

# Health-aware Model Predictive Control of Pasteurization Plant

**Fatemeh Karimi Pour, Vicenç Puig and Carlos Ocampo-Martinez**

Automatic Control Department of the Universitat Politècnica de Catalunya, Institut de Robòtica i Informàtica Industrial (CSIC-UPC). Llorens i Artigas, 4-6, 08028 Barcelona, Spain.

E-mail: {fkarimi, cocampo, vpuig}@iri.upc.edu

**Abstract.** In order to optimize the trade-off between components life and energy consumption, the integration of a system health management and control modules is required. This paper proposes the integration of model predictive control (MPC) with a fatigue estimation approach that minimizes the damage of the components of a pasteurization plant. The fatigue estimation is assessed with the rainflow counting algorithm. Using data from this algorithm, a simplified model that characterizes the health of the system is developed and integrated with MPC. The MPC controller objective is modified by adding an extra criterion that takes into account the accumulated damage. But, a steady-state offset is created by adding this extra criterion. Finally, by including an integral action in the MPC controller, the steady-state error for regulation purpose is eliminated. The proposed control scheme is validated in simulation using a simulator of a utility-scale pasteurization plant.

## 1. Introduction

Pasteurization is a quite common procedure in the food industry. The pasteurization process implies applying heat to some products such as milk, cream, beer and others. Mainly, pasteurization implies that a food product is exposed to some temperature profile during a predetermined period of time, in order to reduce the proportion of microorganisms. This process is distributed into three sections that are heating, cooling and regeneration sections. The most important section is heating, which involves heat exchanger equipment to heat up the temperature of the product at requested setpoint, and then maintaining this temperature during a constant time. An important key in pasteurization plant is controlling and maintaining the temperature of the process. Particularly, in the case of milk pasteurization the heating temperature is in general 72 – 74 °C during 15 – 20 seconds followed by cooling [9]. Therefore, a suitable control system needs to be designed to control the product temperature for keeping the desired product quality. The necessity of a significant control of the process arises from the saving in energy, product and time if an accurate tracking of the setpoint is performed. Because of the high number of load cycles that occur during the life of the pasteurization's pump, fatigue measurement is of particular importance in pasteurization control. For this reason, interest in the integration of control with a fatigue-based prognosis of components has increased in recent years. Fatigue can be taken as a breakdown of the material subject to stress, especially when repeated series of stresses are applied. It has been widely and exhaustively studied from different perspectives [15]. In this paper, the damage rule used to perform fatigue analysis is the Palmgren-Miner linear damage rule [14]. This rule commonly called the Miners rule is being currently used throughout the industry and in academia [12].

Along the last decades, model-based predictive control (MPC) gained its relevant position in the process industry, because of its ability to deal with multivariable control, delays and constraints on system



variables and actuators. Also, MPC has started to attract the attention of academia and industry due to the possibility of dealing with the conflicting power optimization and fatigue load reduction. A data-based MPC strategy that incorporates fatigue estimation was presented in [2]. In the work of [18], an approach including dynamic inflow into the MPC controller is proposed to decrease fatigue load.

This paper propose a health-aware control (HAC) that considers the information about the system health to adapt the objectives of the control law to extend the system remaining useful life (RUL) [5]. Thus, the control actions are generated to fulfill the control objectives/constraints but at the same time to extend the life of the system components. In case that the controller is implemented using MPC, the trade-off is based on modifying the control objective including new terms that take care of the system health. This leads to solving a multi-objective optimization problem where a trade-off between system health and performance should be established [23].

The main contribution of this paper is the integration of MPC with fatigue-based prognosis to minimize the damage of pasteurization components while still, the pasteurization temperature tracks the suitable references related to the products. The integration of a system health management module with MPC control is done by including a fatigue-based model using the rain-flow counting approach and adding an extra criterion in the control objective function that takes into account the accumulated damage. In practice, the extra criterion can lead to steady-state offset unless precautions are taken in the control design. For this reason, the integral action for eliminating steady-state offset involves augmenting the process model to include a constant step disturbance is used in MPC controller. The control scheme is implemented using a high fidelity simulator of a utility-scale pasteurization plant.

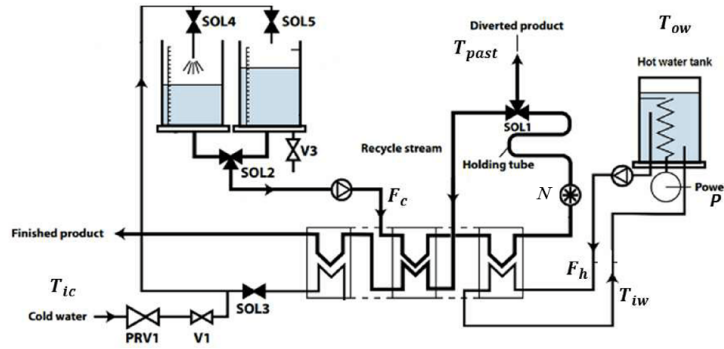
The remainder of the paper is organized as follows. In Section 2, the pasteurization plant and process of this system are represented. Besides, the general statement of the MPC problems is described. Section 3 presents the fatigue analysis applied to pasteurization system and show how to implement health-aware control using MPC. In Section 4, approaches for eliminating steady-state offset in MPC controller are presented. Section 5 describes the case study based on the pasteurization benchmark, where the proposed approach is assessed and the results are analyzed. Finally, in Section 6, the conclusions of this work are drawn and some research lines for future work are proposed.

