Cascade structures of fault-tolerant control schemes with the static and dynamic output controllers

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Abstract. An enhanced approach to fault-tolerant control design is proposed in the paper for linear systems subject to cascade control strategy, while static and dynamic output controllers are employed to maintain the stability of the overall interconnected control structure. The controller gains are solved simultaneously using two-step linear matrix inequality formulation, conditioned by linear matrix equalities. A simulation example, subject to a system matrix parameter fault, demonstrates the effectiveness of the proposed method of design and cascade control technique.

1. Introduction

Operating conditions in modern engineering systems are still exposed to possibility of system failure. Any failure of the sensors, actuators or other system components can drastically change the system behavior. Fault tolerant control (FTC) allows a strategy to improve reliability of the whole system and so many techniques have been proposed especially for sensor and actuator failures with application to a wide range of engineering fields. To recover at least in part the performance of the fault-free control, one of the main ideas to control reconfiguration design is to modify the feedback gain so that the reconfigured system approximates the dynamics of nominal closed-loop system. In the case of complex technological systems, it is possible to predesign controllers for the anticipated fault cases, and switch to the corresponding control law once a fault is detected and isolated.

One of the key reconfigurable control methods is reference model based control, where the main objective is to maintain as much similarity as possible achieved by reassigning the feedback gains to the original designed. Based on modified pseudoinverse methods [6], [7], [8], the lack of stability guarantee limits hardly such use in the design of FTC [15], [16]. Rather than claiming the closed-loop faulty system to have the same model as the nominal system, it can be required to design the control reconfiguration for trajectory following problem in which the reference is the nominal closed-loop system trajectory, or its approximates [2], [4].

Cascade control structures are mainly used to achieve primarily suppression of a disturbance before it propagates to the parts of the plant [13]. To apply in reconfiguration, a cascade state controller structure is proposed in [1], [9], to keep intact the original controller and the nominal model trajectory tracking, while the error between the nominal output and the measurable faulty output variables is proposed as a basis for the correction. Relaxing the existence conditions for reference model state following, the technique proposed in this paper generalizes the design principle in accordance to the model-reference control tenets subject to a system matrix parameter fault [5], [11], [12]. To achieve the desired control objective, the bi-proper dynamic output controller in combination with the static output control part is realized, as a principal novelty, within the reconfiguration cascade structure. A characteristic feature of such structure is that even after the fault occurrence is maintained the inner loop with the nominal control law parameters. The approach utilizes the measurable input and output vector variables, design conditions are based on linear matrix inequality (LMI) technique combined with regularization of bilinear forms by linear matrix inequality approach, which give an effective way to calculate the controller parameters.

The paper is organized as follows. Ensuing the introduction given in Sec. 1, Sec. 2 presents the problem formulation focusing on assumptions about the controller structure properties. In Sec. 3 the main properties of the method exploiting the cascade structure for linear systems with system dynamic faults is presented and the separation principle is proven. Subsequently, Sec. 4 - 6 derive new design results when using the bi-proper dynamic output controller, static output control and static decoupling principle in the fault-tolerant cascade control structure, all in the framework of LMIs and LMEs. Conforming the results, Sec. 7 follows with an illustrative example and simulations and, finally, some concluding remarks are reached in Sec. 8.

Throughout the paper, the following notation was used: $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denotes the transpose of the vector $\boldsymbol{x}$ and the matrix $\boldsymbol{X}$, respectively, rank$(\,\cdot\,)$ remits the rank of a matrix, for a square matrix $\boldsymbol{X} < 0$ means that $\boldsymbol{X}$ is a symmetric negative definite matrix, the symbol $\boldsymbol{I}_n$ indicates the n-th order unit matrix, $I\!R$ notes the set of real numbers, and $I\!R^n$, $I\!R^{n\times r}$ refer to the set of all n-dimensional real vectors and $n \times r$ real matrices, respectively.

2. Problem formulation

In the paper, there are taken into account square linear dynamic systems described in the fault-free conditions as

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t)\,, \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t)\,, \tag{2}$$

where $\boldsymbol{q}(t) \in I\!R^n$ stands up for the system state vector, $\boldsymbol{u}(t) \in I\!R^r$ denotes the input vector, $\boldsymbol{y}(t) \in I\!R^m$ is the measurable output vector, $\boldsymbol{A} \in I\!R^{n\times n}$, $\boldsymbol{B} \in I\!R^{n\times r}$ and $\boldsymbol{C} \in I\!R^{m\times n}$ are real matrices and $r = m$. It is supposed that the system (1), (2) is stabilizable by the the bi-proper dynamic output controller (DOC)

$$\dot{\boldsymbol{p}}(t) = \boldsymbol{J}\boldsymbol{p}(t) + \boldsymbol{L}\boldsymbol{y}(t)\,, \tag{3}$$

$$\boldsymbol{u}(t) = \boldsymbol{M}\boldsymbol{p}(t) + \boldsymbol{N}\boldsymbol{y}(t) - \boldsymbol{W}\boldsymbol{w}(t) \tag{4}$$

of an order p, where it can be accepted $1 \leq p < n$ (reduced order), $p = n$ (full order) and $n < p \leq p_m$ (upgraded order) and where $\boldsymbol{p}(t) \in I\!R^p$ is the vector of the controller state variables, $\boldsymbol{w}(t) \in I\!R^m$ is desired output signal vector and $\boldsymbol{W} \in I\!R^{r\times m}$ is the signal gain matrix. With respect to the real matrices $\boldsymbol{J} \in I\!R^{p\times p}$, $\boldsymbol{L} \in I\!R^{p\times m}$, $\boldsymbol{M} \in I\!R^{r\times p}$, $\boldsymbol{N} \in I\!R^{r\times m}$, the controller parameter notation takes for $\boldsymbol{K}^\circ \in I\!R^{(p+r)\times(p+m)}$ the following prescribed structure

$$\boldsymbol{K}^\circ = \begin{bmatrix} \boldsymbol{J} & \boldsymbol{L} \\ \boldsymbol{M} & \boldsymbol{N} \end{bmatrix}. \tag{5}$$

