

New Denoising Method Based on Empirical Mode Decomposition and Improved Thresholding Function

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Abstract. This paper presents a new denoising method called EMD-ITF that was based on Empirical Mode Decomposition (EMD) and the Improved Thresholding Function (ITF) algorithms. EMD was applied to decompose adaptively a noisy signal into intrinsic mode functions (IMFs). Then, all the noisy IMFs were thresholded by applying the improved thresholding function to suppress noise and improve the signal to noise ratio (*SNR*). The method was tested on simulated and real data and the results were compared to the EMD-Based signal denoising methods using the soft thresholding. The results showed the superior performance of the new EMD-ITF denoising over the traditional approach. The performance were evaluated in terms of *SNR* in *dB*, and Mean Square Error (*MSE*).

1. Introduction

The Empirical Mode Decomposition (EMD) method has recently been pioneered by Huang and al. [1] for analyzing the nonlinear and non-stationary signals. The aim of the EMD method is to adaptively decompose any signal into oscillatory components called intrinsic mode functions (IMFs) using a sifting process. EMD denoising can be based on partial reconstruction or inspired by standard wavelet thresholding [2-8]. The basic principle of wavelet thresholding is to cancel all the coefficients that are lower than a threshold related to the noise level. However, the performance of wavelet approach relies the basis wavelet which is predetermined in advance and on the threshold. A more flexible alternative was performed by EMD approach. Several EMD-Based denoising methods using thresholding were proposed in [6]. Indeed, it was shown that the direct application of wavelet thresholding to IMFs can lead to very bad results for the continuity of the reconstructed signal. Therefore, an EMD interval thresholding was proposed (EMD-IT) in which the thresholding was performed to the zero-crossing interval as a whole. EMD-IT procedure resembles wavelet thresholding more than direct EMD thresholding because wavelet thresholding is applied to the wavelet coefficients [6]. The main factors affecting the quality of Wavelet Threshold Denoising are denoising threshold and selection of the suitable wavelet threshold function. The hard threshold function does not change the local properties of the signal, but it can lead to some fluctuation in the reconstruction of the original signal. The hard threshold function leads to a loss of some high frequency coefficients above the threshold. In order to overcome the drawbacks of the classical threshold functions, Lu Jing-yi et al. proposed an improved threshold function by increasing the adjustment factor [9]. A direct application of wavelet improved threshold function in the EMD case (EMD-ITF) was considered in this paper. Numerical simulation and real data test were performed to evaluate this method, and the results were compared to EMD soft threshold function in terms of *SNR* and *MSE*.



The paper is organized as follows. Section II introduces the EMD algorithm. Section III describes the EMD-Soft thresholding and the new proposed method EMD-ITF. The simulation results are illustrated in section IV. Finally, section V presents the conclusion.

2. EMD algorithm

EMD is an adaptive method to decompose a signal $x(t)$ into a series of IMFs. The IMFs must satisfy the following two conditions: (i) The number of maximum must equal the number of zeros or differ at most by one. (ii) In each period, it is necessary that the signal average is zero.

The EMD algorithm consists of the following steps [1]:

1. Find local maxima and minima in $x(t)$ to construct the upper and lower envelopes respectively using cubic spline interpolation.
2. Calculate the mean envelope $m(t)$ by averaging the upper and lower envelopes.
3. Calculate the temporary local oscillation $h(t) = x(t) - m(t)$.
4. Calculate the average of $h(t)$, if average $h(t)$ is close to zero, then $h(t)$ is considered as the first IMF, named $c_i(t)$ otherwise, repeat steps (1)–(3) while using $h(t)$ for $x(t)$.
5. Calculate the residue $r(t) = x(t) - c_i(t)$.
6. Repeat steps from (1) to (5) using $r(t)$ for $x(t)$ to obtain the next IMF and residue.