## 2. PROBLEM STATEMENT

### 2.1. Pasteurization Model

The considered pasteurization process is the small-scale plant PCT-23 MKII, manufactured by Armfield (UK) [1]. The system emulates an industrial high-temperature short-time (HTST) pasteurization process. The goal of this process is to heat and keep the product, which is typically a liquid, at a predetermined temperature for a minimum time. Throughout the pasteurization process, the liquid is pumped at a predetermined flow rate from one of two storage tanks to an indirect plate heat exchanger. The water heat is transferred to the product inside the first phase of the heat exchanger. The raw product is heated to an intermediate temperature by using missed energy from the pasteurized product. Then, this product is heated from that middle temperature to the full pasteurization temperature, in the second phase, while using a hot-water flow ( $F_h$ ) coming from a closed circuit with a heater. The temperature ( $T_{past}$ ) at the output of the holding tube is used to monitor the product temperature after the pasteurization process. Finally, the product is cooled in the third phase of the heat exchanger, where remaining heat is removed to the entrance product. This last phase does not add anything new to the model, and so it was not considered in this paper.

In order to have a model that can be used with an MPC controller, suitable models of the pasteurization plant are needed. The control-oriented pasteurization model is represented in terms of behavior equations of each subsystem, consisting of power, water pump, heat exchanger and hot water tank. Accordingly, identified models obtained as transfer functions are suitably stated by their equivalent controllable realizations in state space. The controlled inputs are the power of the electrical heater,  $P$ , and water pump speed,  $N$ , respectively. The input temperature of the water heater,  $T_{iw}$ , and temperature of cold water,  $T_{ic}$ , are measured non-controlled inputs (disturbance). Therefore, the continuous time state-space



**Figure 1.** Plant scheme.

model of the pasteurization system can be expressed in the following form:

$$A = \begin{bmatrix} \frac{-1}{\tau_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\tau_2} & 0 & 0 & 0 & 0 & 0 \\ \frac{K_{21}}{\tau_{21}} & \frac{K_{21}}{\tau_{21}} & \frac{-1}{\tau_{21}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_{21}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\tau_{22}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{\tau_f} & 0 \\ 0 & 0 & 0 & \frac{K_{ht}}{\tau_{ht}} & \frac{K_{ht}}{\tau_{ht}} & \frac{K_{ht}}{\tau_{ht}} & \frac{-1}{\tau_{ht}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{K_2}{\tau_2} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{K_{12}}{\tau_{12}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$E = \begin{bmatrix} \frac{K_1}{\tau_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{K_{22}}{\tau_{22}} & 0 & 0 \end{bmatrix}^T, \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where  $K$  is static gain and  $\tau$  is the time constant of the transfer functions of the subsystems. In spite of this, the discretized state space-model form a complete system of pasteurization plant (1) with sampling time  $T_s = 120$  s is used here.

## 2.2. Operational Control

As previously mentioned, the main goal of pasteurization process is to guarantee that the pasteurization temperature is reached and maintained as close as possible to the set-point value. At the same time, reduction of energy consumption and health management of the system expressed as a multi-objective problem. Hence, MPC is suitable technique to control a pasteurization system due to its capability to deal efficiently with dynamic constrained systems and predict the proper action to achieve the optimal performance according to a user defined cost function. In this paper, it is assumed that the system behavior can be described at each time instant  $k \in \mathbb{Z}$  by the following discrete-time difference equation:

$$x(k+1) = Ax(k) + Bu(k) + Ed(k), \quad (2a)$$

$$y(k) = Cx(k), \quad (2b)$$

where  $x \in \mathbb{R}^{n_x}$  is the state of system,  $u \in \mathbb{R}^{n_u}$  is the vector of manipulated variable,  $d \in \mathbb{R}^{n_d}$  is vector of measurable disturbances, and  $y \in \mathbb{R}^{n_y}$  is vector of controlled variables. Moreover,  $A$ ,  $B$ ,  $E$  and  $C$  are time-invariant matrices of adequate dimensions. It also considered that the system is subject to the hard state and input constraints, which can be posed as

$$x(k) \in \mathbb{X} \triangleq \{x(k) \in \mathbb{R}^{n_x} | \underline{x} \leq x(k) \leq \bar{x}, \quad \forall k\}, \quad (3a)$$

$$u(k) \in \mathbb{U} \triangleq \{u(k) \in \mathbb{R}^{n_u} | \underline{u} \leq u(k) \leq \bar{u}, \quad \forall k\}. \quad (3b)$$