The model of the system with a faulted dynamics is taken as [11]

$$\dot{\boldsymbol{q}}_f(t) = \boldsymbol{A}_f\boldsymbol{q}_f(t) + \boldsymbol{B}\boldsymbol{u}^\circ(t)\,, \tag{6}$$

$$\boldsymbol{y}_f(t) = \boldsymbol{C}\boldsymbol{q}_f(t)\,, \tag{7}$$

where $\boldsymbol{q}_f(t) \in I\!R^n$, $\boldsymbol{u}_f(t) \in I\!R^r$, $\boldsymbol{y}_f(t) \in I\!R^m$ denote the faulty system state variables vector, the vector of the input variables and the vector of output variables, respectively, $\boldsymbol{u}^{\circ}(t) \in I\!R^r$ is a composed reference input signal to the cascade structure and $\boldsymbol{A}_f \in I\!R^{n\times n}$ is the faulty system dynamics matrix. That the problem of the cascade structure design is solvable, it is assumed the pair $(\boldsymbol{A}_f, \boldsymbol{B})$ is stabilizable [12].

3. Separation principle

Considering the static output control (SOC) principle so that

$$\boldsymbol{u}^{\circ}(t) = \boldsymbol{K}\boldsymbol{y}_f(t) + \boldsymbol{u}^{\diamond}(t) = \boldsymbol{K}\boldsymbol{C}\boldsymbol{q}_f(t) + \boldsymbol{u}^{\diamond}(t)\,, \tag{8}$$

such control corresponds to the closed-loop state-space representation of the faulty system

$$\dot{\boldsymbol{q}}_f(t) = \boldsymbol{A}_{cr}\boldsymbol{q}_f(t) + \boldsymbol{B}\boldsymbol{u}^{\diamond}(t)\,, \tag{9}$$

$$\boldsymbol{y}_f(t) = \boldsymbol{C}\boldsymbol{q}_f(t)\,, \tag{10}$$

where

$$\boldsymbol{A}_{cr} = \boldsymbol{A}_f + \boldsymbol{B}\boldsymbol{K}\boldsymbol{C}\,, \tag{11}$$

while $\boldsymbol{u}^{\diamond}(t)$ is the signal acting on the input of the faulty system.

Theorem 1 *The state of the faulty system (9), (10) under the control policy*

$$\boldsymbol{u}(t) = \boldsymbol{M}\boldsymbol{p}(t) + \boldsymbol{N}\boldsymbol{y}(t) - \boldsymbol{W}\boldsymbol{w}(t)\,, \tag{12}$$

$$\boldsymbol{u}^{\diamond}(t) = \boldsymbol{M}\boldsymbol{p}_f(t) + \boldsymbol{N}\boldsymbol{y}_f(t) - \boldsymbol{K}\boldsymbol{y}_r(t)\,, \tag{13}$$

asymptotically converge to the state of the reference model if $\boldsymbol{A}_{cn}$, $\boldsymbol{A}_{cr}$, $\boldsymbol{J}$ are Hurwitz, where

$$\boldsymbol{A}_{cn} = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{N}\boldsymbol{C}\,, \quad \boldsymbol{A}_{cr} = \boldsymbol{A}_s + \boldsymbol{B}\boldsymbol{K}\boldsymbol{C}\,, \quad \boldsymbol{A}_s = \boldsymbol{A}_{cn} + \boldsymbol{A}_f - \boldsymbol{A}\,, \tag{14}$$

$\boldsymbol{N} \in I\!R^{r\times m}$ is the dynamic controller gain matrix, $\boldsymbol{K} \in I\!R^{r\times r}$ is the static output control gain matrix, $\boldsymbol{W} \in I\!R^{r\times m}$ is the signal gain matrix and $\boldsymbol{J} \in I\!R^{p\times p}$ is the system matrix of the dynamic controller.

If $\boldsymbol{A}_{cn}$, $\boldsymbol{A}_{cr}$, $\boldsymbol{J}$ are Hurwitz and

$$\boldsymbol{e}_q(t) = \boldsymbol{q}(t) - \boldsymbol{q}_f(t)\,, \quad \boldsymbol{e}_p(t) = \boldsymbol{p}(t) - \boldsymbol{p}_f(t)\,, \quad \boldsymbol{e}_y(t) = \boldsymbol{C}\boldsymbol{e}_q(t)\,, \tag{15}$$

then the autonomous equations

$$\dot{\boldsymbol{q}}_f(t) = \boldsymbol{A}_{cr}\boldsymbol{q}_f(t)\,, \quad \dot{\boldsymbol{e}}_q(t) = \boldsymbol{A}_{cn}\boldsymbol{e}_q(t)\,, \quad \dot{\boldsymbol{p}}_f(t) = \boldsymbol{J}\boldsymbol{p}_f(t)\,, \quad \dot{\boldsymbol{e}}_p(t) = \boldsymbol{J}\boldsymbol{e}_p(t) \tag{16}$$

are asymptotically stable.

