Adaptive backstepping control for three axis microsatellite attitude pointing under actuator faults

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Abstract. This paper presents the design of Low Earth Orbit (LEO) micro-satellite attitude controller using reaction wheels, and under actuator faults. Firstly, a backstepping controller is developed when the actuator is fault-free. Then, a fault tolerant controller is designed to compensate the actuator fault. Two types of this latter are considered (additive and multiplicative faults). The presented control strategy is based on adaptive backstepping technique. The simulation results clearly demonstrate the effectiveness of the presented technique.

1. Introduction

The attitude is the orientation of satellite in the space. In absence of control, it evolves naturally under effect of external disturbances. The attitude control has the role to compensate this disturbing torques.
The attitude control is a very important field of research in space technology. It attracted much attention in the recent years because of many types of space missions.
From automatic point of view, the satellite is a nonlinear dynamic system, it is highly coupled. Therefore, the design of the attitude controllers is usually difficult. Various nonlinear controllers have been proposed to solve this problem. These controllers include sliding mode control [1-2], fuzzy control [3], backstepping control and feedback control [4]. Among these techniques, backstepping control is a recursive method, it is based on Lyapunov theory which ensures the stabilization of each step of synthesis.
The backstepping has been very useful in the space field over the past years. In [5], this technique was used with the inverse optimal control to stabilize spacecraft attitude. In [6], the backstepping was based on similar skew-symmetric structure. [7] presented control by integrator backstepping with internal stabilization. All these controllers are based on the knowledge of the system parameters. However, if faults appear in satellite subsystems such as actuators, the control laws [5-7] induce no desired behaviours, and the consequences of these faults can be catastrophic. Therefore, it is important to develop other control strategies that can tolerate faults.
In [8], a combination between backstepping control, adaptive control and new matrix product (which called the semitensor product) was used to control the spacecraft attitude with unknown external disturbances, but no faults were present. In [9-11] adaptive control was developed in the presence of unknown inertia parameters and also with no faults. In [12] fault tolerant attitude control was designed using adaptive neural network.

In this paper, a controller design for Low Earth Orbit microsatellite is presented in the presence of the combination of two actuator faults: additive and multiplicative faults. The presented controller is developed using the adaptive backstepping technique, which is based on adaptive design of Lyapunov. The paper is organized as follows. Section 2 presents the dynamic and kinematic models used for the satellite. Section 3 presents the controllability test of our system. In the section 4, 5, and 6, we describe the design of the control laws which are presented in this work. In next section, we present the simulation results. Finally, the conclusion of this paper is presented in section 8.

2. Spacecraft attitude model

The dynamics of the spacecraft in inertial space governed by Euler's equations of motion can be expressed as follows in vector form as [13-14],

$$\mathbf{I}\dot{\boldsymbol{\omega}}_{\mathbf{s}}^{\mathbf{I}} = \mathbf{C}_{\mathrm{gg}} + \mathbf{C}_{\mathrm{ext}} + \mathbf{I}\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{I}} \times \left(\mathbf{I}\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{I}} + \mathbf{h}\right) - \dot{\mathbf{h}} \tag{1}$$

where $\boldsymbol{\omega}_{\mathrm{s}}^{\mathrm{I}}, \mathbf{I}$, $\mathbf{C}_{\mathrm{gg}}$ and $\mathbf{C}_{\mathrm{ext}}$ are respectively the inertial referenced body angular velocity vector, moment of inertia of spacecraft, gravity gradient torque vector, and external disturbance torque vector. The expression of the gravity gradient vector in body coordinate is expressed as,

$$C_{ggx} = 3\omega_0^2\left[\left(I_{zz} - I_{yy}\right)A_{23}A_{33} + I_{yx}A_{13}A_{33} + I_{yz}A_{33}^2 - I_{zx}A_{13}A_{23} - I_{zy}A_{23}^2\right] \tag{2.a}$$

$$C_{ggy} = 3\omega_0^2\left[(I_{xx} - I_{zz})A_{13}A_{33} - I_{xy}A_{23}A_{33} - I_{xz}A_{33}^2 + I_{zx}A_{13}^2 + I_{zy}A_{13}A_{23}\right] \tag{2.b}$$

$$C_{ggz} = 3\omega_0^2\left[\left(I_{yy} - I_{xx}\right)A_{13}A_{23} + I_{xy}A_{23}^2 + I_{xz}A_{23}A_{33} - I_{yx}A_{13}^2 - I_{yz}A_{13}A_{33}\right] \tag{2.c}$$

where,
ω_0 mean orbital angular velocity of the satellite;
A_{ij} attitude matrix elements.

