaction with tensor terms the properties of the β-decay of $^{72-80}$Ni are studied. We take into account the effect of unpaired neutron and proton on the ground state properties of odd-odd and even-odd nuclei. It is shown that the calculated Q_β-values and the β-decay half-lives are in a reasonable agreement with experimental data.

A study of the β-decay properties is an interesting problem not only from the nuclear structure point of view but it is very important for the nuclear astrophysics applications. It is desirable to have theoretical models which can describe the data wherever they can be measured, and predict the properties related to spin-isospin modes in the nuclei with extreme N/Z ratio to allow for experimental studies. One of the successful tools for studying charge-exchange nuclear modes is the quasiparticle random phase approximation (QRPA) with the self-consistent mean-field derived from a Skyrme energy-density functional (EDF), see e.g., [1, 2, 3, 4, 5]. These QRPA calculations enable one to describe the properties of the ground states and excited charge-exchange states using the same EDF. Our tool is the QRPA with Skyrme interactions in the finite rank separable approximation (FRSA) [6, 7, 8, 9], allowing one to perform calculations in large configuration spaces. The FRSA model for the charge-exchange excitations and the β-decay was already introduced in Refs. [10, 11] and in Refs. [12, 13], respectively.

The correct description of the Q_β-values is the important ingredient for the reliable prediction of the half-life of the β-decay. To calculate the binding energy of the odd-odd and even-odd nuclei we take into account the effect of the unpaired neutron and proton on the superfluid properties of nuclei, the well-known blocking effect [14, 15]. As an example, the β-decay properties of neutron-rich nuclei ^{72,74,76,80}Ni and the most neutron-rich ($(N-Z)/A = 0.28$) doubly-magic nucleus ^{78}Ni are studied. The β-decay properties of r-process "waiting-point nucleus" ^{78}Ni have attracted a lot of experimental efforts, see e.g., [16, 17, 18].

We use the EDF T45 which takes into account the tensor force [19]. The T45 set is one of 36 parametrizations, covering a wide range of the parameter space of the isoscalar and isovector tensor term added with refitting the parameters of the central interaction, where a fit protocol is very similar to that of the successful SLy parametrizations. This choice of the Skyrme EDF has been selected to reproduce the experimental Q_β value of ^{78}Ni (see Fig. 1) and enough positive value of the spin-isospin Landau parameter ($G'_0 = 0.10$ for T45). It is worth mentioning that the first study of the strong impact of the tensor correlations on the β-decay half-life has been done in [4, 5]. The pairing correlations are generated by a zero-range volume force with a strength of -270 MeVfm^3 and a smooth cut-off at 10 MeV above the Fermi energies [9]. This value of the

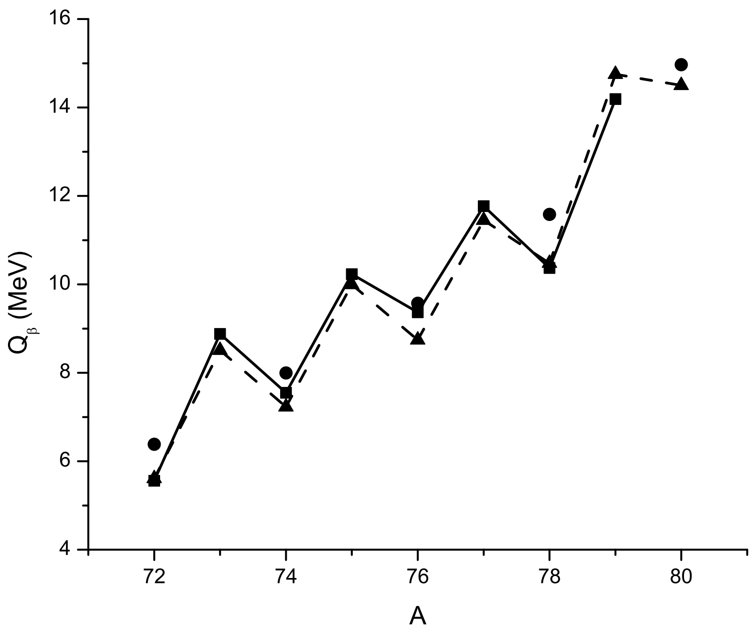

Figure 1. The quasiparticle blocking effect on Q_β-values of $^{72-80}$Ni. Results of the HF-BCS calculations with the blocking effect (triangles) and without the blocking effect (circles) are shown. Experimental data (squares) are from Ref. [26].

pairing strength has been fitted to reproduce the odd-even mass difference in the studied region of nuclei [12, 20].