Proof: Describing the dynamics of the bi-proper dynamic controller in a faulty regime as

$$\dot{\boldsymbol{p}}_f(t) = \boldsymbol{J}\boldsymbol{p}_f(t) + \boldsymbol{L}\boldsymbol{y}_f(t)\,, \tag{17}$$

where $\boldsymbol{p}_f(t) \in I\!R^p$ is the vector of the controller state variables in the faulty regime then, incorporating all state vectors occurring in the control loop, the composed dynamic model becomes

$$\begin{bmatrix} \dot{\boldsymbol{q}}_f(t) \\ \dot{\boldsymbol{q}}(t) \\ \dot{\boldsymbol{p}}_f(t) \\ \dot{\boldsymbol{p}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cr} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{L}\boldsymbol{C} & \boldsymbol{0} & \boldsymbol{J} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}\boldsymbol{C} & \boldsymbol{0} & \boldsymbol{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_f(t) \\ \boldsymbol{q}(t) \\ \boldsymbol{p}_f(t) \\ \boldsymbol{p}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{\diamond}(t) \\ \boldsymbol{u}(t) \end{bmatrix}. \tag{18}$$

To apply the separation principle, the transform matrix $\boldsymbol{T}$ is defined as

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{I}_n & \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{I}_p & \boldsymbol{I}_p \end{bmatrix}, \quad \boldsymbol{T}^{-1} = \begin{bmatrix} \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{I}_n & \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I}_p & \boldsymbol{I}_p \end{bmatrix}, \tag{19}$$

where, carrying out the indicated separation, it yields

$$\boldsymbol{T} \begin{bmatrix} \boldsymbol{q}_f(t) \\ \boldsymbol{q}(t) \\ \boldsymbol{p}_f(t) \\ \boldsymbol{p}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{I}_n & \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{I}_p & \boldsymbol{I}_p \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_f(t) \\ \boldsymbol{q}(t) \\ \boldsymbol{p}_f(t) \\ \boldsymbol{p}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{q}_f(t) \\ \boldsymbol{e}_q(t) \\ \boldsymbol{p}_f(t) \\ \boldsymbol{e}_p(t) \end{bmatrix}, \tag{20}$$

$$\boldsymbol{T} \begin{bmatrix} \boldsymbol{A}_{cr} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{A} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} \end{bmatrix} \boldsymbol{T}^{-1} = \begin{bmatrix} \boldsymbol{A}_{cr} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{A} - \boldsymbol{A}_{cr} & \boldsymbol{A} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} \end{bmatrix}, \tag{21}$$

$$\begin{bmatrix} \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{I}_n & \boldsymbol{I}_n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{I}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{I}_p & \boldsymbol{I}_p \end{bmatrix} \begin{bmatrix} \boldsymbol{B} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B} & \mathbf{0} \\ -\boldsymbol{B} & \boldsymbol{B} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{22}$$

that is (18) can be rewritten in an equivalent form

$$\begin{bmatrix} \dot{\boldsymbol{q}}_f(t) \\ \dot{\boldsymbol{e}}_q(t) \\ \dot{\boldsymbol{p}}_f(t) \\ \dot{\boldsymbol{e}}_p(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{cr} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{A} - \boldsymbol{A}_{cr} & \boldsymbol{A} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_f(t) \\ \boldsymbol{e}_q(t) \\ \boldsymbol{p}_f(t) \\ \boldsymbol{e}_p(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} & \mathbf{0} \\ -\boldsymbol{B} & \boldsymbol{B} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{\diamond}(t) \\ \boldsymbol{u}(t) \end{bmatrix}. \tag{23}$$

Considering the composed cascade structure input signals as (12), (13), it is easily verified that

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{B} & \mathbf{0} \\ -\boldsymbol{B} & \boldsymbol{B} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{M}\boldsymbol{p}_f(t) + \boldsymbol{N}\boldsymbol{y}_f(t) - \boldsymbol{K}\boldsymbol{y}_r(t) \\ \boldsymbol{M}\boldsymbol{p}(t) + \boldsymbol{N}\boldsymbol{y}(t) - \boldsymbol{W}\boldsymbol{w}(t) \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{BM}\boldsymbol{p}_f(t) + \boldsymbol{BN}\boldsymbol{y}_f(t) - \boldsymbol{BK}\boldsymbol{y}_r(t) \\ \boldsymbol{BM}\boldsymbol{e}_p(t) + \boldsymbol{BNC}\boldsymbol{e}_q(t) - \boldsymbol{BW}\boldsymbol{w}(t) + \boldsymbol{BK}\boldsymbol{y}_r(t) \end{bmatrix} \end{aligned} \tag{24}$$

and, consequently,

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{q}}_f(t) \\ \dot{\boldsymbol{e}}_q(t) \\ \dot{\boldsymbol{p}}_f(t) \\ \dot{\boldsymbol{e}}_p(t) \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A}_{cr} + \boldsymbol{BNC} & \mathbf{0} & \boldsymbol{BM} & \mathbf{0} \\ \boldsymbol{A} - \boldsymbol{A}_{cr} & \boldsymbol{A} + \boldsymbol{BNC} & \mathbf{0} & \boldsymbol{BM} \\ \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{LC} & \mathbf{0} & \boldsymbol{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_f(t) \\ \boldsymbol{e}_q(t) \\ \boldsymbol{p}_f(t) \\ \boldsymbol{e}_p(t) \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{0} & -\boldsymbol{BK} \\ -\boldsymbol{BW} & \boldsymbol{BK} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{y}_r(t) \end{bmatrix}. \end{aligned} \tag{25}$$

Because of the block structure of the connected system matrix in (25), it is evident that the separation principle holds.