The kinematic quaternion is expressed as,

$$\dot{\mathbf{q}} = \frac{1}{2}\,\boldsymbol{\Omega}\,\mathbf{q} = \frac{1}{2}\boldsymbol{\Lambda}(\mathbf{q})\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{o}} \tag{3}$$

where,

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \omega_{oz} & -\omega_{oy} & \omega_{ox} \\ -\omega_{oz} & 0 & \omega_{ox} & \omega_{oy} \\ \omega_{oy} & -\omega_{ox} & 0 & \omega_{oz} \\ -\omega_{ox} & -\omega_{oy} & -\omega_{oz} & 0 \end{bmatrix} \tag{4}$$

and,

$$\boldsymbol{\Lambda}(\mathbf{q}) = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{5}$$

$\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{o}} = [\omega_{ox} \quad \omega_{oy} \quad \omega_{oz}]^{\mathrm{T}}$: body angular velocity vector referenced to orbital coordinates.
The angular body rates referenced to the orbit coordinates can be obtained from the inertial referenced body rates by using the transformation matrix **A** as [13],

$$\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{o}} = \boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{I}} - \mathbf{A}\boldsymbol{\omega}_{\mathbf{0}} \tag{6}$$

From equation (1) and equation (3), the satellite mathematical model can be written as,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{h}) + \mathbf{B}\,\mathbf{U} \tag{7.1}$$
$$\mathbf{y} = \mathbf{H}\,\mathbf{x} \tag{7.2}$$

where,

$$\mathbf{f}(\mathbf{x},\mathbf{h}) = \begin{bmatrix} 0.5(\omega_{oz}q_2 - \omega_{oy}q_3 + \omega_{ox}q_4) \\ 0.5(-\omega_{oz}q_1 + \omega_{ox}q_3 + \omega_{oy}q_4) \\ 0.5(\omega_{oy}q_1 - \omega_{ox}q_2 + \omega_{oz}q_4) \\ 0.5(-\omega_{ox}q_1 - \omega_{oy}q_2 - \omega_{oz}q_3) \\ I_x^{-1}\left(C_x - (I_z - I_y)\omega_y\omega_z - \omega_y h_z + \omega_z h_y\right) \\ I_y^{-1}\left(C_y - (I_x - I_z)\omega_x\omega_z + \omega_x h_z - \omega_z h_x\right) \\ I_y^{-1}\left(C_z - (I_y - I_x)\omega_x\omega_y - \omega_x h_y + \omega_y h_x\right) \end{bmatrix} \tag{8}$$

$\mathbf{x} = [q_1, q_2, q_3, q_4, \omega_x, \omega_y, \omega_z]^T$ state vector;
$\mathbf{I} = [I_x \quad I_y \quad I_z]$ moment of inertia of spacecraft;
$\boldsymbol{\omega}_B^I = [\omega_x \quad \omega_y \quad \omega_z]^T$ angular velocity vector in the inertial frame;
$\mathbf{q} = [q_1 q_2 q_3 q_4]$ quaternion;
$\mathbf{h} = [h_x \quad h_y \quad h_z]^T$angular moment vector;
$\mathbf{C}_{ext} = [C_x \quad C_y \quad C_z]^T$external disturbance torque vector;
$\mathbf{B} = \left[\mathbf{0}^{4\times3}\ \mathbf{I}^{3\times3}\right]^T$ control matrix;
$\mathbf{U} = -\dot{\mathbf{h}}$ control input torque;
It assumed that the quaternion measurements are directly available $\mathbf{H} = [\mathbf{I}^{4\times4}\ \ \mathbf{0}^{4\times3}]$.

3. Controllability test

The controllability concept is related to dynamic systems. It has a fundamental importance when studying attitude control algorithms that we will presented in the following sections. For that reason we introduce this concept in what follows.
The controllability test is based on the rank test of the controllability matrix **CO** which is defined as,

$$\mathbf{CO} = \left[\mathbf{B}\quad \mathbf{FB}\quad \mathbf{F}^2\mathbf{B}\quad \mathbf{F}^3\mathbf{B}\quad \mathbf{F}^4\mathbf{B}\quad \mathbf{F}^5\mathbf{B}\quad \mathbf{F}^6\mathbf{B}\right] \tag{9}$$

where,

$$\mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

The system is controllable if the rank of the matrix **CO** is equal to seven.

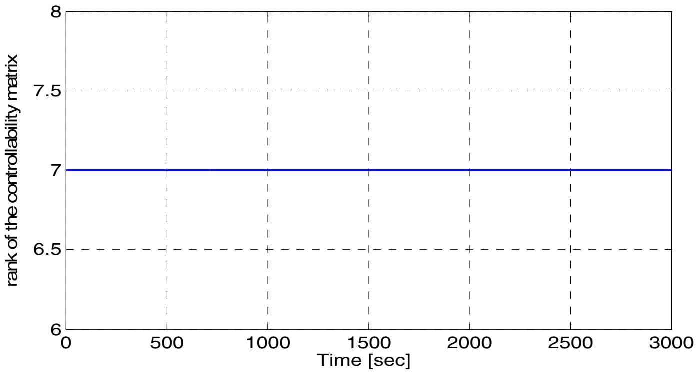

Figure 1. Rank of the controllability matrix.

