Control of coupled localized nonlinear wave solutions

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Abstract. A method of forced localization of non-linear wave by a feedback control is developed for coupled equations accounting for non-linear dynamic processes in complex lattices. It is shown, that the control of the shape and velocity of the wave function of macro-strain allows to achieve localization of the shape of the function describing variations of defects in the lattice. Moreover, change of the sign of the amplitude of the last wave may be achieved by variation of the parameters of the control function but independent of the initial conditions.

Introduction

Exact localized traveling wave solutions to non-linear wave equations may be found by various methods, see, e.g., [1, 2]. Their existence results from a balance between contributions of the terms of the equations which in turn reflect a an influence of physical factors such as nonlinearity, dispersion, dissipation, diffraction. However, existence and stable propagation of localized waves also depend on the choice of initial and boundary conditions. The consistent initial conditions should coincide with the exact solutions and their corresponding derivatives at $t = 0$. For coupled equations, an extra factor concerns relative positions of the inputs for the coupled variables [3, 4, 5]. Variations in the shape of the initial conditions and their initial velocities result in failing of localization, arising of oscillations, serious variations in the shape of the waves.

Among the methods allowing support of stable propagation of localized waves one can note the control methods [6, 7, 8]. Recently a distributive feedback control algorithm has been developed for single non-linear equations [9, 10, 11]. It was found that both the kink-shaped and envelope wave solutions may be achieved for the sine- Gordon equation if both the tendencies to a desired shape and velocity of the wave are included into the control function. The control function is physically reasonable for the sine- Gordon equation describing a crystalline lattice continuum limit if it is related to an external loading. When the wave achieves the shape and the velocity of the desired or target function, the control function becomes very small. Thus its addition to the equation does not seriously change it.

In this paper, a feedback control algorithm is developed for non-linear coupled equations arising for continuum description of highly non-linear dynamic processes in complex lattices [12, 13, 14, 15]. The only one function may be physically reasonable controlled - the function of macro-strain. Another function in the equations accounts for internal variations in the crystalline

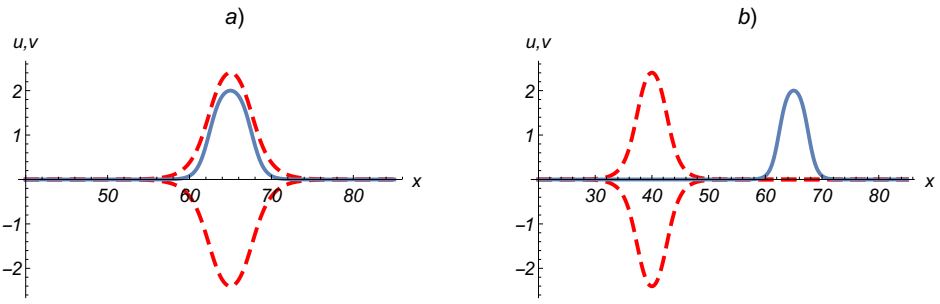

Figure 1. Initial profiles, dashed line for u, solid line for v: a) $x_{11} = x_{12} = 65$; b) $x_{11} = 65$, $x_{12} = 40$.

structure. In Sec. 1 the equations and their known exact traveling wave solutions are discussed. Special attention is paid on variations in the solutions when the positions of the inputs for the functions in the equation do not coincide. Next section is devoted to application of the feedback distributive control algorithm for the macro-strain function and numerical solutions demonstrating recovering of the localization of directly non-controlled function of internal variation in the crystalline structure. Also change of the sign of the amplitude of the localized wave out of the control is achieved by modification of the control of another function in the equations.

1. Non-linear coupled equations for complex lattices

Recently nonlinear coupled equations were obtained in Refs. [12, 13, 14, 15] to account for highly nonlinear dynamic processes in crystals in the continuum approach,

$$v_{tt} - c_L^2 v_{xx} = \frac{S}{\rho}(\cos(u))_{xx}, \tag{1}$$

$$u_{tt} - c_l^2 u_{xx} = \frac{1}{\mu}(Sv - P)\sin(u). \tag{2}$$

Here v is longitudinal macro-strain, u is micro-displacement characterizing movement of defects in a di- atomic crystals [13, 14] or an influence of an additional rotational degree of freedom in an atomic chain [12].

Exact localized traveling wave solution represents coupled functions u, v [15],

$$v = \frac{A}{Q\,\cosh(k\,(x - Vt - x_{11})) + 1}, \tag{3}$$

$$u = \pm \arccos\left(\frac{\rho(V^2 - c_L^2)v^*}{S} + 1\right), \tag{4}$$

where $v = v^*$ and the parameters are

$$A = \frac{4S}{\rho(c_0^2 + c_L^2 - V^2)},\ Q = -\frac{c_L^2 - c_0^2 - V^2}{c_L^2 + c_0^2 - V^2}, k = 2\sqrt{\frac{P}{c_l^2 - V^2}}.$$

One has to note that initial positions of both waves are defined by one and the same parameter x_{11}, see Fig. 1(a), and their phase velocity V is the same. Also the function u may have an amplitude of either sign. The initial conditions may be chosen in the form of exact solution for