The control goal is to minimize a convex multi-objective cost function  $J(x, u) : \mathbb{X} \times \mathbb{U} \rightarrow \mathbb{R}$ , which might tolerate any functional relationship to the economic and safety of the system operation. To do so, the MPC controller design is based on the solution of the following finite-time horizon optimization problem (FHOP):

$$\min_{\mathbf{u}_k} \sum_{i=0}^{N_p-1} [\|x(k+i|k)\|_{w_1}^p + \|u(k+i|k)\|_{w_2}^p + \|\Delta u(k+i|k)\|_{w_3}^p], \quad (4a)$$

subject to:

$$x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k) + Ed(k+i|k), \quad (4b)$$

$$\Delta u(k+i|k) = u(k+i|k) - u(k+i-1|k), \quad (4c)$$

$$u(k+i|k) \in \mathbb{U}, \quad (4d)$$

$$x(k+i|k) \in \mathbb{X}, \quad (4e)$$

for all  $i \in \mathbb{Z}_{[0, N_p-1]}$ , where  $\mathbf{u}_k = \{u_{k+i|k}\}_{i \in \mathbb{Z}_{[0, N_p-1]}}$  is the decision variable, with  $\mathbf{u}_k$  being the sequence of controlled inputs. Furthermore, weighting matrices  $w_1 \in \mathbb{R}^{n_x \times n_x}$ ,  $w_2 \in \mathbb{R}^{n_u \times n_u}$  and  $w_3 \in \mathbb{R}^{n_u \times n_u}$  are used to establish the antecedence of the different control objectives. Also,  $p$  denotes the norm used.

Denote with  $\mathbf{u}_k^*$  the optimal solution of (4) at time step  $k$ . Then, following the MPC receding horizon philosophy, only the first optimal control action is applied, i.e.,  $u_k = u_{k|k}^*$ . Then, the new measurements are accumulated to initialize initial conditions and then the optimization problem (4) is solved again. This procedure is repeated at each time step  $k$ .

### 3. Health-aware MPC Controller

In this section, the rainflow counting algorithm (RFC) is represented and later on it is formulated and integrated by MPC controller for the pasteurization plant as a case study.

#### 3.1. Rainflow counting algorithm (RFC)

The damage accumulation process on a component produced by cyclic loading is called as *fatigue*. Exposing a material to cyclic loading of constant amplitude will cause fatigue failure after a certain number of cycles. A usual method to quantify the fatigue impact of fluctuating loads is the combination of a rainflow counting algorithm and a damage equivalent load approach, enabling the relative comparison of different load samples [13]. RFC method, first introduced by [4], has a complex sequential and nonlinear structure in order to decompose ideal sequences of loads into cycles. Usually, to compute a lifetime estimate from a given structural stress input signal, the RFC method is applied by counting cycles and maxima, jointly with the Palmgren-Miner rule to calculate the expected damage. The input signal is obtained from time history of the loading parameter of interest, such as force, torque, stress, strain, acceleration, or deflection [11].

There are several types of RFC algorithms have been proposed such as [22] and [3], with different rules but providing the same results. The RFC algorithm applied in this paper is presented in [17]. This algorithm computes the stress for each rainflow cycle in four steps [23]:

- the stress history is converted to an extremum sequence of alternating maxima and minima;
- for each local maximum  $M_j$ , the left ( $m_j^-$ ) and right ( $m_j^+$ ) region where all stress values are below  $M_j$  is identified;
- the minimum stress value is processed being  $m_j = \min\{m_j^-, m_j^+\}$ ;
- the equivalent stress per rainflow cycle  $s_j$  related to the  $M_j$  is given by the amplitude  $s_j = M_j - m_j$  or the mean value  $s_j = \frac{M_j - m_j}{2}$ .

The damage,  $D$ , at each stress cycle is calculated by using the  $S - N$  curve [7]. In fact, for many materials there is an explicit relation between the number of cycles to failure and cycle amplitude, which is known as  $S - N$  or Wöhler curves, given as a line in a log-log scale as

$$s^{c_w} N = K, \quad (5)$$

where  $c_w$  and  $K$  are material specific parameters, being the  $c_w$  Wöhler-coefficient, and  $N$  is the number of cycles to failure at a given stress amplitude  $s$ . The damage imposed by a stress cycle with a ranges  $j$  is computed as

$$D_j \equiv \frac{1}{N_j} = \frac{1}{K} s_j^{c_w}. \quad (6)$$

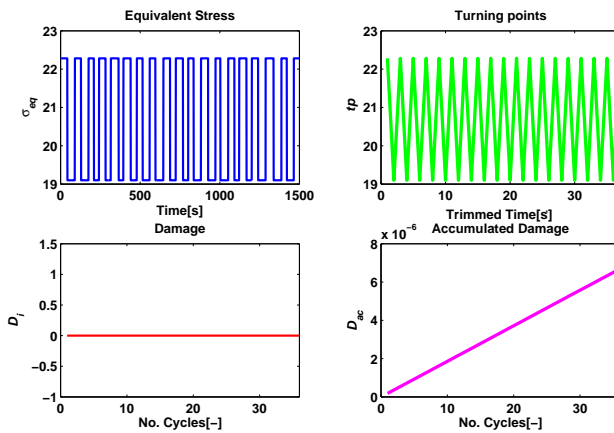
Then, for a time history, the total damage under the linear accumulation damage (Palmgren-Miner) rule is given as

$$D_{ac} = \sum_{j=1}^{\lambda} \frac{1}{N_j} = \sum_{j=1}^{\lambda} \frac{1}{K} s_j^{c_w}, \quad (7)$$

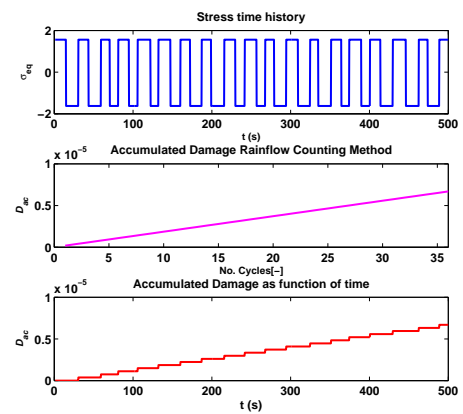
for damage increments  $D_{ac}$  associated to each counted cycle,  $N_j$  the number of cycles to failure associated to stress amplitude  $s_j$ , and the number of all counted cycles  $\lambda$ . These sequences are presented in Figure 2. On the top the input stress is shown, and in the bottom part the instantaneous damage and the accumulated damage are shown.