Figure 1 shows that the rank of the controllability matrix is equal to seven. Therefore, our system is controllable.

4. Backstepping control design

In this section, backstepping control law is developed. This technique is inspired by [15].
The backstepping algorithm is described in two steps as follow.

4.1. Step 1

We define the first and the second variable of backstepping

$$\mathbf{z}_1 = \mathbf{x}_1 = \mathbf{q}_\mathrm{e} = \mathbf{q}_\mathrm{c}\,\mathbf{q} \tag{11}$$
$$\mathbf{z}_2 = \mathbf{x}_2 - \boldsymbol{\alpha}_1 \tag{12}$$

where,
$\mathbf{x}_2 = \boldsymbol{\omega}_\mathrm{s}^\mathrm{o}$;
$\boldsymbol{\alpha}_1$ is a virtual control law.
The time derivative of $\mathbf{z}_1$ is expressed as,

$$\dot{\mathbf{z}}_1 = \dot{\mathbf{x}}_1 = \dot{\mathbf{q}}_\mathrm{e} = \mathbf{q}_\mathrm{c}\left(\frac{1}{2}\boldsymbol{\Lambda}(\mathbf{q})\boldsymbol{\omega}_\mathrm{s}^\mathrm{o}\right) \tag{13}$$

The first Lyapunov function is defined as,

$$\mathrm{V}_1(\mathbf{z}_1) = \mathbf{z}_1^\mathrm{T}\,\mathbf{z}_1 \tag{14}$$

Its time derivative is expressed as,

$$\dot{\mathrm{V}}_1 = 2\mathbf{z}_1^\mathrm{T}\dot{\mathbf{z}}_1 = \mathbf{z}_1^\mathrm{T}\mathbf{G}(\mathbf{q})\mathbf{z}_2 + \mathbf{z}_1^\mathrm{T}\mathbf{G}(\mathbf{q})\boldsymbol{\alpha}_1 \tag{15}$$

where,

$$\mathbf{G}(\mathbf{q}) = \mathbf{q}_\mathrm{c}\;\boldsymbol{\Lambda}(\mathbf{q}) \tag{16}$$

To make $\dot{\mathrm{V}}_1$ negative, $\boldsymbol{\alpha}_1$ is chosen as,

$$\boldsymbol{\alpha}_1 = -\mathbf{k}_1\mathbf{G}(\mathbf{q})^\mathrm{T}\mathbf{z}_1 \tag{17}$$

where,
$\mathbf{k}_1$: is a positive gain matrix.

The time derivative of V_1 becomes

$$\dot{V}_1 = -\mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{k}_1\mathbf{G}(\mathbf{q})^T\mathbf{z}_1 + \mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{z}_2 \tag{18}$$

The term $\mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{z}_2$ will be eliminated in the next step.

4.2. Step 2

The time derivative of $\mathbf{z}_2$ is expressed as,

$$\dot{\mathbf{z}}_2 = \dot{\boldsymbol{\omega}}_s^o - \dot{\boldsymbol{\alpha}}_1 \tag{19}$$

where,

$$\boldsymbol{\omega}_s^o = \boldsymbol{\omega}_s^I - \mathbf{A}\boldsymbol{\omega}_0 \tag{20}$$

We replace the equation (20) in (19), we obtain

$$\dot{\mathbf{z}}_2 = \dot{\boldsymbol{\omega}}_s^I - \dot{\mathbf{A}}\boldsymbol{\omega}_0 - \dot{\boldsymbol{\alpha}}_1 \tag{21}$$
$$\mathbf{I}\dot{\mathbf{z}}_2 = \mathbf{I}\dot{\boldsymbol{\omega}}_s^I - \mathbf{I}\dot{\mathbf{A}}\boldsymbol{\omega}_0 - \mathbf{I}\dot{\boldsymbol{\alpha}}_1 \tag{22}$$

The second Lyapunov function is defined as,

$$V_2(\mathbf{z}_1, \mathbf{z}_2) = V_1(\mathbf{z}_1) + \frac{1}{2}\mathbf{z}_2^T\mathbf{I}\,\mathbf{z}_2 \tag{23}$$

Its time derivative is expressed as,

$$\dot{V}_2 = \dot{V}_1 + \mathbf{z}_2^T\mathbf{I}\,\dot{\mathbf{z}}_2 \tag{24}$$
$$\begin{aligned}\dot{V}_2 &= -\mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{k}_1\mathbf{G}(\mathbf{q})^T\mathbf{z}_1 + \mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{z}_2 \\ &+\mathbf{z}_2^T\left[\left(\mathbf{C}_{gg} + \mathbf{C}_{ext} - \boldsymbol{\omega}_s^I \times \left(\mathbf{I}\boldsymbol{\omega}_s^I + \mathbf{h}\right) - \dot{\mathbf{h}}\right) - \mathbf{I}\dot{\mathbf{A}}\boldsymbol{\omega}_0 - \mathbf{I}\dot{\boldsymbol{\alpha}}_1\right]\end{aligned} \tag{25}$$

