

Mathematical modelling of risk reduction in reinsurance

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Abstract. The paper presents a mathematical model of efficient portfolio formation in the reinsurance markets. The presented approach provides the optimal ratio between the expected value of return and the risk of yield values below a certain level. The uncertainty in the return values is conditioned by use of expert evaluations and preliminary calculations, which result in expected return values and the corresponding risk levels. The proposed method allows for implementation of computationally simple schemes and algorithms for numerical calculation of the numerical structure of the efficient portfolios of reinsurance contracts of a given insurance company.

1. Introduction

Reinsurance is a system of financial relations, during which the insurer, while forming the insurance portfolio, delegates part of the responsibility for the risk on certain agreed terms to other insurers to create a balanced portfolio of own insurance, ensuring financial stability and profitability of insurance operations within the generated portfolio. Consequently, the financial reinsurance principle is redistribution of the primary insurance fund among insurance companies [1, 2]. Reinsurance agreements differ in the ways risk is shared among insurers and reinsurers. There are three risk transfer systems for reinsurance: quota share, surplus and combined. In the case of quota share reinsurance, the reinsurer is involved in all the risks of the insurer in a certain proportion. Consequently, the quota share reinsurance system provides a great opportunity to create an insurance portfolio consisting of shares of the aggregate sums insured. In the case of the surplus reinsurance system the insurer passes to the reinsurer the insurance amount in excess of the limit value specified in the contract. If the limit value is, for example, 10 million rubles, the implied risks of an insurance amount lower than the limit value are not transferred to reinsurer and fully retained by the insurer. According to the insurance liabilities, when the sum insured exceeds the limit, the insurer reduces their risk to 10 million rubles, while the remainder transfers to the reinsurer. Usually in practice surplus reinsurance agreements are concluded for large objects, such as power lines, gas pipelines, oil pipelines, nuclear power plants i.e. when the insurance value is high. A mixed system resides in establishing quota share insurance up to a certain limit of the insurance value beyond which the transfer of risk is accomplished by means of the surplus system [1–6]. Reinsurance market contributes to balancing insurance portfolio and the financial stability of the effective performance of insurance companies and their operations. The development of the reinsurance market, and mutual confidence of the insurance companies allows the reinsurance market to



ensure the efficient functioning of insurance markets and, as a result, has a positive effect on the insurance market and the industries involved in the insurance process. International economic relations allow Russian insurance companies to work in international markets. Therefore, the reinsurance market is a regulator of the market of insurance services in Russian Federation [1–6]. Since the main purpose of the effective portfolio formation of an insurance company is the redistribution of risk in order to reduce its aggregate value, while maintaining an acceptable level of profitability, the task of forming an effective composition of portfolios is one of the main tasks of analytical management in insurance companies.

2. The setting of the problem

The problem of efficient portfolio formation in the reinsurance market, considered in this paper is states as follows (see also [7]):

$$\begin{aligned} & \sum_{i=1}^n x_i w_i R_i - \max, \sum_{i=1}^n x_i w_i r_i - \min \\ & 0 \leq x_i \leq \beta_i \leq 1, \quad i = 1, \dots, n \\ & \sum_{i=1}^n x_i V_i = S, \quad \sum_{i=1}^n \beta_i V_i > S, \quad w_i = \frac{V_i}{V}, \quad V = \sum_{i=1}^n V_i, \end{aligned} \quad (1)$$

where $x_i, i = 1, \dots, n$ - share of the insurance coverage in the i - th contract of reinsurance included in the portfolio; $R_i, i = 1, \dots, n$ - the expected value of return in the i - th contract of reinsurance (for example, profitability or normalized return); $r_i, i=1, n$ - the risk value of an insured event in the i - th contract of reinsurance (for example, the possibility of income deficiency); $V_i, i = 1, \dots, n$ - the total insured amount in the i - th contract of reinsurance; S - the total insured amount of the portfolio ($0 < S < V$), n - the number of potential contracts for reinsurance, which are tested for their possible inclusion in the contract portfolio of the insurance company.

The values of $R_i, i = 1, \dots, n$ $r_i, i = 1, \dots, n$ are determined according to expert evaluations of the uncertainties in return of insurance contracts or based on the processing of statistical data [8]. This problem has two criteria; Pareto solutions correspond to efficient portfolios of contracts of the insurance company.

To calculate the Pareto solutions of this problem it is sufficient to find the solutions of the two linear programming problems with linear objective functions

$$\sum_{i=1}^n x_i w_i R_i$$

and

$$\sum_{i=1}^n x_i w_i r_i,$$

respectively, and then to take their linear combination with an acceptable level of compromise between the expected values of profitability and risk generated by the portfolio.

Thus, finding efficient portfolios for reinsurance contracts in the statement (1) is reduced to single criterion solutions of two problems:

the problem of maximizing the efficiency of the portfolio

$$\begin{aligned} & \sum_{i=1}^n x_i w_i R_i - \max \\ & 0 \leq x_i \leq \beta_i \leq 1, \quad i = 1, \dots, n \\ & \sum_{i=1}^n x_i V_i = S, \quad \sum_{i=1}^n \beta_i V_i > S, \quad w_i = \frac{V_i}{V}, \quad V = \sum_{i=1}^n V_i, \end{aligned} \quad (2)$$

and the problem of minimizing portfolio risk

$$\begin{aligned} & \sum_{i=1}^n x_i w_i r_i - \min \\ & 0 \leq x_i \leq \beta_i \leq 1, \quad i = 1, \dots, n \\ & \sum_{i=1}^n x_i V_i = S, \quad \sum_{i=1}^n \beta_i V_i > S, \quad w_i = \frac{V_i}{V}, \quad V = \sum_{i=1}^n V_i, \end{aligned} \quad (3)$$

3. Solutions

Problems (2),(3) are linear programming problems, each of which has a unique solution \vec{x}_1^*, \vec{x}_2^* , respectively

$$\begin{aligned} \vec{x}_1^* &= (x_{11}^*, \dots, x_{1n}^*)^T \\ \vec{x}_2^* &= (x_{21}^*, \dots, x_{2n}^*)^T \end{aligned}$$

The entire set of Pareto solutions of (1) defined by the equality

$$\vec{x}^*(\alpha) = \alpha \vec{x}_1^* + (1 - \alpha) \vec{x}_2^* \quad (4)$$

where $\alpha \in [0, 1]$.

