

DOA estimation based on multiple beamspace measurements sparse reconstruction for manoeuvring towed array

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Abstract. The port-starboard ambiguity in the conventional single towed linear array sonar is one of the most deceiving obstacles which exist in the way of development of spatial spectrum estimation. In order to improve the performance of target detection and Direction of Arrival (DOA) estimation, this paper proposes a novel spatial spectrum sparse reconstruction method based on multiple beamspace measurements (MBM-SR). An array sparse signal model for manoeuvring towed array is established. Then the Mutual Incoherent Property (MIP) is analyzed to ensure the proposed algorithm possessing better spatial spectrum reconstruction property. Simulation results demonstrate that, compared with Conventional Beam Forming (CBF) algorithm, the proposed algorithm has evident advantage in ambiguity suppression ratio (ASR) and estimation performance.

1. Introduction

Towed hydrophone arrays are useful for detecting submerged submarines and monitoring vocalizing marine mammals [1]. The conventional towed arrays comprise multiple omni-directional hydrophone elements drawn behind a vessel. When towing an in-line hydrophone array from a vessel sailing in a straight line, it is impossible to determine which side of the vessel an underwater sound source originates from, because the signal time difference of arrival (TDOA) is same from either side, referred to as ‘port-starboard ambiguity’. Port-starboard ambiguity can be resolved by changing the heading of the vessel, but in many applications, the array shape is treated as a straight line which can result in large variance in the bearing estimates and influence the stable target tracking during vessel turning. In order to resolve the port-starboard ambiguity effectively, various towed arrays are studied such as twin-line array [2], array with hydrophone triplets [3] and acoustic vector sensor array [4]. But these arrays are not used widely in military and industrial operations because of increased costs, sophisticated technologies and limited deployment possibilities. Therefore, how to develop methods of signal processing to realize DOA estimation without port-starboard ambiguity for towed hydrophone array is still a challenging research topic. In practice, when the vessel manoeuvres, the towed array may not be a true line array. The distorted array shape induced by the turning of vessel is utilized for resolving port-starboard ambiguity [5, 6], because the symmetric feature of TDOA is destroyed by the distorted array. However, the distorted array can result in reduced signal gain in the beamforming process and limited snapshot number can restrict the use of many adaptive covariance based approaches.



Compressive sensing is a new signal sparse reconstruction technique which requires fewer measurements to reconstruct the signal, which is sparse in some basis vectors [7-10]. In this paper, a novel approach to DOA estimation in beamspace for manoeuvring towed array is proposed. Utilizing multiple beamspace measurements, the sparse reconstruction algorithm results in high ambiguity suppression ratio and good estimation performance.

The outline of the paper is organized as follows. In section 2, we present the array signal model for manoeuvring towed array. In section 3, the details of spatial spectrum sparse reconstruction algorithm based on multiple beamspace measurements is discussed. Simulation results are presented in section 4. Section 5 concludes the paper.

2. Array sparse signal model for manoeuvring towed array

2.1. Spatial spectrum sparse reconstruction in beamspace

To simplify the subsequent discussion, the coordinate system of manoeuvring towed array is illustrated in figure 1. The towed array is made up with M sensors with equal separation d . Narrowband far field source from the direction θ_q impinges on the towed array. The first sensor that is the closest to the towing vessel is located at original point.

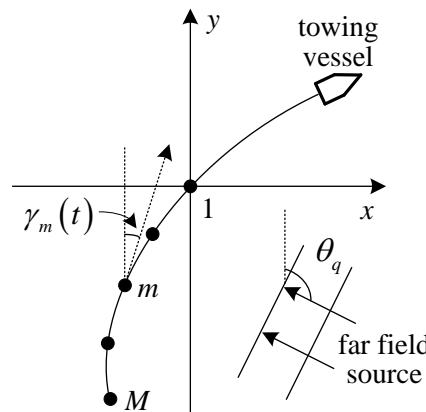


Figure 1. Coordinate system of manoeuvring towed array.