To make $\dot{V}_2$ negative, the control law $\dot{\mathbf{h}}$ is chosen as,

$$\dot{\mathbf{h}} = \mathbf{k}_2\mathbf{z}_2 + \mathbf{G}(\mathbf{q})^T\mathbf{z}_1 + \left(\mathbf{C}_{gg} + \mathbf{C}_{ext} - \boldsymbol{\omega}_s^I \times \left(\mathbf{I}\boldsymbol{\omega}_s^I + \mathbf{h}\right)\right) - \mathbf{I}\dot{\mathbf{A}}\boldsymbol{\omega}_0 - \mathbf{I}\dot{\boldsymbol{\alpha}}_1 \tag{26}$$

5. Adaptive control based on Lyapunov theory

The adaptive design of Lyapunov is presented in this section as an introduction of the adaptive backstepping method.
Consider the following nonlinear system

$$\dot{\mathbf{x}} = \mathbf{u} + \boldsymbol{\varphi}(\mathbf{x})^T\boldsymbol{\theta} \tag{27}$$

where,
$\boldsymbol{\theta}$ is a vector of unknown parameters.

We seek to find the control law $\mathbf{u}(\mathbf{x},\boldsymbol{\theta})$ which ensures the stability of the system (27). Therefore, we choose the following Lyapunov function

$$\mathrm{V}_1(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\,\mathbf{x} \tag{28}$$

which is definite positive, its derivative is expressed as,

$$\dot{\mathrm{V}}_1 = \mathbf{x}^{\mathrm{T}}\dot{\mathbf{x}} = \mathbf{x}^{\mathrm{T}}[\mathbf{u} + \boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\boldsymbol{\theta}] \tag{29}$$

The control law is chosen as,

$$\mathbf{u}(\mathbf{x},\boldsymbol{\theta}) = -\boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\boldsymbol{\theta} - \mathbf{k}_1\mathbf{x} \tag{30}$$

where, $\mathbf{k}_1 > \mathbf{0}$
Therefore, two cases arise:

- $\boldsymbol{\theta}$ is known: the control law (30) can be realized, which makes the system (25) stable.
- $\boldsymbol{\theta}$ is unknown : the controller law (30) cannot be realized. We propose to replace it by its equivalent $\widehat{\boldsymbol{\theta}}$.

$$\mathbf{u} = -\boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\,\widehat{\boldsymbol{\theta}} - \mathbf{k}_1\mathbf{x} \tag{31}$$

The derivative of the Lyapunov function will be expressed as,

$$\dot{\mathrm{V}}_1 = \mathbf{x}^{\mathrm{T}}\left[-\mathbf{k}_1\mathbf{x} + \boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\,\widetilde{\boldsymbol{\theta}}\right] = -\mathbf{x}^{\mathrm{T}}\mathbf{k}_1\mathbf{x} + \mathbf{x}^{\mathrm{T}}\boldsymbol{\varphi}(\mathbf{x})^{\mathrm{T}}\,\widetilde{\boldsymbol{\theta}} \tag{32}$$

where, $\widetilde{\boldsymbol{\theta}}$ represents the estimation error $\left(\boldsymbol{\theta} - \widehat{\boldsymbol{\theta}}\right)$.
This expression contains an unknown term $\widetilde{\boldsymbol{\theta}}$, its sign is indefinite, and no conclusion can be drawn for the stability of the system. Therefore, we define a new Lyapunov function by adding to the initial function (28) a quadratic term according to the estimation error $\widetilde{\boldsymbol{\theta}}$.

$$\mathrm{V}_2\left(\mathbf{x},\widetilde{\boldsymbol{\theta}}\right) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\,\mathbf{x} + \frac{1}{2}\widetilde{\boldsymbol{\theta}}^{\mathrm{T}}\,\boldsymbol{\Gamma}^{-1}\,\widetilde{\boldsymbol{\theta}} \tag{33}$$

where, $\boldsymbol{\Gamma}$ is a gain matrix (definite positive) which represents the adaptation gain.
The derivative of this function becomes:

$$\dot{\mathrm{V}}_2 = \mathbf{x}^{\mathrm{T}}\dot{\mathbf{x}} + \widetilde{\boldsymbol{\theta}}^{\mathrm{T}}\,\boldsymbol{\Gamma}^{-1}\dot{\widetilde{\boldsymbol{\theta}}} = -\mathbf{x}^{\mathrm{T}}\mathbf{k}_1\mathbf{x} + \widetilde{\boldsymbol{\theta}}^{\mathrm{T}}\,\boldsymbol{\Gamma}^{-1}\left(\dot{\widetilde{\boldsymbol{\theta}}} + \boldsymbol{\tau}\right) \tag{34}$$

where,

$$\boldsymbol{\tau} = \boldsymbol{\Gamma}\boldsymbol{\varphi}(\mathbf{x})\,\mathbf{x} \tag{35}$$