Using the terms given above, it can write

$$\boldsymbol{A}_{cr} + \boldsymbol{BNC} = \boldsymbol{A}_f + \boldsymbol{BKC} + \boldsymbol{BNC} = \boldsymbol{A} + \boldsymbol{BNC} + \boldsymbol{A}_f - \boldsymbol{A} + \boldsymbol{BKC}\,, \tag{26}$$

$$\boldsymbol{A}_{cr} + \boldsymbol{BNC} = \boldsymbol{A}_{cn} + \boldsymbol{A}_f - \boldsymbol{A} + \boldsymbol{BKC} = \boldsymbol{A}_s + \boldsymbol{BKC} = \boldsymbol{A}_{sc}\,, \tag{27}$$

respectively, and (27) gives (14).

Since the separation principle yields, to obtain an asymptotic stable solution of (25), the matrices $\boldsymbol{A}_{cn}$, $\boldsymbol{A}_{cr}$ of the structures (14) and the matrix $\boldsymbol{J}$ have to be Hurwitz. This concludes the proof. ■

4. Dynamic controller parameter design

Exploiting the separation principle, it can write for (15) and the error subsystem autonomous block in (25) that

$$\begin{bmatrix} \dot{\boldsymbol{e}}_q(t) \\ \dot{\boldsymbol{e}}_p(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} + \boldsymbol{BNC} & \boldsymbol{BM} \\ \boldsymbol{LC} & \boldsymbol{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_q(t) \\ \boldsymbol{e}_p(t) \end{bmatrix}, \tag{28}$$

$$\boldsymbol{e}_y(t) = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_p \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_q(t) \\ \boldsymbol{e}_p(t) \end{bmatrix}. \tag{29}$$

Introducing the notations

$$\boldsymbol{e}^{\circ T}(t) = \begin{bmatrix} \boldsymbol{e}_q^T(t) & \boldsymbol{e}_p^T(t) \end{bmatrix}, \tag{30}$$

$$\boldsymbol{A}^{\circ} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \ \boldsymbol{B}^{\circ} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{B} \\ \boldsymbol{I}_p & \boldsymbol{0} \end{bmatrix}, \ \boldsymbol{C}^{\circ} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_p \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix}, \ \boldsymbol{I}^{\circ} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix}, \tag{31}$$

where $\boldsymbol{A}^{\circ} \in I\!R^{(n+p)\times(n+p)}$, $\boldsymbol{B}^{\circ} \in I\!R^{(n+p)\times(p+r)}$, $\boldsymbol{C}^{\circ} \in I\!R^{(p+m)\times(n+p)}$, $\boldsymbol{I}^{\circ} \in I\!R^{m\times(p+m)}$, than the state-space equations (28), (29) take the forms

$$\dot{\boldsymbol{e}}^{\circ}(t) = \boldsymbol{A}_c^{\circ}\boldsymbol{e}^{\circ}(t)\,, \tag{32}$$

$$\boldsymbol{y}^{\circ}(t) = \boldsymbol{I}^{\circ}\boldsymbol{C}^{\circ}\boldsymbol{e}^{\circ}(t)\,, \tag{33}$$

where, with $\boldsymbol{K}^{\circ}$ given in (5),

$$\boldsymbol{A}_c^{\circ} = \boldsymbol{A}^{\circ} + \boldsymbol{B}^{\circ}\boldsymbol{K}^{\circ}\boldsymbol{C}^{\circ}\,. \tag{34}$$

In the sequel, it is supposed that the pair $(\boldsymbol{A}^{\circ}, \boldsymbol{B}^{\circ})$ is stabilizable and the couple $(\boldsymbol{A}^{\circ}, \boldsymbol{C}^{\circ})$ is detectable [3].

Theorem 2 *The error equation (32) converges asymptotically to their equilibrium if there exist a symmetric positive definite matrix $\boldsymbol{Q}^{\circ} \in I\!R^{(n+p)\times(n+p)}$, a regular matrix $\boldsymbol{H}^{\circ} \in I\!R^{(p+m)\times(p+m)}$ and a matrix $\boldsymbol{Y}^{\circ} \in I\!R^{(p+r)\times(p+m)}$ such that*

$$\boldsymbol{Q}^{\circ} = \boldsymbol{Q}^{\circ T} > 0\,, \tag{35}$$

$$\boldsymbol{A}^{\circ}\boldsymbol{Q}^{\circ} + \boldsymbol{Q}^{\circ}\boldsymbol{A}^{\circ T} + \boldsymbol{B}^{\circ}\boldsymbol{Y}^{\circ}\boldsymbol{C}^{\circ} + \boldsymbol{C}^{\circ T}\boldsymbol{Y}^{\circ T}\boldsymbol{B}^{\circ T} < 0\,, \tag{36}$$

$$\boldsymbol{C}^{\circ}\boldsymbol{Q}^{\circ} = \boldsymbol{H}^{\circ}\boldsymbol{C}^{\circ}. \tag{37}$$

When the above conditions hold then the composed gain matrix of the bi-proper dynamic controller is given as

$$\boldsymbol{K}^{\circ} = \boldsymbol{Y}^{\circ}(\boldsymbol{H}^{\circ})^{-1} = \begin{bmatrix} \boldsymbol{J} & \boldsymbol{L} \\ \boldsymbol{M} & \boldsymbol{N} \end{bmatrix}. \tag{38}$$

Proof: Defining the Lyapunov function candidate as follows

$$v(\boldsymbol{e}^\circ(t)) = \boldsymbol{e}^{\circ T}(t)\boldsymbol{P}^\circ\boldsymbol{e}^\circ(t) > 0\,, \tag{39}$$

where $\boldsymbol{P}^\circ > 0$ is a symmetric positive definite matrix, then it has to be satisfied

$$\dot{v}(\boldsymbol{e}^\circ(t)) = \dot{\boldsymbol{e}}^{\circ T}(t)\boldsymbol{P}^\circ\boldsymbol{e}^\circ(t) + \boldsymbol{e}^{\circ T}(t)\boldsymbol{P}^\circ\dot{\boldsymbol{e}}^\circ(t) < 0\,. \tag{40}$$