When the towing vessel is turning, the shape of array is distorted. Then with the AR model, the shape of array can be described as [11]

$$\boldsymbol{\gamma}(t) = \boldsymbol{\Gamma}\boldsymbol{\gamma}(t-1) + \boldsymbol{v}(t) + \boldsymbol{z}(t) \quad (1)$$

$$\boldsymbol{\gamma}(t) = [\gamma_2(t), \gamma_3(t), \dots, \gamma_M(t)]^T \quad (2)$$

$$\boldsymbol{\Gamma} = (1-\rho)\mathbf{I}_{M-1} + \rho \begin{bmatrix} 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{(M-1) \times (M-1)} \quad (3)$$

$$\boldsymbol{v}(t) = \rho [\gamma_1(t), 0, \dots, 0]_{(M-1) \times 1}^T \quad (4)$$

where $\gamma_m(t)$ denotes heading angle of sensor m at time instant t . $\boldsymbol{v}(t)$ is an update item which is obtained from heading angle of first sensor. ρ is the ratio of current sensor displacement and

intersensor spacing. When $\rho = 1$, the heading angle of the current sensor is the same with that of the previous sensor close to it. When $\rho = 0$, the heading angle of sensor is independent from each other. $\mathbf{z}(t)$ denotes process white noise.

2.2. The sparse received signal model

As illustrated in figure 1, the position of the sensor $m \in \{2, 3, \dots, M\}$ at time instant t can be written in vector form as

$$\mathbf{r}_m = \left[-\sum_{i=2}^m d \sin[\gamma_i(t)], -\sum_{i=2}^m d \cos[\gamma_i(t)] \right]^T \quad (5)$$

Consider a source located at θ_q , the direction unit vector can be written as

$$\mathbf{u}_q = -[\cos(\theta_q), \sin(\theta_q)]^T \quad (6)$$

Then we obtain the steering vector as

$$\mathbf{a}(\theta_q) = [1, \exp(-i2\pi \mathbf{r}_2^T \mathbf{u}_q / \lambda), \dots, \exp(-i2\pi \mathbf{r}_M^T \mathbf{u}_q / \lambda)]^T \quad (7)$$

where λ is wavelength. Based on the theory of sparse reconstruction, the bearing space is uniformly discretized into Q angular directions, and the steering matrix can be written as

$$\mathbf{A}(\Theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_Q)] \quad (8)$$

where $\theta_1, \theta_2, \dots, \theta_Q$ are all possible directions.

Then we obtain the sparse received signal model of the towed array as

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n} \quad (9)$$

where \mathbf{y} is received complex data vector at the M sensors, \mathbf{x} is signal vector of all discredited directions, the components of \mathbf{x} can be modelled as complex Gaussian with zero mean that is uncorrelated each other, \mathbf{n} is independent and identically distributed complex Gaussian noise vector.

3. MBM spatial spectrum sparse reconstruction algorithm

3.1. Spatial spectrum sparse reconstruction in beamspace

The beamspace sparse-regularization approach was proposed by Malioutov [9]. The compressive measurement data is transformed from sensor space to beamspace using steering manifold vectors. Based on the conventional beamforming principle, the beam output power can be written as

$$b(\varphi) = E \left\{ |\mathbf{w}^H(\varphi) \cdot \mathbf{y}|^2 \right\} \quad (10)$$

where $\mathbf{w}(\varphi) = \frac{1}{M} [1, \exp(-i2\pi \mathbf{r}_2^T \mathbf{u} / \lambda), \dots, \exp(-i2\pi \mathbf{r}_M^T \mathbf{u} / \lambda)]^T$ is beamforming steering weighting vector for angle φ and $\mathbf{u} = -[\cos(\varphi), \sin(\varphi)]^T$. Using equation (9) in equation (10) gives