This derivative remains always indefinite, but this time the choice of the update law can cancel the second term of the equation.

$$\dot{\widehat{\boldsymbol{\theta}}} = -\dot{\widetilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}\boldsymbol{\varphi}(\mathbf{x})\,\mathbf{x} \tag{36}$$

6. Fault tolerant adaptive backstepping control design

The adaptive backstepping is designed from the fusion of the adaptive design of Lyapunov presented in section (5) and the no adaptive backstepping technique presented in section (4). The direct combination of these two methods gives a triplet (Lyapunov function, control law, adaptation law). Figure 2 presents the adaptive backstepping schematic diagram.

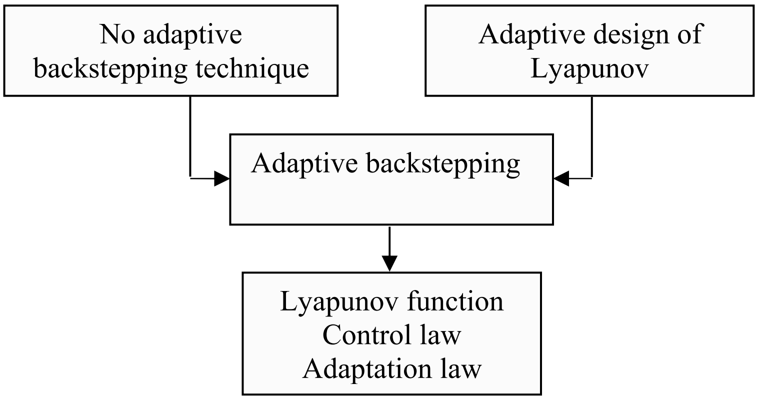

Figure 2. Adaptive backstepping schematic diagram.

This technique is used to develop a controller which can tolerate actuator faults. The design of this controller is described as follow [16],
In the presence of actuator faults, the dynamic equation model is rewritten as,

$$\mathbf{I}\dot{\boldsymbol{\omega}}_{\mathbf{s}}^{\mathbf{I}} = \mathbf{C}_{\mathrm{gg}} + \mathbf{C}_{\mathrm{ext}} + \mathbf{I}\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{I}} \times \left(\mathbf{I}\boldsymbol{\omega}_{\mathbf{s}}^{\mathbf{I}} + \mathbf{h}\right) - \left(\mathbf{f}_{\mathrm{m}}\dot{\mathbf{h}} + \mathbf{f}_{\mathrm{a}}\right)$$

where,
$\mathbf{f}_{\mathrm{a}}$: additive fault;
$\mathbf{f}_{\mathrm{m}}$: multiplicative fault.

6.1. Step 1

We define the first variable of backstepping $\mathbf{z}_1 = \mathbf{x}_1 = \mathbf{q}_{\mathrm{e}}$, and the estimated fault errors $\tilde{\mathbf{f}}_{\mathrm{a}} = \hat{\mathbf{f}}_{\mathrm{a}} - \mathbf{f}_{\mathrm{a}}$, $\tilde{\mathbf{f}}_{\mathrm{m}} = \hat{\mathbf{f}}_{\mathrm{m}} - \mathbf{f}_{\mathrm{m}}$.
where,
$\hat{\mathbf{f}}_{\mathrm{a}}$: estimated additive fault;
$\hat{\mathbf{f}}_{\mathrm{m}}$: estimated multiplicative fault;
$\tilde{\mathbf{f}}_{\mathrm{a}}$: additive fault error;
$\tilde{\mathbf{f}}_{\mathrm{m}}$: multiplicative fault error.

The second variable of backstepping is:

$$\mathbf{z}_2 = \mathbf{x}_2 - \boldsymbol{\alpha}_1 = \boldsymbol{\omega}_{\mathrm{s}}^{\mathrm{o}} - \boldsymbol{\alpha}_1 \tag{37}$$

where, $\boldsymbol{\alpha}_1$ is a virtual control law.

The first subsystem of the mathematic model is independent of faults. Therefore, the design of the first step of the adaptive backstepping technique is the same as the traditional backstepping and the derivative of the first Lyapunov function does not change.

$$\dot{V}_1 = -\mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{k}_1\mathbf{G}(\mathbf{q})^T\mathbf{z}_1 + \mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{z}_2 \tag{38}$$

The term $\mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{z}_2$ will be eliminated in the next step.