Assuming the spherical symmetry for the nuclei considered here, the starting point of the method is the self-consistent HF-BCS calculation [21] for the ground state properties of the even-even parent nucleus (N,Z). The continuous part of the single-particle spectrum is discretized by diagonalizing the HF hamiltonian on a harmonic oscillator basis. In the particle-hole channel we use the Skyrme interaction with the tensor components and their inclusion leads to the modification of the spin-orbit potential [19, 22].

The ground state of the odd-odd daughter nucleus (N-1, Z+1) can be obtained as the neutron-quasiparticle proton-quasiparticle state. The neutron and proton quasiparticles can be simultaneously blocked [23]. Using the blocking effect for unpaired nucleons [14, 15, 21] we get the following secular equations,

$$\Delta_j = \frac{1}{2}\sum_{j'\neq j_2} V_{jj'}\frac{(2j'+1)\Delta_{j'}}{\sqrt{\Delta_{j'}^2+(E_{j'}-\lambda)^2}} + \frac{1}{2}V_{jj_2}\frac{(2j_2-1)\Delta_{j_2}}{\sqrt{\Delta_{j_2}^2+(E_{j_2}-\lambda)^2}}, \quad (1)$$

where the indexes j denote the quantum numbers nlj. The indexes j_2 emphasize the blocked neutron subshell and the blocked proton subshell near the Fermi energies. For ^{72,74,76,78}Cu the neutron quasiparticle blocking is based on filling the $1g_{9/2}$ subshell and the $2d_{5/2}$ subshell should be blocked for ^{80}Cu. The proton $2p_{3/2}$ and $1f_{5/2}$ subshells are chosen to be blocked in the cases of ^{72,74,76}Cu and ^{78,80}Cu, respectively. It is worth pointing out that there is the closeness of the proton single-particle energies $2p_{3/2}$, $1f_{5/2}$ for ^{76}Cu. The quasiparticle blocking calculations are discussed in more detail in Ref. [24].

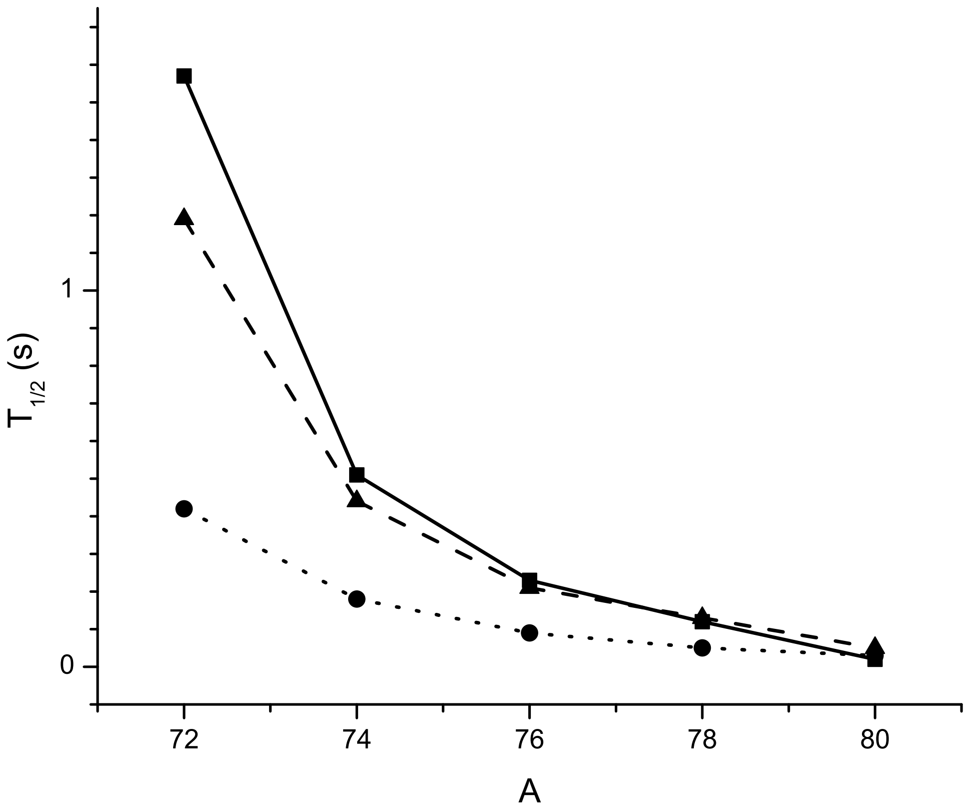

Figure 2. Same as Fig. 1, for the half-lives of the β-decay of ^{72,74,76,78,80}Ni. Experimental data are taken from Refs. [16, 29].