The decomposition process stops when the residue $r(t)$ becomes a monotonic function or a constant that no longer satisfies the conditions of an IMF.

$$x(t) = \sum_{i=1}^N c_i(t) + r_N(t) \quad (1)$$

3. EMD Based denoising

3.1. EMD Soft thresholding

Having a noisy signal $y(t)$ given by:

$$y(t) = x(t) + \eta(t) \quad (2)$$

Where $x(t)$ is the noiseless signal and $\eta(t)$ is independent noise of finite amplitude. In EMD-Soft thresholding method, the noisy signal $y(t)$ was first decomposed into noisy IMFs: $c_{ni}(t)$. These noisy IMFs was thresholded by soft function in order to obtain an estimation of the noiseless IMFs $\hat{c}_i(t)$ of the noiseless signal. In this work the universal threshold is used proposed in [10] and it identified as follows:

$$\tau_i = C\sqrt{E_i 2\ln(n)} \quad (3)$$

Where C is a constant depending of the type of signal that was set to 0.5 in this work, n is the length of the signal and E_i is given by [4]:

$$\hat{E}_k = \frac{E_1^2}{0.719} 2.01^{-k}, \quad k = 2, 3, 4, N \quad (4)$$

Where E_1^2 is the energy of the first IMF defined by:

$$E_1^2 = \left(\frac{\text{median}(|c_{n1}(t)|)}{0.6745} \right)^2 \quad (5)$$

A direct application of wavelet soft thresholding [11] in the EMD case:

$$\hat{c}_i(t) = \begin{cases} c_{ni} - \tau_i & \text{if } c_{ni} \geq \tau_i \\ 0 & \text{if } |c_{ni}| < \tau_i \\ c_{ni} + \tau_i & \text{if } c_{ni} \leq -\tau_i \end{cases} \quad (6)$$

A reconstruction of the denoised signal is given by:

$$\hat{x}(t) = \sum_{i=1}^N \hat{c}_i(t) + r_N(t) \quad (7)$$

3.2. EMD-ITF Denoising

A direct application of wavelet improved threshold function [9] in the EMD case:

$$\hat{c}_i(t) = \begin{cases} \text{sgn}(c_{ni}(t)) \left[|c_{ni}(t)| - \frac{\tau_i}{\exp\left(\frac{3\alpha(c_{ni}(t)-\tau_i)}{\tau_i}\right)} \right] & \text{if } |c_{ni}| \geq \tau_i \\ 0 & \text{if } |c_{ni}| < \tau_i \end{cases} \quad (8)$$

A reconstruction of the denoised signal is given by equation (7).

4. Simulation results

In this section, we assess our proposed denoising algorithm compared to EMD-soft thresholding denoising. The new EMD-ITF approach was applied to five test signals (Doppler, Blocks, Bumps, Heavysine and Piece-Regular). The size of the signals was equal to 2048. The method was also tested on real *ECG* signal using the MIT-BIH database [12]. For simulated signals, the variance of the additive white Gaussian noise was set so that the *SNR* before denoising was maintained at 15 *dB*. The original signals and the corresponding noisy versions are depicted in Fig. 1. The *SNR* before denoising of the real *ECG* signal was 20 *dB*. Each noisy signals were decomposed into IMFs using EMD process and all IMFs are thresholded by the Improved Thresholding Function and soft thresholding. The performance of the proposed method was affected by the choice of the α value as shown in Fig.2 that depicts the *SNR* after denoising as function of α . The α values for which the the *SNR* after denoising are maximum are 0.4, 0.2, 0.2, 0.1, 0.1 for *ECG*, bumps, heavysine, Piece Regular, blocks, and Doppler signals respectively. Indeed, the maximum *SNR* after denoising depends on the signal and α . A comparative study of two methods considered in this work is presented in Table 1. This table shows that the proposed method provides the best results in terms of *SNR* and *MSE* for all test signals. Therefore, the proposed EMD-ITF outperforms totally the conventional EMD-soft thresholding denoising. Fig.3 shows the denoising results of applying EMD Soft and EMD- ITF to simulated signals. Fig.4 displays the denoising results of real *ECG* signal using EMD-Soft and EMD- ITF.

Table 1. *SNR* before and *SNR* after denoising of the different signals.