For real-time applications, applying the customary rainflow counting algorithm is quite challenging and computationally heavy. Considerable amounts of data must be stored and processed periodically to establish a magnitude of the data in equivalent regular cycles. Besides the algorithm must be applied to a stored set of data. One of the objectives of this paper is to analyze the fatigue due to pump load of the pasteurization system. Loads in the pump structure arise from several factors, being the primary cause of failure in the pump is bearing failure. While there are two influential and important factors such as pressure (PSI) and speed (RPM) which are related to the pump, and in this case, the speed of the pump is used as fatigue stress.

Utilizing the RFC algorithm the accumulated damage is obtained as a function of the cycles of the pump speed stress signal. In order to have available an accumulated damage variable that can be integrated with a linear MPC model, a simplified approach to compute fatigue on time series signal is proposed based on RFC theory explained before. The outcome of this approach is that the accumulated damage is obtained as a function of time instead of the number of cycles.



**Figure 2.** Rainflow counting damage estimation.



**Figure 3.** Accumulated Damage Comparison.

The proposed approach finds the changes of the sign which corresponds to a cycle in the stress time signal. The obtained function at each sample step  $k$  is the following:

$$D(k) = \begin{cases} 0 & \text{if } I(k) = I(k-1), \\ \frac{1}{K}(s(k))^{c_w} & \text{if } I(k) \neq I(k-1), \end{cases} \quad (8)$$

where  $s(k)$  is the stress at time  $k$

$$s(k) = \frac{1}{L} \sum_{q=k-L}^k N_s(q), \quad (9)$$

and  $L$  is the number of samples per cycle,  $N_s$  is the pump speed moment,  $q$  is difference between the number of samples per cycle and sample step and  $I(k)$  is the signal adapted to indicate cycle (10)

$$I(k) = N_s(k) - s(k). \quad (10)$$

Then, the accumulated damage is calculated by

$$D_{acc}(k) = D_{acc}(k-1) + D(k). \quad (11)$$

Notice that, at the end of the scenario, the accumulated damage is practically the same. The difference, as explained before, depends on the fact that the damage obtained by RFC technique is represented as a function of the cycles count while the other is a function of time.

### 3.2. MPC with health-aware objective

As shown in Section 3.1, the degradation process of the heat pump of pasteurization plant can be assessed using the water pump speed sensor information. In order to decrease the accumulated damage, a new objective is considered in the MPC controller that the rainflow counting model is approximated by means of a linear model in this objective.

As first approximation, later on observing that the proposed approach gives a quite close approximation of the accumulated damage obtained by the RFC algorithm (see Figure 3), the slope  $m$  of the accumulated damage curve in function of time is calculated and then employed as one of the parameters in the linear fatigue-damage model proposed in this paper. According to the [21], an experimental model that relates the values of the pump speed and flow signals in steady state was proposed. Therefore, it can be shown the relation between the pump speed moment as a function of the flow. The proposed model for pump speed dynamics is a first order pump speed model with an slope of  $\alpha_1$  plus a constant value  $\alpha_0$ , i.e.,

$$N_s(k) = \alpha_1 F(k) + \alpha_0. \quad (12)$$

Briefly, a linear fatigue damage model is suggested as a function of the flow value signal while there is a relation between the accumulated damage of the pump and the control signal expressed as:

$$z(k+1) = z(k) + \frac{m}{L}(\alpha_1 F(k) + \alpha_0), \quad (13)$$

where  $z(k+1)$  is the accumulated damage of the pump. This damage model (13) can be included into the MPC as a new state of model and additional objective is increased in the MPC cost function (4) to minimize the accumulated damage. Taking into account (13), the MPC problem (4) can be represented as follows:

$$\min_{\mathbf{u}_k} \sum_{i=0}^{N_p-1} [\|x(k+i|k)\|_{w_1}^p + \|u(k+i|k)\|_{w_2}^p + \|\Delta u(k+i|k)\|_{w_3}^p + z(k+i|k)\|_{w_4}^p], \quad (14a)$$

subject to:

$$x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k) + Ed(k+i|k), \quad (14b)$$

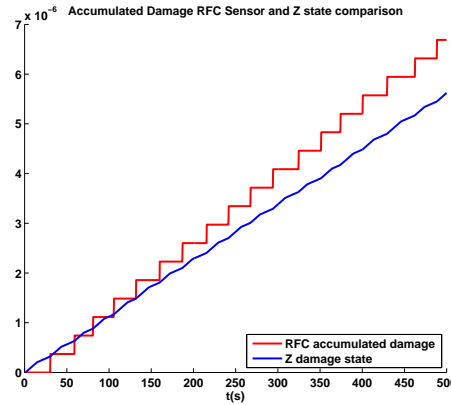
$$\Delta u(k+i|k) = u(k+i|k) - u(k+i-1|k), \quad (14c)$$

$$z(k+1) = z(k) + \frac{m}{L}(\alpha_1 F(k) + \alpha_0), \quad (14d)$$

$$u(k+i|k) \in \mathbb{U}, \quad (14e)$$

$$x(k+i|k) \in \mathbb{X}, \quad (14f)$$

where the new objective with the corresponding matrix weight  $w_4$  is appended in the MPC cost function to minimize the accumulated damage. The degree of fitting between the RFC approximation as a function of time presented in Section 3.1 and the linear  $z$  state damage model are described in (13) can be compared in Figure 4.