$$b(\varphi) = E \left\{ \left| \mathbf{w}^H(\varphi) \cdot (\mathbf{A}\mathbf{x} + \mathbf{n}) \right|^2 \right\} = E \left\{ \mathbf{w}^H(\varphi) \mathbf{A} \cdot \mathbf{x} \mathbf{x}^H \cdot \mathbf{A}^H \mathbf{w}(\varphi) + \mathbf{w}^H(\varphi) \cdot \mathbf{n} \mathbf{n}^H \cdot \mathbf{w}(\varphi) \right\} \quad (11)$$

$$= \mathbf{h}(\varphi) \cdot \mathbf{p} + \left\| \mathbf{w}(\varphi) \right\|^2 \cdot \sigma_n^2$$

where $\mathbf{h}(\varphi) = \left[\left| \mathbf{w}^H(\varphi) \cdot \mathbf{a}(\theta_1) \right|^2, \left| \mathbf{w}^H(\varphi) \cdot \mathbf{a}(\theta_2) \right|^2, \dots, \left| \mathbf{w}^H(\varphi) \cdot \mathbf{a}(\theta_Q) \right|^2 \right]$, Q denotes the number of all potential directions. $\mathbf{p} = \left[E\{|x_1|^2\}, E\{|x_2|^2\}, \dots, E\{|x_Q|^2\} \right]^T$ is sparse signal power vector, σ_n^2 is sensor noise power. Then the beamspace measurement vector is

$$\mathbf{b}(\Phi) = \mathbf{H} \cdot \mathbf{p} + \left[\left\| \mathbf{w}(\varphi_1) \right\|^2, \dots, \left\| \mathbf{w}(\varphi_K) \right\|^2 \right]^T \cdot \sigma_n^2 \quad (12)$$

where $\mathbf{H} = \left[\mathbf{h}^T(\varphi_1) \quad \mathbf{h}^T(\varphi_2) \quad \dots \quad \mathbf{h}^T(\varphi_K) \right]^T$ is sensing matrix.

The sparse signal power vector can be recovered by the following ℓ_1 minimization problem

$$\hat{\mathbf{p}} = \arg \min_{\mathbf{p} \in \mathbb{C}^Q} \left\| \mathbf{p} \right\|_1 \quad \text{s.t.} \quad \left\| \mathbf{H}\mathbf{p} - \mathbf{b} \right\|_2 \leq \zeta \quad (13)$$

where $\zeta = \sigma_n^2 M^{-1/2}$.

3.2. MIP analysis of sensing matrix for manoeuvring towed array

To ensure good performance of sparse reconstruction, the sensing matrix needs to satisfy reasonable conditions such as Mutual Incoherent Property (MIP) [12] or Restricted Isometry property (RIP) [13]. The mutual coherent between any two columns of \mathbf{H} is defined as

$$\eta(\mathbf{H}) = \max_{k \neq l} \mathbf{G}_{kl} \quad (14)$$

where $\mathbf{G}_{kl} = \left| \mathbf{h}_k^H \times \mathbf{h}_l \right| \cdot \left(\left\| \mathbf{h}_k \right\|_2 \times \left\| \mathbf{h}_l \right\|_2 \right)^{-1}$ is the absolute Gram matrix.

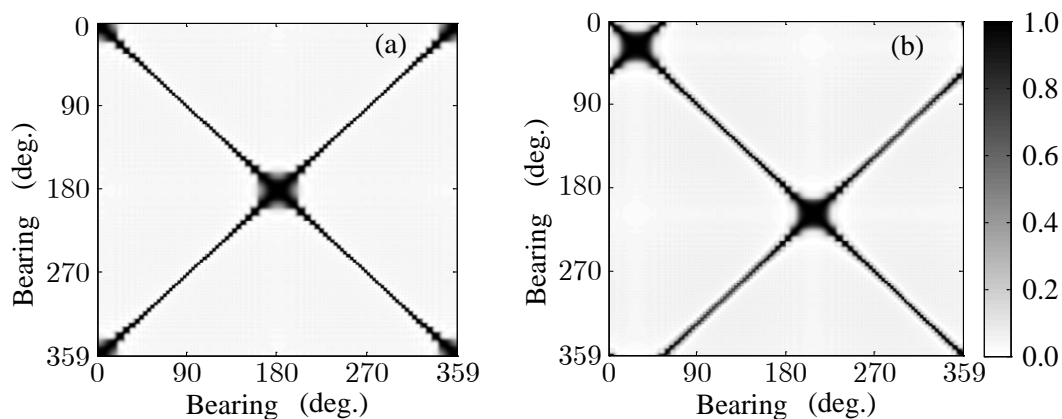


Figure 2. Gram matrix in bearing angle for a manoeuvring towed array. (a) Straight array shape. (b) Distorted array shape.