Substituting (32) into (40) results

$$\dot{v}(\boldsymbol{e}^\circ(t)) = \boldsymbol{e}^{\circ T}(t)(\boldsymbol{A}_c^{\circ T}\boldsymbol{P}^\circ + \boldsymbol{P}^\circ\boldsymbol{A}_c^\circ)\boldsymbol{e}^\circ(t) < 0 \tag{41}$$

and, evidently, (41) implies

$$\boldsymbol{P}^\circ\boldsymbol{A}_c^\circ + \boldsymbol{A}_c^{\circ T}\boldsymbol{P}^\circ < 0\,. \tag{42}$$

Using (34), the open structure of (42) takes the form of the bilinear matrix inequality

$$\boldsymbol{P}^\circ(\boldsymbol{A}^\circ + \boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{C}^\circ) + (\boldsymbol{A}^\circ + \boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{C}^\circ)^T\boldsymbol{P}^\circ < 0\,, \tag{43}$$

which can be rewritten as

$$(\boldsymbol{A}^\circ + \boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{C}^\circ)\boldsymbol{Q}^\circ + \boldsymbol{Q}^\circ(\boldsymbol{A}^\circ + \boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{C}^\circ)^T < 0\,, \tag{44}$$

where $\boldsymbol{Q}^\circ = (\boldsymbol{P}^\circ)^{-1}$. Writing now

$$\boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{C}^\circ\boldsymbol{Q}^\circ = \boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{H}^\circ(\boldsymbol{H}^\circ)^{-1}\boldsymbol{C}^\circ\boldsymbol{Q}^\circ, \tag{45}$$

where $\boldsymbol{H}^\circ$ is a regular square matrix, and defining the equality

$$(\boldsymbol{H}^\circ)^{-1}\boldsymbol{C}^\circ = \boldsymbol{C}^\circ(\boldsymbol{Q}^\circ)^{-1}, \tag{46}$$

then it yields

$$\boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{C}^\circ\boldsymbol{Q}^\circ = \boldsymbol{B}^\circ\boldsymbol{K}^\circ\boldsymbol{H}^\circ\boldsymbol{C}^\circ = \boldsymbol{B}^\circ\boldsymbol{Y}^\circ\boldsymbol{C}^\circ, \tag{47}$$

where

$$\boldsymbol{Y}^\circ = \boldsymbol{K}^\circ\boldsymbol{H}^\circ. \tag{48}$$

Thus, with (47) then (44) imply (36) and the equality (46) gives (37). This concludes the proof. ■

5. Static output control parameter design

Theorem 3 *The controlled state in the closed-loop structure with faulty system is stable if there exist a positive definite symmetric matrix $\boldsymbol{Q} \in I\!R^{n\times n}$, a regular matrix $\boldsymbol{H} \in I\!R^{m\times m}$ and matrix $\boldsymbol{Y} \in I\!R^{m\times m}$ such that*

$$\boldsymbol{Q} = \boldsymbol{Q}^T > 0\,, \tag{49}$$

$$\boldsymbol{A}_s\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{A}_s^T + \boldsymbol{B}\boldsymbol{Y}\boldsymbol{C} + \boldsymbol{C}^T\boldsymbol{Y}^T\boldsymbol{B}^T < 0\,, \tag{50}$$

$$\boldsymbol{C}\boldsymbol{Q} = \boldsymbol{H}\boldsymbol{C}\,, \tag{51}$$

where

$$\boldsymbol{A}_s = \boldsymbol{A}_{cn} + \boldsymbol{A}_f - \boldsymbol{A}\,, \quad \boldsymbol{A}_{cn} = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{N}\boldsymbol{C}\,. \tag{52}$$

When the above conditions hold, the static output control gain matrix is computed as

$$\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{H}^{-1}. \tag{53}$$

Proof: Using the separation principle, (25), (27) imply the following equation, describing the autonomous regime of the closed-loop system state with the faulty system matrix $\boldsymbol{A}_f$,

$$\dot{\boldsymbol{q}}_f(t) = (\boldsymbol{A}_s + \boldsymbol{BKC})\boldsymbol{q}_f(t)\,, \tag{54}$$

where $\boldsymbol{A}_s$ is prescribed in (52).

Considering the Lyapunov function candidate in the form

$$v(\boldsymbol{q}_f(t)) = \boldsymbol{q}_f^T(t)\boldsymbol{P}\boldsymbol{q}_f(t) > 0\,, \tag{55}$$

where $\boldsymbol{P} \in \mathbb{R}^{n\times n}$ is a square, symmetric, positive definite matrix, then the time derivative of (55) along a trajectory of (54) takes the form

$$\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}_f^T(t)\boldsymbol{P}\boldsymbol{q}_f(t) + \boldsymbol{q}_f^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}_f(t) < 0\,. \tag{56}$$

Therefore, using (54), the inequality (56) can be written as

$$\dot{v}(\boldsymbol{q}_f(t)) = \boldsymbol{q}_f^T(t)\boldsymbol{P}_s\boldsymbol{q}_f(t) < 0\,, \tag{57}$$

where

$$\boldsymbol{P}_s = (\boldsymbol{A}_s + \boldsymbol{BKC})^T\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A}_s + \boldsymbol{BKC}) < 0\,. \tag{58}$$