**Figure 4.** Accumulated damage RFC as a function of time and  $z$  fatigue damage state.

#### 4. Condition for Offset Elimination in MPC Controller

As it described before, the new objective in cost function of MPC controller can be lead to steady-state offset unless precautions are taken into control design. Elimination of steady-state offset is carried out in two basic manners. The first method involves modifying the controller objective to include integration of the tracking error [16]. The tracking error state and process model are augmented and incorporated as an integral term into the MPC structure. But, this augmentation can significantly increase the computational cost of the dynamic optimization which grows in proportion to the cube of the state dimension for some complex system. Also, this method is needed an anti-windup algorithm for the integral term, and sometimes costly, performance penalty [20]. A second general method, by augmenting the process model that is included a constant step disturbance can be eliminated the steady-state offset, whereas this disturbance, which is estimated from the measured process variables, is generally assumed to remain constant in the future and its effect on the controlled variables is eliminated by shifting the steady-state target for the controller. In this paper, by using the disturbance model remove the steady-state offset which is created with the new state in the model.

Now, it is considered the general type in which the particular recent MPC controllers eliminate the offset by means of disturbance models [19], [16], [6]. Hence, a general way to include disturbance sub-model given from the state-space model is as follows [16]:

$$\begin{bmatrix} x(k+1) \\ d(k+1) \\ g(k+1) \end{bmatrix} = \begin{bmatrix} A & G_d & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \\ g(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(k), \quad y(k) = \begin{bmatrix} C & 0 & G_g \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \\ g(k) \end{bmatrix}, \quad (15)$$

where the output and state disturbance are  $g(k) \in \mathbb{R}^{s_g}$  and  $d(k) \in \mathbb{R}^{s_d}$ , respectively; and  $s_g$  and  $s_d$  are the numbers of augmented output and state disturbance. In addition,  $G_g$  and  $G_d$  are matrices that determine the effect of the disturbance on the output and the states. In the general form of considering the disturbance, if  $G_d = 0$  and  $G_g = I$  model (15) have an output disturbance, on the other hand, if  $G_d = B$  and  $G_g = 0$  model (15) have an input disturbance. According to the complete model of pasteurization system (1), the matrix  $E$  is utilized as disturbance states in this paper. The states of the process model and the unmeasured disturbance model must be estimated simultaneously using the augmented system model. The state estimation can be performed within a stochastic framework using a Kalman filter or a deterministic framework using a Luenberger observer. In both cases, a stable filter gains  $L$  can be determined using standard methods provided the augmented system is detectable [10]. Thus, the steady-state observer equation is given by

$$\hat{x}(k) = A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}} + L_x[y(k) - C(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}})], \quad (16)$$

$$\begin{bmatrix} \hat{d}_{\bar{k}} \\ \hat{g}_{\bar{k}} \end{bmatrix} = \begin{bmatrix} \hat{d}_{\bar{k}} \\ \hat{g}_{\bar{k}} \end{bmatrix} + \begin{bmatrix} L_d \\ L_g \end{bmatrix} [y(k) - C(A\hat{x}_{\bar{k}} + Bu_{\bar{k}} + G_d\hat{d}_{\bar{k}}) - G_g\hat{g}_{\bar{k}}], \quad (17)$$

where  $L_x$ ,  $L_d$  and  $L_g$  are the observer gains that corresponds to the states estimation  $\hat{x}_{\bar{k}}$ , and the disturbance estimation  $\hat{d}_{\bar{k}}$  and  $\hat{g}_{\bar{k}}$ , respectively.

The states from the augmented system (15) which corresponding to the disturbances, that cannot be controlled by means of the input  $u$ . In that case, the MPC regulator makes use of the original model and shifts the steady-state targets in order to remove the estimated disturbance effect. Moreover, the dynamic control is separated from the stationary computation: the target calculus block is completely devoted to obtain the stationary values, while the MPC regulator block is dedicated to guiding the states  $\hat{x}(k)$  to its corresponding targets. Then, the optimization problem that must be solved in the regulator block is represented by

$$\min_{\tilde{u}_k} \sum_{i=0}^{N_p-1} [\|C\tilde{x}(k+i|k)\|_{w_1}^p + \|\tilde{u}(k+i|k)\|_{w_2}^p + \|\Delta\tilde{u}(k+i|k)\|_{w_3}^p + z(k+i|k)\|_{w_4}^p], \quad (18a)$$

subject to:

$$\tilde{x}(k+i+1|k) = A\tilde{x}(k+i|k) + B\tilde{u}(k+i|k) + Ed(k+i|k), \quad (18b)$$