For example, a 40-sensors towed array ($d/\lambda = 0.45$), the matrix \mathbf{G} as a function of bearing angle is presented in figures 2. When the towed array is drawn in a straight line along y axis, the Gram matrix is depicted in figure 2(a). The coherence is high along minor diagonal which is caused by port-starboard ambiguity. When the towing vessel changes course with a steady angular speed, the Gram

matrix at some time instant is depicted in figure 2(b), the high coherence moves parallel to the minor diagonal which denotes the ambiguity angle is time-varying during vessel turning.

3.3. MBM spatial spectrum sparse reconstruction

According to the characteristic of port-starboard ambiguity at different time instant, the solution to spatial spectrum estimation of towed array can be imagined by stacking together several error norms of beamspace measurements, each one corresponding to a time-varying array shape. Then equation (13) can be rewritten as

$$\hat{\mathbf{p}} = \arg \min_{\mathbf{p} \in \mathbb{C}^Q} \|\mathbf{p}\|_1 \quad \text{s. t.} \quad \sum_{i=t-N}^t \|\mathbf{H}_i \mathbf{p} - \mathbf{b}_i\|_2 \leq \zeta \quad (15)$$

where N is the number of beamspace measurement. The equation (15) is a disciplined convex optimization problem which can be solved efficiently using cvx toolbox.

4. Simulation

In the following simulation, we consider an ULA with $M=40$ sensors with spacing 1.5 m. The frequency of far field narrowband signal is 450 Hz and the speed of towing vessel is 6 knots. Initially, the towing vessel heads North (0°) and performs a 40° turn toward East.

To evaluate the performance of the algorithm, the following scenario is generated: two signals of interest are placed at 98° and 165° with magnitudes 12 dB and 9 dB respectively. The number of snapshots is $L=5$, the number of beamspace measurement is $N=20$ and the number of grid division is $Q=360$, searching from 0° to 359° . At SNR= -5 dB, the performance of spatial spectrum estimation and ambiguity suppression with MBM-SR and CBF is showed in figure 3. The number of independent Monte Carlo trials is 100 at each SNR.