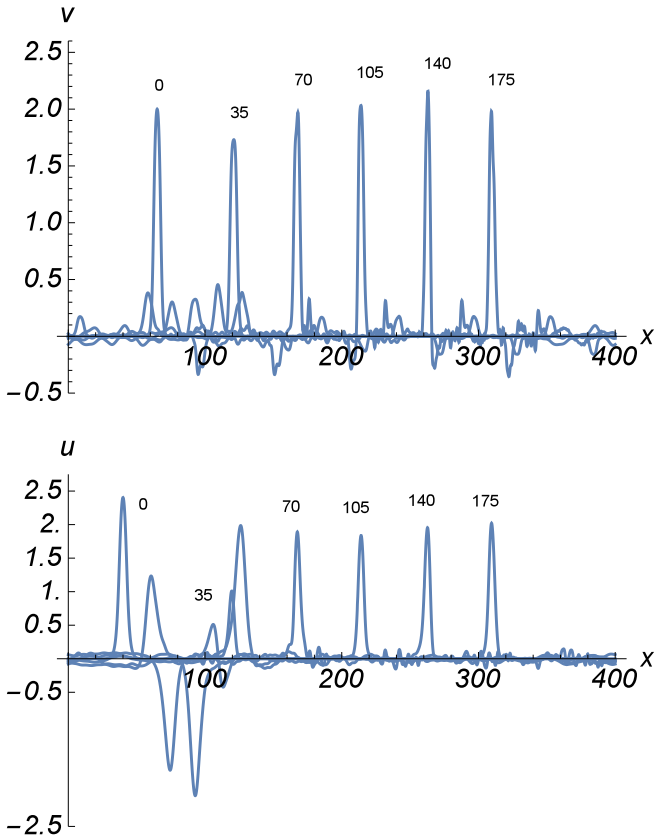

Figure 2. Evaluation of the waves at $x_{11} = 65$, $x_{12} = 40$ without control. Times are noted at corresponding maxima/minima of the wave profiles.

v (3), but with initial function for u with

$$v^* = \frac{A}{Q\,\cosh(k\,(x - Vt - x_{ij})) + 1}, \tag{5}$$

where x_{ij} may be chosen not equal to x_{11}, say, equal to x_{12}. This is shown in Fig. 1(b). Numerical simulations in [3, 4, 5] revealed failing of stable propagation of the exact traveling wave solution (3), (4) at initial condition with v^* defined by Eq. (5). In particular change of the sign of the amplitude of the wave u happens depending on the difference between x_{11} and x_{12}.

Consider the case with the values of the coefficients $c_L = 1.6, c_l = 2, c_0 = 1, S = 1, P = 1, \rho = 1$ and with the values of the parameters of the initial conditions $V = 1.3, x_{11} = 65, x_{12} = 40$. Numerical simulations are performed using the NDSolve tool of the Wolfram Mathematica 11. Shown in Fig. 2 is the evolution of the waves that demonstrates an influence of the difference between initial positions on stable propagation of localized waves. Left profiles correspond to the initially mutually spaced localized inputs. One can see variation in the amplitudes for the wave v and appearance of oscillations. The profile for the wave u suffers even temporal change of the sign of the amplitude. Finally the maxima of the wave come to the same position, and the waves propagate together with one and the same velocity. Similar behavior is observed for negative amplitude input for u.

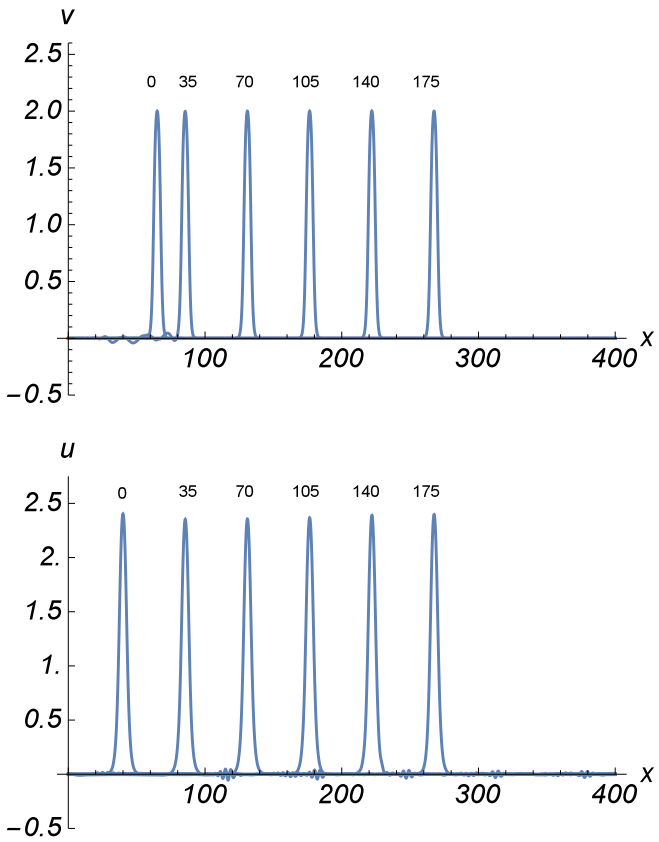

Figure 3. Support of the waves localization by the control. Times are noted at corresponding maxima/minima of the wave profiles.