6.2. Step 2

$$\mathbf{z}_2 = \boldsymbol{\omega}_s^o - \boldsymbol{\alpha}_1 \tag{39}$$

Its time derivative is expressed as,

$$\dot{\mathbf{z}}_2 = \dot{\boldsymbol{\omega}}_s^o - \dot{\boldsymbol{\alpha}}_1 \tag{40}$$
$$\boldsymbol{\omega}_s^o = \boldsymbol{\omega}_s^I - \mathbf{A}\boldsymbol{\omega}_0 \tag{41}$$

We replace the equation (41) in (40), we obtain

$$\dot{\mathbf{z}}_2 = \dot{\boldsymbol{\omega}}_s^I - \dot{\mathbf{A}}\boldsymbol{\omega}_0 - \dot{\boldsymbol{\alpha}}_1 \tag{42}$$
$$\mathbf{I}\dot{\mathbf{z}}_2 = \mathbf{I}\dot{\boldsymbol{\omega}}_s^I - \mathbf{I}\dot{\mathbf{A}}\boldsymbol{\omega}_0 - \mathbf{I}\dot{\boldsymbol{\alpha}}_1 \tag{43}$$

We choose the following Lyapunov function

$$V_2(\mathbf{z}_1, \mathbf{z}_2, \tilde{\mathbf{f}}) = V_1(\mathbf{z}_1) + \frac{1}{2}\mathbf{z}_2^T\mathbf{I}\,\mathbf{z}_2 + \frac{1}{2}\tilde{\mathbf{f}}_a^T\,\boldsymbol{\Gamma}^{-1}\tilde{\mathbf{f}}_a + \frac{1}{2}\tilde{\mathbf{f}}_m^T\,\boldsymbol{\Gamma}^{-1}\,\tilde{\mathbf{f}}_m \tag{44}$$

where,
$\boldsymbol{\Gamma}$ is a design parameter which must be positive.

The time derivative of the Lyapunov function is expressed as,

$$\dot{V}_2 = \dot{V}_1 + \mathbf{z}_2^T\mathbf{I}\,\dot{\mathbf{z}}_2 + \tilde{\mathbf{f}}_a^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{f}}}_a + \tilde{\mathbf{f}}_m^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{f}}}_m \tag{45}$$

$$\begin{aligned}\dot{V}_2 = {} & \mathbf{z}_2^T\left[\left(\mathbf{C}_{gg} + \mathbf{C}_{ext} - \boldsymbol{\omega}_s^I \times \left(\mathbf{I}\boldsymbol{\omega}_s^I + \mathbf{h}\right) - \mathbf{f}_m\dot{\mathbf{h}} - \mathbf{f}_a\right) - \mathbf{I}\dot{\mathbf{A}}\boldsymbol{\omega}_0 - \mathbf{I}\dot{\boldsymbol{\alpha}}_1\right] - \mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{k}_1\mathbf{G}(\mathbf{q})^T\mathbf{z}_1 + \\ & \mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{z}_2 + \tilde{\mathbf{f}}_a^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{f}}}_a + \tilde{\mathbf{f}}_m^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{f}}}_m\end{aligned} \tag{46}$$

$$\begin{aligned}\dot{V}_2 = {} & -\mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{k}_1\mathbf{G}(\mathbf{q})^T\mathbf{z}_1 + \mathbf{z}_1^T\mathbf{G}(\mathbf{q})\mathbf{z}_2 \\ & +\mathbf{z}_2^T\left[\left(\mathbf{C}_{gg} + \mathbf{C}_{ext} - \boldsymbol{\omega}_s^I \times \left(\mathbf{I}\boldsymbol{\omega}_s^I + \mathbf{h}\right) - \hat{\mathbf{f}}_m\dot{\mathbf{h}} - \hat{\mathbf{f}}_a\right) - \mathbf{I}\dot{\mathbf{A}}\boldsymbol{\omega}_0 - \mathbf{I}\dot{\boldsymbol{\alpha}}_1\right] + \tilde{\mathbf{f}}_a^T\left[\mathbf{z}_2 + \boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{f}}}_a\right] \\ & +\tilde{\mathbf{f}}_m^T\left[\dot{\mathbf{h}}\mathbf{z}_2 + \boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{f}}}_m\right]\end{aligned} \tag{47}$$

To make this derivative negative, we choose the control law $\dot{\mathbf{h}}$ as follow,

$$\dot{\mathbf{h}} = \hat{\mathbf{f}}_m^{-1}\left[\mathbf{k}_2\mathbf{z}_2 + \mathbf{G}(\mathbf{q})^T\mathbf{z}_1 + \left(\mathbf{C}_{gg} + \mathbf{C}_{ext} - \boldsymbol{\omega}_s^I \times \left(\mathbf{I}\boldsymbol{\omega}_s^I + \mathbf{h}\right)\right) - \hat{\mathbf{f}}_a - \mathbf{I}\dot{\mathbf{A}}\boldsymbol{\omega}_0 - \mathbf{I}\dot{\boldsymbol{\alpha}}_1\right] \tag{48}$$

The update laws of the estimated faults are expressed as,

$$\dot{\hat{\mathbf{f}}}_a = -\boldsymbol{\Gamma}\,\mathbf{z}_2 \tag{49}$$
$$\dot{\hat{\mathbf{f}}}_m = -\boldsymbol{\Gamma}\,\dot{\mathbf{h}}\,\mathbf{z}_2 \tag{50}$$

7. Simulation results

In this section, we present the simulation results obtained for the controllers exposed previously. These results are obtained using the following parameters.