$$\Delta\tilde{u}(k+i|k) = \tilde{u}(k+i|k) - \tilde{u}(k+i-1|k), \quad (18c)$$

$$z(k+1) = z(k) + \frac{m}{L}(\alpha_1 F(k) + \alpha_0), \quad (18d)$$

$$\tilde{u}(k-1) = u(k-1) - u_s \quad (18e)$$

$$\tilde{u}(k+i) = 0 \quad (18f)$$

$$\tilde{x}(k) = \hat{x}(k) - x_s \quad (18g)$$

$$u_{min} \leq \tilde{u}(k+i) + u_s \leq u_{max} \quad u(k+i|k) \in \mathbb{U}, \quad (18h)$$

$$\Delta u_{min} \leq \Delta\tilde{u}(k+i) \leq \Delta u_{max} \quad x(k+i|k) \in \mathbb{X}, \quad (18i)$$

where it can be seen that the state and input are led to the targets  $x_s$  and  $u_s$ . The role of this part is to steer the shifted states and input to zero, which means that the ability of this strategy to eliminate the offset depends on the computation of the targets  $x_s$  and  $u_s$ . For obtaining this operation, the following optimization problem is solved:

$$\min_{u_s, x_s} V_t(k) \triangleq \{(y_{sp} - y_t^a)^T Q(y_{sp} - y_t^a) + (u_s - u_{sp})^T R(u_s - u_{sp})\}, \quad (19a)$$

subject to:

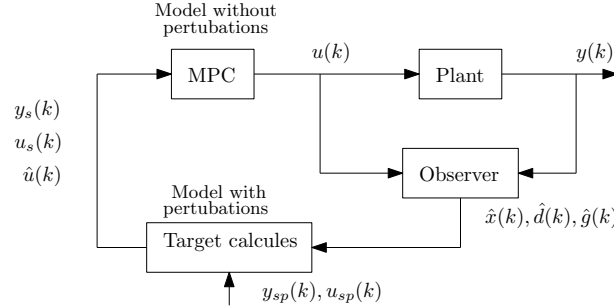
$$x_s = Ax_s + Bu_s + G_d\hat{d}(k), \quad (19b)$$

$$y_t^a \triangleq Cx_s + G_g\hat{g}(k), \quad (19c)$$

$$u_{min} \leq u_s \leq u_{max}, \quad (19d)$$



where  $y_{sp}$  stands for the output set points, and  $y_t^a$  is the achievable stationary output. Also,  $Q$  and  $R$  are positive definite weighting matrices.



**Figure 5.** Representative block diagram of the predictive control system with linear regulator.

Following the strategy describe in Figure 5, two optimization problems must be solved at each sample time  $k$ . Assume that constraints (19b) and (19c) hold true for the stationary disturbances  $\hat{d}$  and  $\hat{g}$  presented by the observer and in that case, for a time  $\bar{k}$  large enough, the states and input target satisfies,

$$x_s = Ax_s + Bu_s + G_d \hat{d}_{\bar{k}}, \quad (20a)$$

$$y_{sp} \triangleq Cx_s + G_g \hat{g}_{\bar{k}}. \quad (20b)$$

On the contrary, from (17), the states and disturbances provided by the observer satisfy

$$\hat{x}_{\bar{k}} = A\hat{x}_{\bar{k}} + B\hat{u}_{\bar{k}} + G_d \hat{d}_{\bar{k}}, \quad (21a)$$

$$y_{\bar{k}} = C\hat{x}_{\bar{k}} + G_g \hat{g}_{\bar{k}}. \quad (21b)$$

Afterward, by subtracting (20a) from (21a) can be driven as

$$(\hat{x}_{\bar{k}} - x_s) = A(\hat{x}_{\bar{k}} - x_s) + B(\hat{u}_{\bar{k}} - u_s), \quad (22)$$

which relates to the original system considered by the target tracking optimization. If the original model given by  $A$ ,  $B$  and  $C$  can be stabilized, then the regulator will lead the states  $\tilde{x}(k)$  to zero, that is,

$$(\hat{x}_{\bar{k}} - x_s) = 0. \quad (23)$$

Now, subtracting (20b) from (21b) can be written as

$$(y_{\bar{k}} - y_{sp}) = C(\hat{x}_{\bar{k}} - x_s), \quad (24)$$

and, from (23) and (24), implies

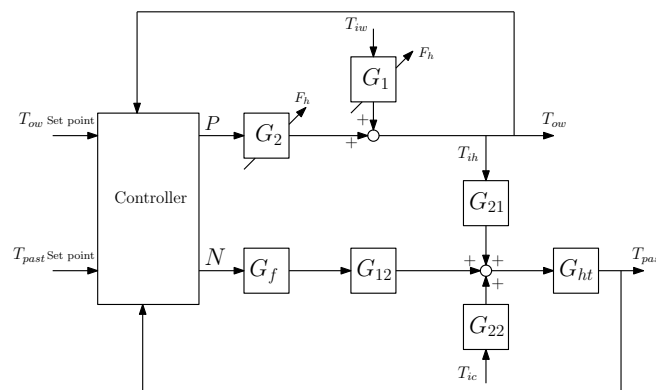
$$y_{\bar{k}} = y_{sp}. \quad (25)$$

To sum up, that the use of an augmented model requires the simultaneous solution of both optimization problems. This method can increase the computational cost of the algorithm, especially if the system has a large number of inputs and outputs.