2. Control of localization of the coupled waves

Physically reasonable control concerns only the macro-strain function v that may be explained as an external loading. The control of the internal structure variable u is unlikely. Then a control function $w(x,t)$ is introduced only in one equation (1),

$$v_{tt} - c_L^2 v_{xx} = \frac{S}{\rho}(\cos(u))_{xx} + w(x,t), \tag{6}$$

while Eq. (2) does not contain any control term. The function w controls only the behavior of the function v.

Following Refs. [9, 10, 11] let us choose the goal function in the form of exact solution for v (3), but with initial phase defined as x_{ij} in Eq. (5). The function $v_t^*(x,t)$ is defined as the first temporal derivative of Eq.(5). Then one defines

$$e(x,t) = v(x,t) - v^*(x,t), \tag{7}$$

$$e_t(x,t) = v_t(x,t) - v_t^*(x,t). \tag{8}$$

and

$$\epsilon(x,t) = \alpha e(x,t) + e_t(x,t), \tag{9}$$

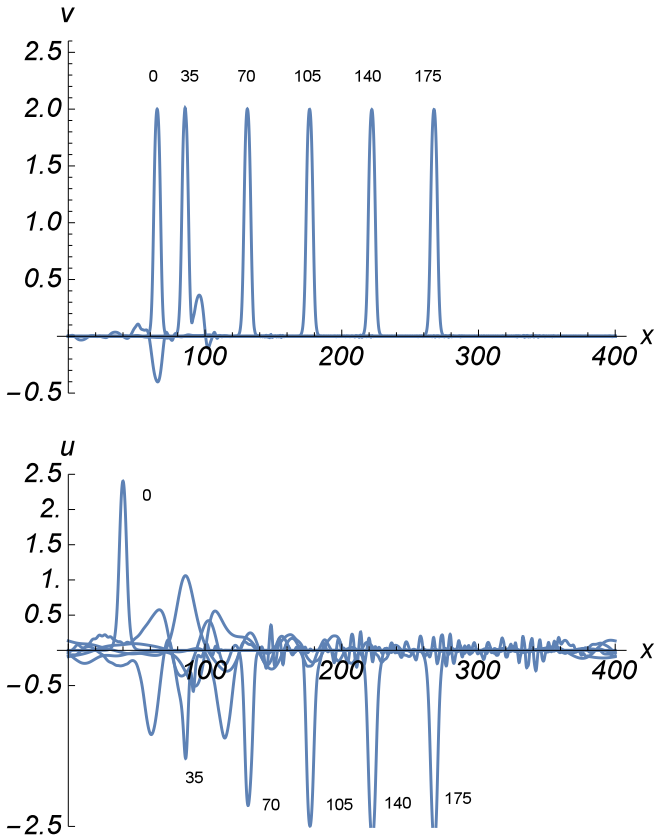

Figure 4. Change of the sign of the amplitude of localized wave due to the control. Times are noted at corresponding maxima/minima of the wave profiles.

Like in [9, 10, 11], the function w is defined as

$$w(x,t) = -\gamma\epsilon(x,t), \tag{10}$$

where $\gamma > 0$.

Numerical simulations are shown in Fig.3 for the choice of the control parameters $\gamma = 0.16, \alpha = 160$. Comparing with Fig.2, one can see recovering of stable propagation of localized wave u with no more variations in the sign of the amplitude, and no more variations in the amplitude of v is seen. The oscillations almost disappear on the profiles of both functions.

One can see an interval for evolution of u in Fig. 2 when the amplitude changes its sign. However, then the original sign of the amplitude is recovered. One can switch on the control at time corresponding to the interval of the negative sign by modifying the control function,

$$w(x,t) = -\gamma\epsilon(x,t)H(t-t_b), \tag{11}$$

where H is the unit-step function, t_b is time for switch on. Shown in Fig. 4 is numerical simulations with modified control function (11) and with $t_b = 19$. One can see that the wave u changes its sign and continue to propagate like in the case without control when the initial condition is chosen with negative sign, see Fig. 1. The choice of t_b is sensitive to small variations. In particular, choice of $t_b = 18$ results only in a more longer negative amplitude wave propagation than in Fig. 2 with forthcoming recovering of the sign of the amplitude. An increase in the value of t_b, say, $t_b = 23$, gives rise to the scenario similar to that shown in Fig. 3 without change of the sign of the amplitude of wave u.

3. Conclusions

A distributive feedback control may support localization of nonlinear coupled equations even if only one of the functions is controlled. The sign of the localized wave may be changed by the control independent of the initial conditions. For the equations considered, it means that an external loading plays the role of the the control of the shape and velocity of the macro-strain wave in the lattice, while another coupled function accounting for the movement of defects of the crystalline structure also suffers serious variations.

Acknowledgments

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