Pre-multiplying the left-hand side and post-multiplying the right-hand side of (58) by the matrix $\boldsymbol{Q} = \boldsymbol{P}^{-1}$ leads to the bilinear matrix inequality

$$(\boldsymbol{A}_s + \boldsymbol{BKC})\boldsymbol{Q} + \boldsymbol{Q}(\boldsymbol{A}_s + \boldsymbol{BKC})^T < 0\,. \tag{59}$$

Writing, analogously as above,

$$\boldsymbol{BKCQ} = \boldsymbol{BKHH}^{-1}\boldsymbol{CQ} = \boldsymbol{BYC}\,, \tag{60}$$

where $\boldsymbol{H}$ is a regular square matrix, and

$$\boldsymbol{H}^{-1}\boldsymbol{C} = \boldsymbol{CQ}^{-1}, \quad \boldsymbol{Y} = \boldsymbol{KH}\,, \tag{61}$$

then (59) implies (50) and the equalities in (61) gives (51), (53). This concludes the proof. ■

6. Signal gain matrix design

Naturally, the control law (12) inherently defines a forced regime, where $\boldsymbol{w}(t) \in \mathbb{R}^m$ is desired output signal vector and $\boldsymbol{W} \in \mathbb{R}^{m\times m}$ is the signal gain matrix. Using the static decoupling principle, the conditions to design the signal gain matrices $\boldsymbol{W}_n$, $\boldsymbol{W}_f$ can be derived considering that $\boldsymbol{y}_r(t) = \boldsymbol{0}$ by using the principle of superposition.

Theorem 4 *If the system (1), (2) is stabilizable by the control policy (3), (4), and [17]*

$$rank\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} = rank\begin{bmatrix} \boldsymbol{A}_f & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{0} \end{bmatrix} = n + m\,, \tag{62}$$

then the matrix $\boldsymbol{W}$ takes, in the dependency on the system regime, the forms

$$\boldsymbol{W}_n = \Big(\boldsymbol{C}(\boldsymbol{A} - \boldsymbol{BMJ}^{-1}\boldsymbol{LC} + \boldsymbol{BNC})^{-1}\boldsymbol{B}\Big)^{-1}\,. \tag{63}$$

$$\boldsymbol{W}_f = \Big(\boldsymbol{C}(\boldsymbol{A}_f - \boldsymbol{BMJ}^{-1}\boldsymbol{LC} + \boldsymbol{BNC})^{-1}\boldsymbol{B}\Big)^{-1}\,. \tag{64}$$

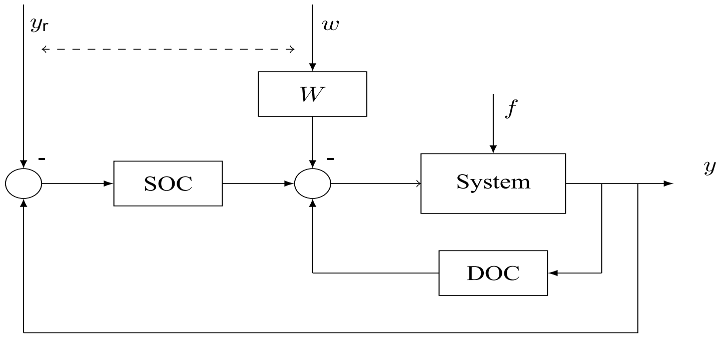

Figure 1. Cascade control reconfiguration scheme

Proof: In a steady-state, the system equations (1), (2) and the control (3), (4) imply

$$\mathbf{0} = \boldsymbol{A}\boldsymbol{q}_o + \boldsymbol{B}\boldsymbol{u}_o, \tag{65}$$

$$\mathbf{0} = \boldsymbol{J}\boldsymbol{p}_o + \boldsymbol{L}\boldsymbol{C}\boldsymbol{q}_o, \tag{66}$$

$$\boldsymbol{u}_o = \boldsymbol{M}\boldsymbol{p}_o + \boldsymbol{N}\boldsymbol{C}\boldsymbol{q}_o - \boldsymbol{W}\boldsymbol{w}_o\,, \tag{67}$$

where $\boldsymbol{q}_o$, $\boldsymbol{u}_o$, $\boldsymbol{p}_o$, $\boldsymbol{w}_o$ are the steady-state values of the vectors $\boldsymbol{q}(t)$, $\boldsymbol{u}(t)$, $\boldsymbol{p}(t)$, $\boldsymbol{w}(t)$, respectively.

Since (66), (67) in the steady-state give

$$\boldsymbol{u}_o = (-\boldsymbol{M}\boldsymbol{J}^{-1}\boldsymbol{L}\boldsymbol{C} + \boldsymbol{N}\boldsymbol{C})\boldsymbol{q}_o - \boldsymbol{W}\boldsymbol{w}_o\,, \tag{68}$$

then, substituting (68) into (65), it yields

$$\mathbf{0} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{M}\boldsymbol{J}^{-1}\boldsymbol{L}\boldsymbol{C} + \boldsymbol{B}\boldsymbol{N}\boldsymbol{C})\boldsymbol{q}_o - \boldsymbol{B}\boldsymbol{W}\boldsymbol{w}_o\,, \tag{69}$$

$$\boldsymbol{q}_o = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{M}\boldsymbol{J}^{-1}\boldsymbol{L}\boldsymbol{C} + \boldsymbol{B}\boldsymbol{N}\boldsymbol{C})^{-1}\boldsymbol{B}\boldsymbol{W}\boldsymbol{w}_o\,, \tag{70}$$

respectively, and

$$\boldsymbol{y}_o = \boldsymbol{C}(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{M}\boldsymbol{J}^{-1}\boldsymbol{L}\boldsymbol{C} + \boldsymbol{B}\boldsymbol{N}\boldsymbol{C})^{-1}\boldsymbol{B}\boldsymbol{W}\boldsymbol{w}_o\,. \tag{71}$$

Thus, considering $\boldsymbol{y}_o = \boldsymbol{w}_o$, then (71) implies (63).