Methods	<i>SNR</i> (dB)(after denoising)					<i>ECG</i> 20dB
	<i>Blocks</i> 15dB	<i>Bumps</i> 15dB	<i>Heavysine</i> 15dB	<i>Doppler</i> 15dB	<i>Piece- Regular</i> 15dB	
EMD-Soft	20.1370	20.4058	26.4670	22.2368	21.6292	26.2561
EMD-ITF	20.6421	23.3905	26.6830	24.4025	22.3292	28.1154
Methods	MSE					

EMD-Soft	0.0854	0.0295	0.0214	0.0005	0.0220	0.0023
EMD-ITF	0.0760	0.0148	0.0204	0.0003	0.0187	0.0015

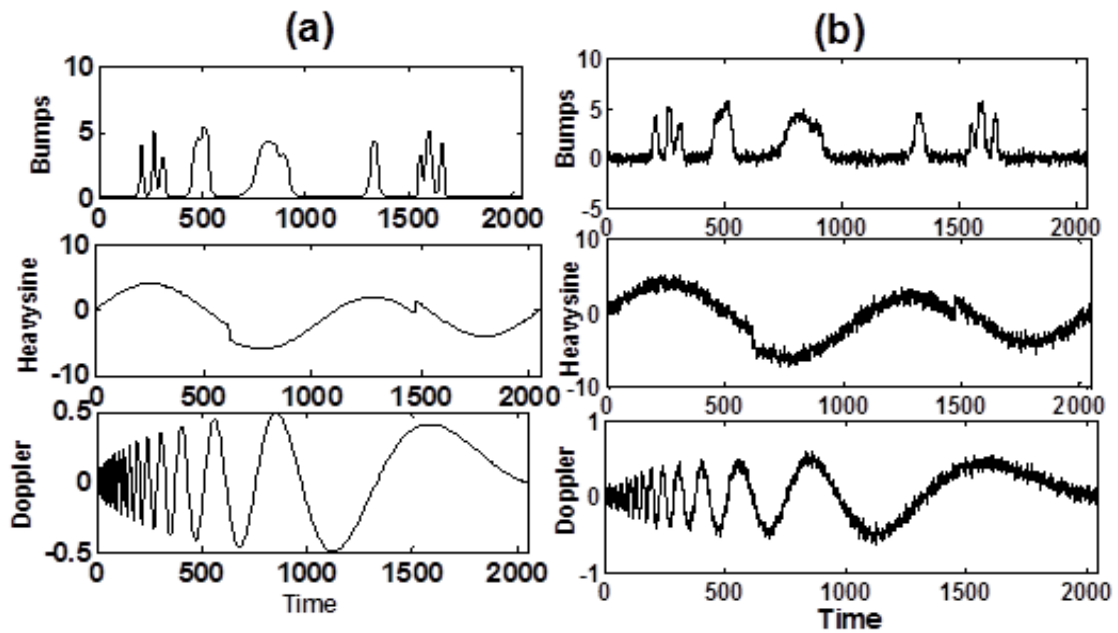


Figure 1. (a). Test signals with $n=2048$. (b). Noisy test signals $SNR=15dB$

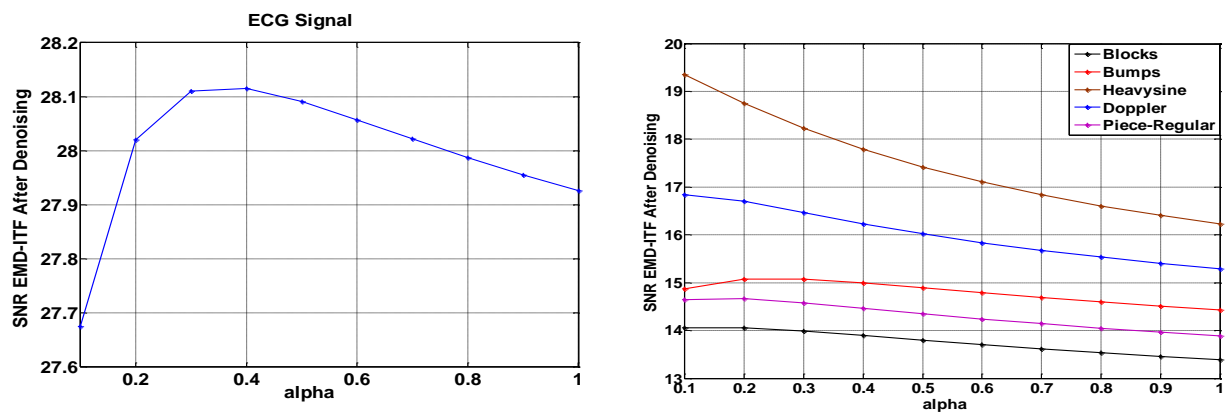


Figure 2. Performance evaluation of EMD-ITF denoising method.

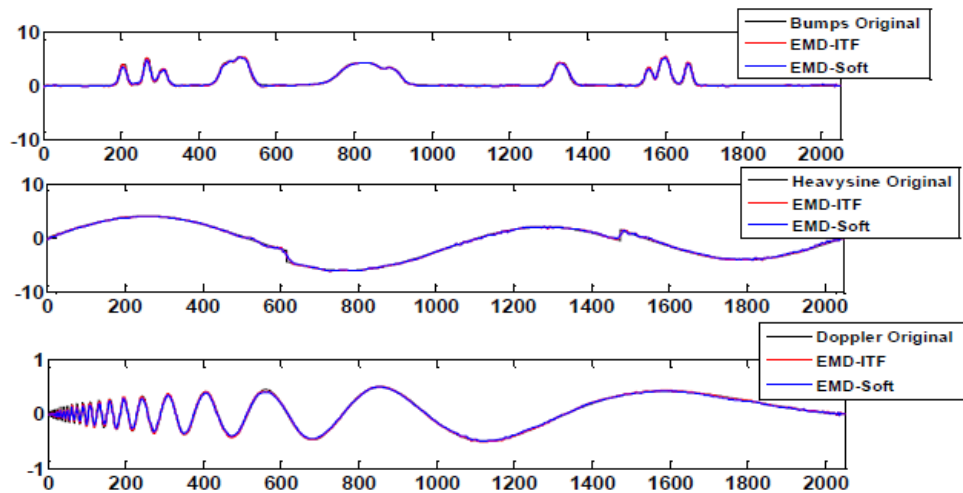


Figure 3. Denoising results in $SNR = 15dB$ of test signals corrupted by Gaussian noise.