Table1. Satellite simulation parameters.

Parameter	Value
Inertia [kg.m^2]	$\begin{bmatrix} 12 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 10 \end{bmatrix}$
Orbit [km]	686
Inclination [deg]	98
Initial attitude [deg]	$[5 \quad 10 \quad -10]$
Initial attitude rate	$[0 \quad -0.06 \quad 0]$
External Torques [N.m]	$\begin{bmatrix} 10^{-7}(5\cos(\omega_0 t)+1) \\ 10^{-7}(5\cos(\omega_0 t)+2\sin(\omega_0 t)) \\ 10^{-7}(5\cos(\omega_0 t)+1) \end{bmatrix}$

Table 2. Desired attitude for the Euler angles.

Desired attitude [deg]
$[10 \quad 30 \quad 20]$

Table 3. Backstepping parameters.

$\mathbf{k_1}$	$\mathbf{k_2}$
$0.01 * \mathrm{eye}(3)$	$8 * \mathrm{eye}(3)$

Table 4. Adaptive backstepping parameters.

$\mathbf{k_1}$	$\mathbf{k_2}$	$\mathbf{\Gamma}$
$0.01 * \mathrm{eye}(3)$	$8 * \mathrm{eye}(3)$	$50 * \mathrm{eye}(3)$

Two actuator faults are considered simultaneously. We introduce at the time t=400 sec an additive fault $\boldsymbol{f_a(t)}$.

$$\mathbf{f_a(t)} = \begin{cases} 0 & \mathrm{t} < 400\ sec \\ 2 + 0.01\cos(0.5\pi \mathrm{t}) & \mathrm{t} \geq 400 \text{ sec} \end{cases}$$

Then, at the time t=600 sec a multiplicative fault $\boldsymbol{f_m(t)}$.

$$\mathbf{f_m(t)} = \begin{cases} 1 & \mathrm{t} < 600\ sec \\ 0.5 + 0.07\cos(0.5\pi \mathrm{t}) & \mathrm{t} \geq 600 \text{ sec} \end{cases}$$

The simulation results are presented as follow.

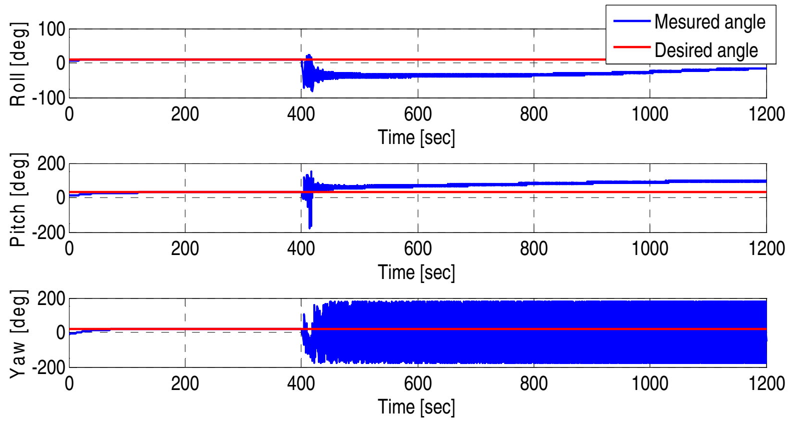

Figure 3. Actual and estimated attitude - No adaptive backstepping.

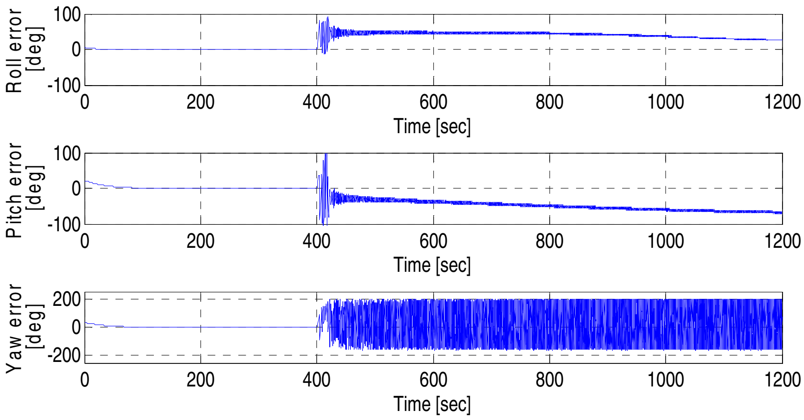

Figure 4. Errors of estimated attitude-No adaptive backstepping.