Analogously can be derived (64). This concludes the proof. ■

The cascade reconfiguration control scheme is presented in Fig. 1, where $\boldsymbol{W} = \boldsymbol{W}_n$ under nominal control and $\boldsymbol{W} = \boldsymbol{W}_f$ after reconfiguration activation. To obtain a forced mode structure, the reference input signal $\boldsymbol{y}_r(t)$ has to be connected to $\boldsymbol{w}(t)$.

7. Illustrative example

In the example, there is considered the system (1), (2) in the state-space representation, where the system matrices are [10]

$$\boldsymbol{A} = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.290 & 0.000 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0.000 & 0.000 \\ 5.679 & 0.000 \\ 1.136 & -3.146 \\ 1.136 & 0.000 \end{bmatrix},$$

$$\boldsymbol{A}_f = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.290 & 0.000 & 0.675 \\ 2.134 & 8.546 & -13.308 & 11.786 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 0 & 2 \end{bmatrix}.$$

Considering the order of dynamic controller $p = 2$ then, solving (35)-(37) using Self-Dual-Minimization (SeDuMi) package [14] for MATLAB, the task is feasible and

$$\boldsymbol{Q}^{\circ} = \begin{bmatrix} 0.2498 & -0.0036 & 0.0193 & 0.1547 & 0.0000 & 0.0000 \\ -0.0036 & 0.2515 & 0.0142 & 0.0000 & 0.0000 & 0.0000 \\ 0.0193 & 0.0142 & 0.1773 & 0.0387 & 0.0000 & 0.0000 \\ 0.1547 & 0.0000 & 0.0387 & 0.1437 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5865 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.5865 \end{bmatrix},$$

$$\boldsymbol{Y}^{\circ} = \begin{bmatrix} -0.4337 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & -0.4337 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0033 & -0.2246 \\ 0.0000 & 0.0000 & 0.1453 & 0.1304 \end{bmatrix}, \quad \boldsymbol{H}^{\circ} = \begin{bmatrix} 0.5865 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.5865 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.2547 & 0.6573 \\ 0.0000 & 0.0000 & 0.0387 & 0.1437 \end{bmatrix}.$$

The dynamic controller gain matrices are computed using (38) as follows

$$\boldsymbol{J} = \begin{bmatrix} -0.7395 & 0.0000 \\ 0.0000 & -0.7394 \end{bmatrix}, \quad \boldsymbol{L} = 10^{-3} \begin{bmatrix} -0.0131 & 0.1031 \\ 0.0369 & -0.3260 \end{bmatrix},$$

$$\boldsymbol{M} = 10^{-4} \begin{bmatrix} -0.1101 & -0.0180 \\ -0.1553 & -0.1631 \end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix} 0.8193 & -5.3100 \\ 1.4158 & -5.5686 \end{bmatrix},$$

and guaranties the stable closed-loop state convergence, where the system matrix eigenvalues spectrum is

$$\rho(\boldsymbol{A}_c^{\circ}) = \{-0.7394 \ \ -0.7395 \ \ -3.6642 \pm 3.1351\,\mathrm{i} \ \ -6.9476 \pm 11.6631\,\mathrm{i}\}.$$

Moreover, evidently, the structure of $\boldsymbol{J}$ implies that the bi-proper dynamic controller is stable.

To obtain the parameters for the static output controllers, (49)-(51) is solved with the result

$$\boldsymbol{Q} = \begin{bmatrix} 0.2249 & -0.0143 & 0.0248 & 0.1534 \\ -0.0143 & 0.2553 & 0.0571 & 0.0000 \\ 0.0248 & 0.0571 & 0.1317 & 0.0384 \\ 0.1534 & 0.0000 & 0.0384 & 0.1455 \end{bmatrix},$$

$$\boldsymbol{Y} = \begin{bmatrix} 0.0023 & -0.0128 \\ 0.0650 & 0.2597 \end{bmatrix}, \quad \boldsymbol{H} = \begin{bmatrix} 0.2311 & 0.6521 \\ 0.0384 & 0.1455 \end{bmatrix}.$$

The obtained gain matrix $\boldsymbol{K}$ is

$$\boldsymbol{K} = \begin{bmatrix} 0.0956 & -0.5161 \\ -0.0590 & 2.0498 \end{bmatrix}$$

which gives that all eigenvalues of $\boldsymbol{A}_{sc} = \boldsymbol{A}_s + \boldsymbol{BKC}$ are stable, where

$$\rho(\boldsymbol{A}_{sc}) = \{ \ -2.8442 \ \ -11.9300 \ \ -6.6978 \pm 9.5406\,\mathrm{i} \ \}.$$

One can verify the stable eigenvalue spectrum of the system matrix in (25), where

$$\rho(\boldsymbol{A}_{cs}^{\circ}) = \left\{ \begin{array}{c} -0.7394 \ \ -0.7394 \ \ -0.7395 \ \ -0.7395 \ \ -2.8442 \ \ -11.9300 \\ -3.6642 \pm 9.5406\,\mathrm{i} \ \ -6.6978 \pm 9.5406\,\mathrm{i} \ \ -6.9476 \pm 11.6631\,\mathrm{i} \end{array} \right\},$$

which certify stability of the cascade control structure.