## 5. Case Study

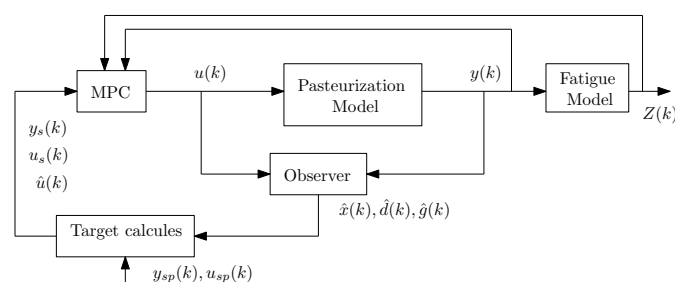
### 5.1. Benchmark description

As discussed in Section 2, the pasteurization system is used as a case study which the considered pasteurization process is the benchmark PCT-23 MKII, manufactured by Armfield (UK). This laboratory plant is the small version (1.2 m, 0.6 m, 0.6 m) of several real-time industrial pasteurization processes. Moreover, the model of the pasteurization plant are obtained from experimental data according to [8] also the identified models obtained as transfer functions are suitably formulated by their equivalent controllable realizations in state-space model.



**Figure 6.** Block diagram of the control loop.

Besides, the size of handle tube of pasteurization pumps is 1.6 mm with wall thickness from 0.5 mm to 8.0 mm bore. The rotor speed is 400 rpm while the flow rate is 2000 ml/min. Figure 6 introduce a block diagram of the pasteurization simulation model, provided with the benchmark, including the feedback loops corresponding to the flow and power. Also, the MPC controller, fatigue monitoring and integral action block that will be designed in an integrated manner are presented in this Figure 7.



**Figure 7.** Representative block diagram of the predictive control system with linear regulator and fatigue model.

### 5.2. Implementation of MPC with health-aware objective and integral action

The health-aware MPC, designed as described in Section 3, is obtained adding the accumulated damage model presented before, as a new state introduced in (13). In addition, for removing steady-state offset that is obtained with the new objective in the health-aware MPC cost function is used disturbance model.

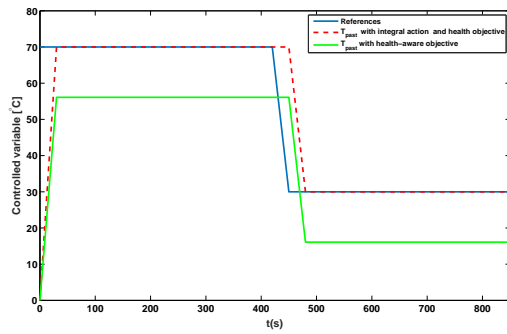
In brief then, the MPC with the new augmented model uses the following model:

$$x(k+1) = A_n x(k) + B_n u(k) + E_n d(k), \quad (26a)$$

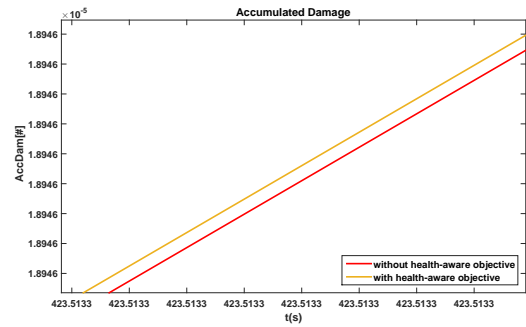
$$y(k) = C_n x(k), \quad (26b)$$

where  $A_n$ ,  $B_n$ ,  $C_n$ , and  $E_n$  are state matrices of proper dimensions with included accumulated damage and disturbance model given by (13) and (15), respectively. The MPC controller has been implemented in simulation where the weight matrices are  $Q = 1$ ,  $R = 0.01$ . The prediction horizon is chosen  $N_p = 400$ .

The control objective of the MPC controller for the pasteurization system is pasteurization temperature,  $T_{past}$ , tracking the setpoint, where their references change from 70 °C to 40 °C and in the same time, the accumulated damaged evaluated as (13) is minimized and the power of the electrical heater,  $P$ , and water pump speed,  $N$ , are minimized. Therefore, it can be said the cost and degradations are decreased while the temperature is achieved the references without steady-state offset.



**Figure 8.** Evolution of pasteurization temperature with (red) and without (green) integral action in the MPC.



**Figure 9.** Evolution of accumulated damages with (yellow) and without (red) health-aware objective in the MPC.

### 5.3. Results

In order to test the behavior of the proposed health-aware MPC control scheme regarding tracking, several simulations were carried out and the results obtained are presented in this part. The operating point for the flow is chosen as 200 ml/min.

The behavior of controlled temperature  $T_{past}$  from the pasteurization plant under the health-aware MPC controller with and without integral action for elimination steady-state offset is presented in Figure 8 with its corresponding references, while the controlled variables track the references. As it can be seen from the Figure 8, the temperature of the pasteurization process  $T_{past}$  (green line) wants to follow the references but, due to the adding the accumulated damage state as new states in the model generated same steady state error between the reference and output. However, by including an integral action in the MPC controller, the steady-state is eliminated (red line).

In Figure 9, it is provided the evolution of the accumulated damages obtained with (yellow) and without a (red) health-aware objective in the MPC controller. From Figure 9, it can be seen that the inclusion of the fatigue objective relieves 0.11 of the accumulated damage. At the same time, the pasteurization temperature,  $T_{past}$ , tracking the setpoint, where their references change from 70 °C to 40 °C is shown in Figure 8.