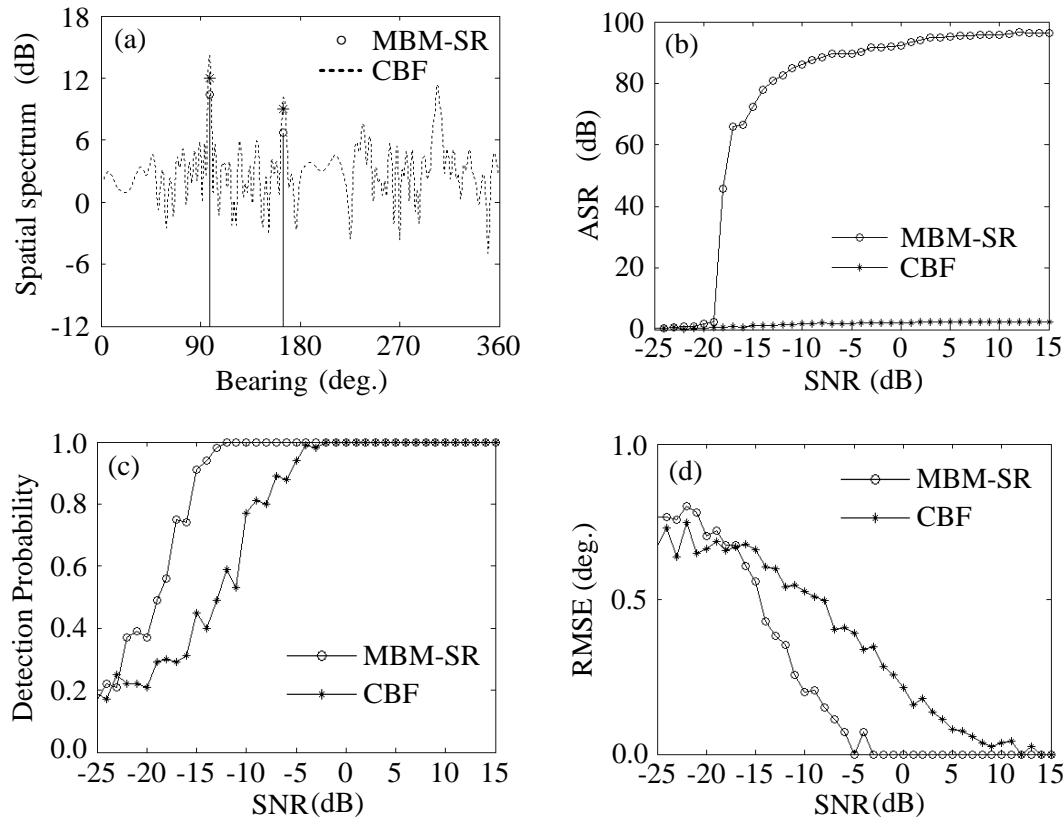


Figure 3. Spatial spectrum of two signals with DOAs of 98° and 165° (SNR= -5 dB).

Figure 3(a) shows that MBM-SR algorithm can suppress two ambiguity targets that located at 237° and 304° . While with CBF, the two ambiguity peaks is still there. The true signals are indicated with asterisks. Figure 3(b) shows that the ambiguity suppression ratio (ASR) of MBM-SR is higher than 90 dB when SNR > 0 dB, while the suppression ratio of CBF is low even in high SNR. So MBM-SR is a valid algorithm in ambiguity suppression. Figure 3(c) is the detection probability of two algorithms. It shows that the detection probability of MBM-SR is higher than that of CBF when SNR < -5 dB. Figure 3(d) gives the DOA RMSE of two algorithms. It can be see that the RMSE of MBM-SR is lower than that of CBF, except for very low SNR.

In figure 4, we consider two targets placed at 98° and 100° respectively with SNR is 10 dB. The angle between them is smaller than the Rayleigh limit. Figure 4 shows that MBM-SR can distinguish two adjacent targets and CBF can't provide enough angle resolution to identify them. Therefore, the proposed algorithm possesses higher resolution.

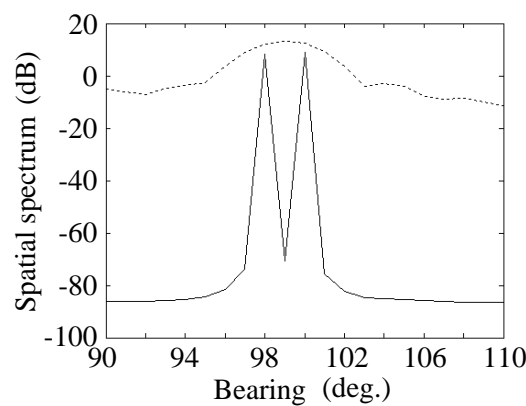


Figure 4. Spatial spectrum of two signals with DOAs of 98° and 100° (SNR = 10 dB).

5. Summary

In this paper, based on multiple beamspace measurements, a novel spatial spectrum sparse reconstruction algorithm for manoeuvring towed array is proposed. The spatial spectrum estimation and ambiguity suppression performance with MBM-SR is then evaluated in a simulation study. It is concluded as following: (1) compared with CBF, MBM-SR can provide evident higher ambiguity suppression ratio; (2) the detection probability and DOA RMSE of MBM-SR is better than that of CBF; (3) the proposed algorithm possesses higher resolution than CBF.

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