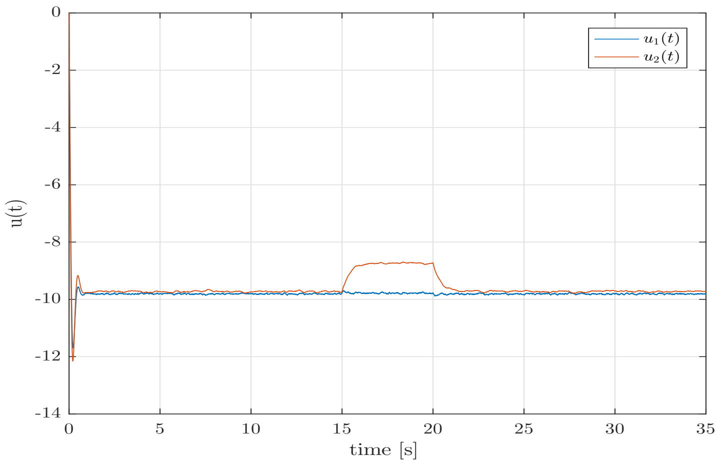

Figure 2. System input signals

Simulations are realized in the forced mode and the noise environment where, to set up the equal working point of the closed-loop system in the forced mode for nominal and faulty regimes, the signal gains matrices were computed, and the system noise input is given so that

$$\boldsymbol{W}_n = \begin{bmatrix} 0.7880 & -5.3257 \\ 1.1933 & -5.7555 \end{bmatrix}, \ \boldsymbol{W}_f = \begin{bmatrix} 0.7880 & -5.3257 \\ 0.9820 & -5.9368 \end{bmatrix},$$

$$\boldsymbol{V}_d^T = \begin{bmatrix} 1.136 & 1.136 & 5.679 & 3.146 \end{bmatrix}, \ \sigma_d^2 = 0.08\,.$$

The desired output values are prescribed by the vector $\boldsymbol{w}(t)$ as $\boldsymbol{w}^T(t) = [\,1\ 2\,]$ (in simulation set as $-\boldsymbol{w}(t)$ because the cascade structure is in a dual form - see Fig. 1), the system initial state is $\boldsymbol{q}(0) = \boldsymbol{0}$ and the dynamic controller initial state is $\boldsymbol{p}(0) = \boldsymbol{0}$.

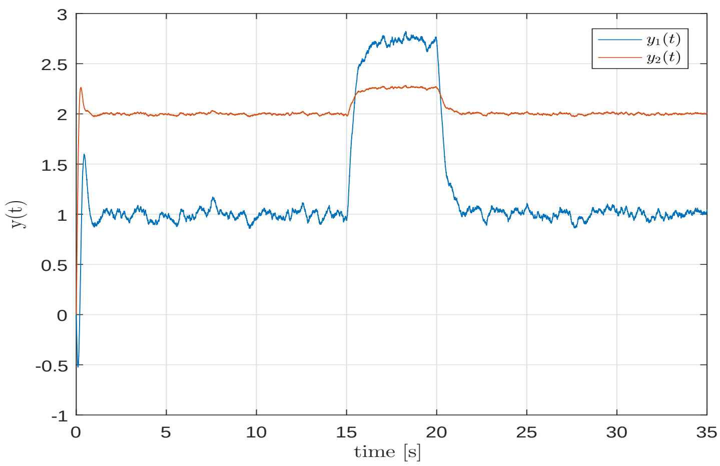

Figure 3. System output response

As results, Fig. 2 and Fig. 3 show the input and output of the closed-loop system in the cascade reconfiguration structure and forced mode, when the fault occurs in the matrix $\boldsymbol{A}$ at the time instant $t = 15$s. The control reconfiguration law applied to the cascade structure with the faulty system at the time instant $t = 20$s (reflecting the fault detection and isolation time delay lasted approximately 5 s). Practically, it means that the control law parameters of the cascade structure are still the same and, with unchanged $\boldsymbol{w}(t)$, there $\boldsymbol{W}_n$ is switched to $\boldsymbol{W}_f$. It is obvious that the system fault impact on the system output was compensated by using the cascade reconfiguration control structure.

8. Concluding remarks

Exploiting the reference model control design principle, the influence of the system matrix parameter faults on the system output is analyzed for control reconfiguration. To achieve the desired control objective, the bi-proper dynamic output controller, in combination with the static output control part, is realized within the reconfiguration cascade structure. A specific feature of this is that even after the fault occurrence is maintained the inner loop with the nominal control loop parameters, so it was necessary to use as a reference model a state form based on the faulty system description. Because dynamic and static output controllers are exploited, in terms of synthesis it has proven advantageous to use dual cascade control structure.

To optimize the cascade reconfiguration structure properties, the gain matrix parameters design conditions are designed to reflect the stability of both regimes, verifying them separately in the sense of Lyapunov asymptotic stability. In consequence, the design requires solution of two set of consistent linear matrix inequalities, every one combined with an associated matrix equality.

Adapting to the faulty plant and the fault detection subsystem with a time delay, the reconfigured gain matrix parameter is changed after a fault occurrence, keeping the rated control intact. The reconfiguration is so conveyed as an autonomous formula that may be designed and organized online. Because the faulty mode is optimized and bi-proper dynamic output control is used, large system input and output signal peeks, occurring at the starting interval of control when the standard cascade structure is used, are substantially reduced.

Presented method for linear continuous-time systems provides useful and easily implementable structure in process of control reconfiguration for the system matrix parameter faults. A simulation example, subject to given type of failures, demonstrates the effectiveness of the proposed form of the fault tolerant control design technique.

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