Choosing a numerical value priority α , we obtain the specific structure of the effective reinsurance portfolio. In particular, in the case where $\alpha = 1$, we obtain the structure of the effective portfolio of reinsurance company, that corresponds to the maximum expected value of return, and in the case where $\alpha = 0$, we obtain the structure of the effective reinsurance portfolio, that corresponds to the minimum value of risk.

The numerical solution of linear programming problems (2),(3) can be realized with the help of, the two-stage method for solving linear programming problems under group constraints, developed by one of the authors of this paper [9].

Table 1 shows the results of numerical solution of problems (2),(3) for the following initial data: $V = 750$ million rubles, $S = 150$ million rubles;

$$\vec{W} = (0.05, 0.15, 0.1, 0.03, 0.07, 0.12, 0.03, 0.01, 0.04, 0.11, 0.06, 0.23)^T;$$

$$\vec{R} = (0.13, 0.05, 0.07, 0.1, 0.11, 0.12, 0.07, 0.09, 0.08, 0.13, 0.14, 0.04)^T;$$

$$\vec{r} = (0.05, 0.03, 0.07, 0.09, 0.15, 0.1, 0.06, 0.15, 0.25, 0.12, 0.08, 0.14)^T;$$

$$\vec{\beta} = (0.25, 0.5, 0.35, 0.25, 0.3, 0.2, 0.25, 0.1, 0.15, 0.1, 0.5, 0.3)^T.$$

The problem of efficient portfolio formation in the reinsurance market (1) can be considered in the case of the more severe group restrictions under which the mathematical model of efficient portfolios is stated as (5).

Table 1. Results of numerical solution.

	$x_{1_i}^*$	$x_{2_i}^*$	$x_{1_i}^* \cdot V_i$, mln. RUB	$x_{2_i}^* \cdot V_i$, mln. RUB
1	0	0.25	0	9.375
2	0.29(3)	0.5	33	56.25
3	0	0.35	0	26.25
4	0	0.25	0	5.625
5	0.3	0.3	15.75	15.75
6	0.2	0.0375	18	3.375
7	0	0.25	0	5.625
8	0.1	0.1	0.75	0.75
9	0	0.15	0	4.5
10	0.1	0	8.25	0
11	0.5	0.5	22.5	22.5
12	0.3	0	51.75	0
Total			$S = 150$	$S = 150$

$$\begin{aligned}
& \sum_{i=1}^n x_i w_i R_i - \max, \sum_{i=1}^n x_i w_i r_i - \min, \\
& 0 \leq \alpha_j \leq x_{j_1} + x_{j_2} + \dots + x_{j_{n_j}} \leq \beta_j \leq 1, \quad j = 1, \dots, m, \\
& \sum_{j=1}^m \beta_j \geq 1, \sum_{j=1}^m \alpha_j \leq 1, \sum_{j=1}^m n_j = n,
\end{aligned} \tag{5}$$

where m is the number of groups.

Then, instead of problems (2),(3) we approach, respectively, problems (6),(7):

$$\begin{aligned}
& \sum_{i=1}^n x_i w_i R_i - \max, \\
& 0 \leq \alpha_j \leq x_{j_1} + x_{j_2} + \dots + x_{j_{n_j}} \leq \beta_j \leq 1, \quad j = 1, \dots, m, \\
& \sum_{j=1}^m \beta_j \geq 1, \sum_{j=1}^m \alpha_j \leq 1, \sum_{j=1}^m n_j = n,
\end{aligned} \tag{6}$$

$$\begin{aligned}
& \sum_{i=1}^n x_i V_i = S, \sum_{i=1}^n \beta_i V_i \geq S, \quad w_i = \frac{V_i}{V}, \quad V = \sum_{i=1}^n V_i, \\
& \sum_{i=1}^n x_i w_i r_i - \min, \\
& 0 \leq \alpha_j \leq x_{j_1} + x_{j_2} + \dots + x_{j_{n_j}} \leq \beta_j \leq 1, \quad j = 1, \dots, m, \\
& \sum_{j=1}^m \beta_j \geq 1, \sum_{j=1}^m \alpha_j \leq 1, \sum_{j=1}^m n_j = n, \\
& \sum_{i=1}^n x_i V_i = S, \sum_{i=1}^n \beta_i V_i \geq S, \quad w_i = \frac{V_i}{V}, \quad V = \sum_{i=1}^n V_i,
\end{aligned} \tag{7}$$

Numerical solutions of specific problems of linear programming type (6),(7) can also be computed using the effective two-stage method [9].

4. Conclusion

The balance of the insurance portfolio leads to financial sustainability and efficient operation of insurance companies. The development of the reinsurance market, and mutual confidence of insurance companies ensure reinsurance market its efficient functioning, which will positively impact the insurance market. The internationalization of economic relations makes it possible for Russian insurance companies to operate not only in the domestic market, but also on the international reinsurance markets, creating efficient portfolios of reinsurance contracts with the inclusion of foreign market participants.

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