## 6. Conclusion

This paper has presented the integration of MPC with fatigue-based prognosis to minimize the damage of the components of a pasteurization plant. The integration of a system health management module with MPC control has provided the pasteurization plant with a mechanism to operate safely and optimize the

trade-off between components life and savings energy, product if an accurate following of the setpoint is performed. The MPC controller objective has been modified by adding an extra criterion that takes into account the accumulated damage. Finally, by including an integral action into the MPC controller design, the steady-state offset has been eliminated. The control scheme has been satisfactorily implemented using simulator of a utility-scale pasteurization plant. The results obtained show that there exists a trade-off between the minimization of the accumulated damage and tracking setpoint that is for saving energy. As future research, the proposed MPC control scheme to reduce the accumulated damage more than this approach and designing the health-aware nonlinear MPC based on the real model of the pasteurization system.

## References

- [1] Armfield. *Process Plant trainer PTC23-MKII, Instruction Manual*. 2015.
- [2] J. de Jesus Barradas-Berglind, R. Wisniewski, and M. Soltani. Fatigue damage estimation and data-based control for wind turbines. *IET Control Theory & Applications*, 9(7):1042–1050, 2015.
- [3] S. D. Downing and D. F. Socie. Simple rainflow counting algorithms. *International journal of fatigue*, 4(1):31–40, 1982.
- [4] T. Endo, K. Mitsunaga, and H. Nakagawa. Fatigue of metals subjected to varying stress prediction of fatigue lives. In *Preliminary Proceedings of the Chugoku-Shikoku District Meeting, Japanese Society of Mechanical Engineers, Tokyo*, pages 41–44, 1967.
- [5] T. Escobet, J. Quevedo, V. Puig, and F. Nejjari. Combining health monitoring and control. *Diagnostics and Prognostics of Engineering Systems: Methods and Techniques: Methods and Techniques*, page 230, 2012.
- [6] AH. González, EJ. Adam, and JL. Marchetti. Conditions for offset elimination in state space receding horizon controllers: A tutorial analysis. *Chemical Engineering and Processing: Process Intensification*, 47(12):2184–2194, 2008.
- [7] K. Hammerum, P. Brath, and N. K. Poulsen. A fatigue approach to wind turbine control. In *Journal of Physics: Conference Series*, volume 75, page 012081. IOP Publishing, 2007.
- [8] J. Ibarrola, J. Sandoval, M. Garcia-Sanz, and M. Pinzolas. Predictive control of a high temperature–short time pasteurisation process. *Control Engineering Practice*, 10(7):713–725, 2002.
- [9] M. T. Khadir. Enthalpy predictive functional control of a pasteurisation plant based on a plate heat exchanger. *International Journal of Modelling, Identification and Control*, 13(1-2):78–87, 2011.
- [10] H. Kwakernaak and R. Sivan. *Linear optimal control systems*. Wiley-interscience New York, 1972.
- [11] Y. L. Lee. *Fatigue testing and analysis: theory and practice*, volume 13. Butterworth-Heinemann, 2005.
- [12] J. C. Marin, A. Barroso, F. Paris, and J. Canas. Study of damage and repair of blades of a 300kW wind turbine. *Energy*, 33(7):1068–1083, 2008.
- [13] S. Martinen, I. Carlén, K. Nilsson, S. P. Breton, and S. Ivanell. Analysis of the effect of curtailment on power and fatigue loads of two aligned wind turbines using an actuator disc approach. In *Journal of Physics: Conference Series*, volume 524, page 012182. IOP Publishing, 2014.
- [14] M. A Miner et al. Cumulative damage in fatigue. *Journal of applied mechanics*, 12(3):159–164, 1945.
- [15] M. Musallam and C. M. Johnson. An efficient implementation of the rainflow counting algorithm for life consumption estimation. *IEEE Transactions on reliability*, 61(4):978–986, 2012.
- [16] K. R. Muske and T. A. Badgwell. Disturbance modeling for offset-free linear model predictive control. *Journal of Process Control*, 12(5):617–632, 2002.
- [17] A. Niesłony. Determination of fragments of multiaxial service loading strongly influencing the fatigue of machine components. *Mechanical Systems and Signal Processing*, 23(8):2712–2721, 2009.
- [18] P. F. Odgaard, T. Knudsen, A. Overgaard, H. Steffensen, and M. Jørgensen. Importance of dynamic inflow in model predictive control of wind turbines. *IFAC-Congress*, 48(30):90–95, 2015.
- [19] G. Pannocchia and J. B. Rawlings. Disturbance models for offset-free model-predictive control. *AIChE journal*, 49(2):426–437, 2003.
- [20] C. V. Rao, S. J. Wright, and J. B. Rawlings. Application of interior-point methods to model predictive control. *Journal of optimization theory and applications*, 99(3):723–757, 1998.
- [21] A. Rosich and C. Ocampo-Martínez. Real-time experimental implementation of predictive control schemes in a small-scale pasteurization plant. In *Developments in Model-Based Optimization and Control*, pages 255–273. Springer, 2015.
- [22] I. Rychlik. A new definition of the rainflow cycle counting method. *International journal of fatigue*, 9(2):119–121, 1987.
- [23] H. Sanchez, T. Escobet, V. Puig, and P. F. Odgaard. Health-aware model predictive control of wind turbines using fatigue prognosis. *IFAC-Congress*, 48(21):1363–1368, 2015.