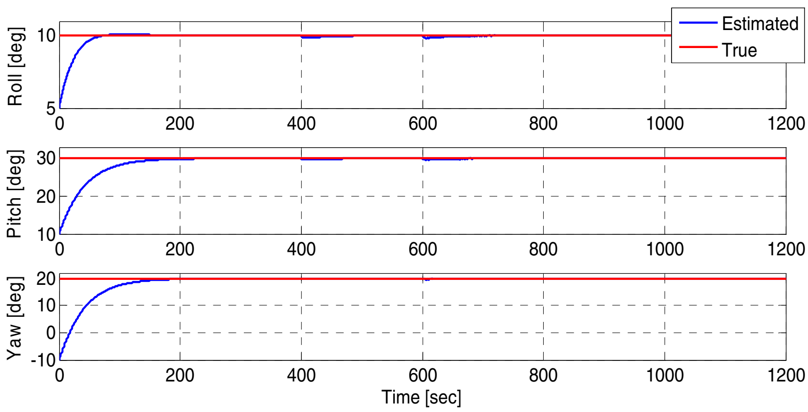

Figure 5. Actual and estimated attitude - Adaptive backstepping

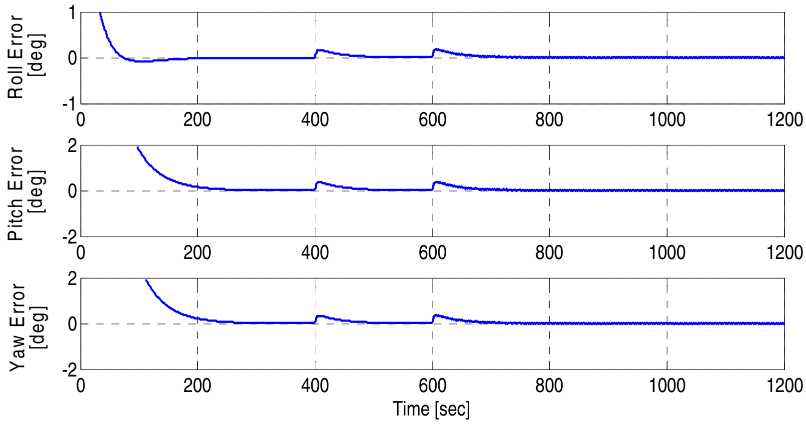

Figure 6. Errors of estimated attitude - Adaptive backstepping.

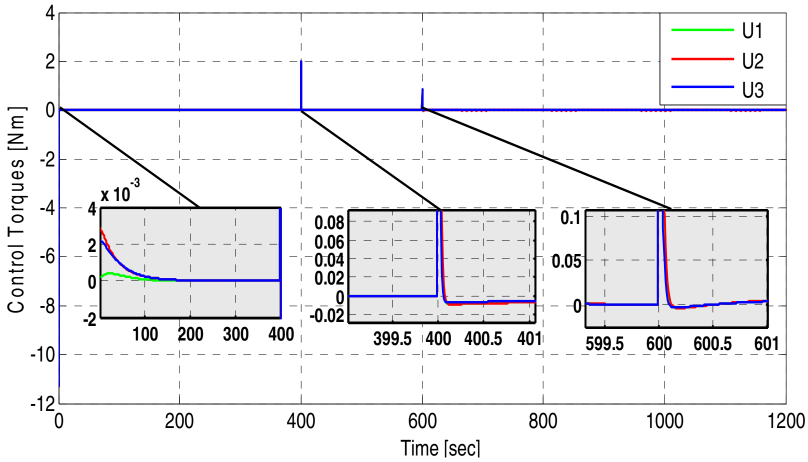

Figure 7. Control torques.

The figures 3-6 show the estimated attitude and its error of the two controllers (no adaptive and adaptive backstepping). These results are obtained using the same simulation conditions. We clearly observe in figures 2-3 that the actuator faults are not compensated by the backstepping controller, the consequence of these faults is catastrophic, the attitude diverges and the Euler angle errors are very large (200 deg). However, in figures 5-6 the desired attitude tracking is assured even the occurrence of the actuator faults at t=400 sec and t=600. In addition, the control torques converges to zero.
From all of the above, it is clearly found that the presented controller can guarantees the control performance, it succeed despite the combination of these faults.

8. Conclusion

In this work, we presented a synthesis of control laws for Low Earth Orbit (LEO) micro-satellite attitude stabilization using three axis controls by reaction wheels, and under actuator faults. All these laws are based on the Lyapunov theorem. Firstly, a nominal backstepping controller was developed when the actuator is fault-free. Then, a fault tolerant controller is designed to compensate the actuator faults. Two types of this latter were considered (additive and multiplicative faults). The presented control strategy is based on adaptive backstepping technique. This latter is designed by combining two methods (the adaptive design of Lyapunov and the no adaptive backstepping technique).
The results of this work demonstrate the effectiveness of the presented technique. It is found that the presented controller can guarantees the control performance despite the combination of these faults.

As a future research, we will develop other control techniques to compensate the actuator faults such as the super twisting algorithm.

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