It is interesting to study the blocking effect on Q_β value. The Q_β value can be obtained by the binding-energy difference between the daughter and parent nuclei,

$$Q_\beta = \Delta M_{n-H} + B(Z+1, N-1) - B(Z, N). \quad (2)$$

$\Delta M_{n-H} = 0.782$ MeV is the mass difference between the neutron and the hydrogen atom. As proposed in Ref. [25], the Q_β value of the even-even nucleus can be calculated without the blocking effect,

$$Q_\beta \approx \Delta M_{n-H} + \mu_n - \mu_p - E_{2qp,lowest}, \quad (3)$$

where $E_{2qp,lowest}$ corresponds the lowest two-quasiparticle energy, μ_n and μ_p are the neutron and proton chemical potentials. The calculated Q_β values in the neutron-rich Ni isotones are compared with existing experimental data [26] in Fig.1. The results of the HF-BCS calculation with the blocking effect are in a reasonable agreement with the experimental data. There is a remarkable the odd-even isotope staggering. This feature observed in the calculation occurs because for an odd-neutron nucleus the odd neutron contributes strongly to the β decay. This contribution is absent in an even-even nucleus. For even-even nuclei, the Q_β analysis within the approximation (3) can help to clarify the blocking effect. We find that the blocking effect induces a reduction of the Q_β values and it results in a improvement of the Q_β description, see Fig.1.

To build the QRPA equations on the basis of HF-BCS quasiparticle states of the parent nucleus is the standard procedure [27]. Using the FRSA model the QRPA eigenvalues (E_k) are obtained as the roots of the relatively simple secular equation [10, 11] and we carry out QRPA calculations in very large two-quasiparticle spaces. The cut-off of the discretized continuous part of the single-particle spectra is at the energy of 100 MeV. This is sufficient to exhaust the Ikeda sum rule $3(N-Z)$ for the Gamow-Teller (GT) transitions.

In the allowed GT approximation, the β^--decay half-life is expressed by summing the probabilities (in units of $G_A^2/4\pi$) of the energetically allowed transitions ($E_k^{\mathrm{GT}} \leq Q_\beta$) weighted with the integrated Fermi function

$$T_{1/2}^{-1} = D^{-1}\left(\frac{G_A}{G_V}\right)^2 \sum_k f_0(Z+1, A, E_k^{\mathrm{GT}}) B(GT)_k, \tag{4}$$

$$E_k^{\mathrm{GT}} = Q_\beta - E_{1_k^+}, \tag{5}$$

where G_A/G_V=1.25 and D=6147 s [28]. $E_{1_k^+}$ denotes the excitation energy of the 1_k^+ state of the daughter nucleus. As proposed in Ref. [25], this energy can be estimated by the following expression:

$$E_{1_k^+} \approx E_k - E_{2qp,lowest}. \tag{6}$$

It is worth mentioning that the spin-parity of the lowest two-quasiparticle state is, in general, different from 1^+.

The properties of the low-lying 1^+ states in the daughter nuclei ^{72,74,76,78,80}Cu are studied. There is the gradual reduction of β-decay half-lives with increasing neutron number [16, 29], see Fig.2. One can see that our results calculated with the blocking effect reproduce this behaviour. As expected, the largest contribution (>60%) in the calculated half-life comes from the 1_1^+ state. QRPA results indicate that the dominant configuration of the 1_1^+ wave function is $\{\pi 2p\frac{3}{2}\nu 2p\frac{1}{2}\}$ whose contribution is about 99 % in all five nuclei. The inclusion of the blocking effect for the Q_β calculation reduces the transition energies (5) and this energy shift produces a sizable impact on the β-decay half-life.

In summary, by starting from the Skyrme HF-BCS calculations the Q_β-window has been studied within the model including the blocking effect of unpaired neutron and proton in cases of the even-odd and odd-odd nuclei. Using the EDF T45 containing the tensor terms, we analyze the effect on the β-transition rates of the neutron-rich nuclei $^{72-80}$Ni. It is shown that the inclusion of this effect definitely improves the description of the Q_β-value and the β-decay half-life. A further systematic study of the blocking effect on the β-decay properties is clearly necessary and is in progress.

Acknowledgments
We thank N. N. Arsenyev and I. N. Borzov for useful discussions. This work is partly supported by from the Russian Science Foundation (Grant No. RSF-16-12-1016).

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