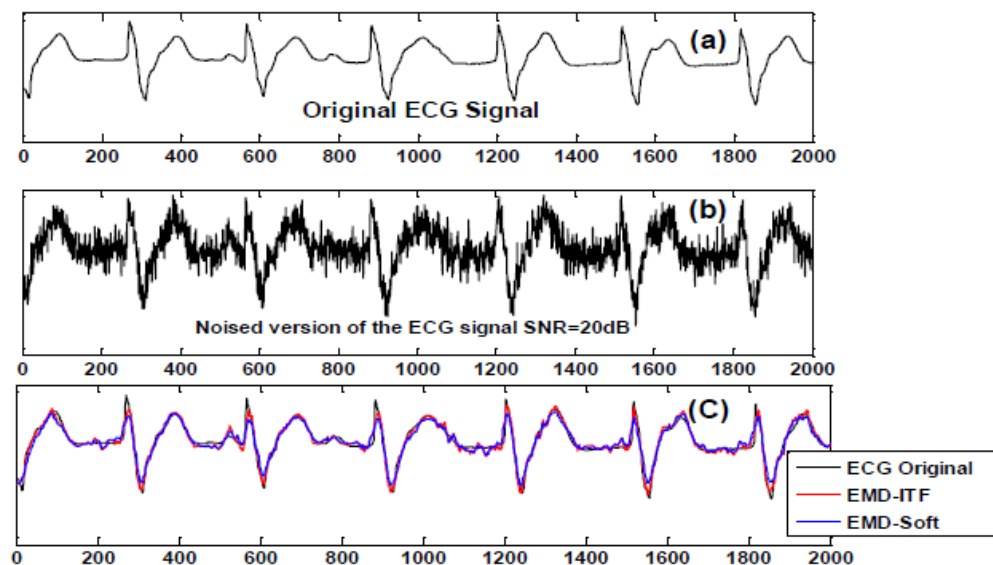


Figure 4. Denoising results in SNR (20dB) of real ECG signal

5. Conclusion

A new signal denoising method is proposed based on empirical mode decomposition and the Improved Thresholding Function to suppress noise in the signal and improve the output SNR . The proposed method was tested on real *ECG* signal and simulated signals (Doppler, Blocks, Bumps Heavysine, and Piece-regular) corrupted by white Gaussian noise. Based on SNR and MSE , the simulation results are in favour of the new EMD-ITF denoising method. We showed that the new approach is useful for removing noise

References

- [1] N.E. Huang, Z. Shen, S.R. Long, M.C. Wu, H.H. Shin, Q. Zheng, N.C. Yen, C.C. Tung and H.H. Liu, "The Empirical Mode Decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis," *Proceedings of the Royal Society of London*, 454:903–995, 1998.
- [2] A .O. Boudraa, J. C. Cexus, Z. Saidi, "EMD-based signal noise reduction, *Int. J. Signal Process.* 1(1), 33–37, 2004.

- [3] A. O. Boudraa, J. C. Cexus, “Denoising via empirical mode decomposition”, in Proceedings of the IEEE International Symposium on Control, Communications and Signal Processing (ISCCSP '06), p. 4, Marrakech, Morocco, March 2006.
- [4] P. Flandrin, G. Rilling, P. Goncalves, “EMD equivalent filter banks, from interpretations to application,” in *Hilbert-Huang Transform and its Application*, N. E. Huang and S. Shen, Eds., 1st ed. Singapore: World Scientific, 2005.
- [5] A. O. Boudraa, J. C. Cexus, “EMD-Based Signal Filtering”, IEEE Transactions on Instrumentation and Measurement, Vol. 56, NO. 6, pp.2196-2202, 2007.
- [6] Y. Kopsinis, S. McLaughlin, “Development of EMD-based denoising methods inspired by wavelet thresholding”, IEEE Trans.Signal Process.57(4) 1351–1362, 2009.
- [7] Y. Kopsinis, S. McLaughlin, “Empirical Mode Decomposition Based Soft-Thresholding,” in *Proc. 16th Eur. Signal Process. Conf.(EUSIPCO)*, Lausanne, Switzerland, Aug. 25–29, 2008.
- [8] G.S. Tsolis and T. D. Xenos, “Signal Denoising Using Empirical Mode Decomposition and Higher Order Statistics”, International Journal of Signal Processing, Image Processing and Pattern Recognition Vol. 4, No. 2, pp.91-106, 2011.
- [9] L. Jing-yi, L. Hong, Y. Dong, and Z. Yan-sheng “A New Wavelet Threshold Function and Denoising Application”, Mathematical Problems in Engineering; Vol. 2016, 8 pages.
- [10] D. L. Donoho and I. M. Johnstone, “Ideal spatial adaptation by wavelet shrinkage,” *Biometrika*, vol. 81, no. 3, pp. 425 – 455, Aug. 1994.
- [11] Donoho DL, “De-noising by soft-thresholding,” IEEE Trans Inform Theory, Vol.14, No.3, pp. 612-627, 1995.
- [12] A.L. Goldberger, L.A.N. Amaral, L. Glass, J.M. Hausdorff, P.C. Ivanov, R.G. Mark, J.E. Mietus, G.B. Moody, C.-K. Peng, H.E. Stanley, PhysioBank, PhysioToolkit, and PhysioNet: components of a new research resource for complex physiologic signals, *Circulation*, Vol. 101, N° 23, pp. 215–220, 2000.