

Performance comparison of attitude determination, attitude estimation, and nonlinear observers algorithms

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Abstract. This paper presents a brief synthesis and useful performance analysis of different attitude filtering algorithms (attitude determination algorithms, attitude estimation algorithms, and nonlinear observers) applied to Low Earth Orbit Satellite in terms of accuracy, convergence time, amount of memory, and computation time. This latter is calculated in two ways, using a personal computer and also using On-board computer 750 (OBC 750) that is being used in many SSTL Earth observation missions. The use of this comparative study could be an aided design tool to the designer to choose from an attitude determination or attitude estimation or attitude observer algorithms. The simulation results clearly indicate that the nonlinear Observer is the more logical choice.

1. Introduction

The objective of the attitude filtering algorithms is to calculate the satellite orientation, using measurement data of satellite sensors such as star sensor, sun sensor, infrared horizon sensor, gyroscope and the magnetometer, etc. Among these, Sun sensors or star trackers are very used in small satellites because of the limitations of small satellite missions in terms of power and structure [1].

There are many algorithms for determining the attitude of the satellite. These algorithms are divided into three families: attitude determination algorithms (such as TRIAD, Q-Method, Quest, Least Squares, etc.), attitude estimation algorithms (Kalman Filter, H-infinity Filter, Particle Filter, etc.), and nonlinear observers (Sliding Mode Observer, Luenberger observer, Backstepping observer, etc.). Each of these algorithms is able to determinate the attitude, and it has an advantages and disadvantages.

The attitude determination algorithms have an advantage of low execution time. This is due to its independence of dynamic models, but they are able to estimate only the attitude but not other system states such as angular rate, biases and noise measurement (not presented here). So, what do we do if the angular rates are also calculated from angular sensor measurements? Because we can't use the attitude determination algorithms to estimate this system state, to solve this problem we use the attitude estimation algorithms.

The attitude estimation algorithms are characterized by good performances in term of accuracy, but they have a high time consuming because its structure uses the dynamic models. This point presents a disadvantage in comparison with attitude determination algorithms. In addition, the attitude estimation algorithms are able to estimate the attitude and the angular rate using the sensors installed in the



satellite body. Among these sensors, the gyroscope is a sensor which provides the angular rate measurements but it is costly and heavy. Therefore, this sensor presents a problem in small satellites. To eliminate the use of this sensor, and to solve the problem of the time consuming, we use nonlinear observers such as sliding mode observer.

The sliding mode observers are helpful for micro satellite when gyro is unavailable or gyro fails, and they have robustness proprieties similar to sliding mode controllers. They are characterized by high performances in terms of time consuming and accuracy at the same time with respect to attitude determination and attitude estimation algorithms.

The attitude determination and control system (ADCS) is the more important subsystems of satellite. The main aim of the designer of this subsystem is the choice of the best algorithm to extract the full attitude of spacecraft (Euler angles or quaternions or Rodriguez parameters and angular velocities) in term of accuracy or execution time or both, which depends on the mission requirement during the satellite design. Therefore, many questions will be asked, among the algorithms that are presented at the previous paragraph, which algorithm must be used? how does the designer choose this algorithm? and why do we use nonlinear observers while it is possible to use attitude estimation or attitude determination algorithms? Therefore, it is important to give a comparative study between the three filtering algorithms and help the ADCS designer to choose the best algorithm.

[2-3] gave a comparative study between attitude determination and estimation algorithms and did not discuss of nonlinear observers.

In this paper, we give a comparative performance analysis between attitude determination algorithms (Least Squares, Quest method and Optimized TRIAD), attitude estimation algorithms (Extended Kalman Filter and Unscented Kalman Filter), and nonlinear observer (Sliding Mode Observer based) to be applied to Low Earth Orbit satellite, we choose those particular algorithms because they are the most applied in spatial field. The selected algorithms are compared to find the required accuracy, computation time, convergence time, and amount memory, the software implementation of these algorithms is executed in a personal computer and in the On-board Computer 750 (OBC 750) that was developed in SSTL to function as the primary flight computer for application requiring processing On-board.

The presented study will help the designer to determine which type of algorithms should be used to extract spacecraft attitude.

The paper is organized as follows. Section 2 presents the dynamic and kinematic models used for the satellite. Section 3 presents the observability test of our system. In the section 4, 5, and 6, we describe the design of the different algorithms which are presented at the beginning. In next section, we present the simulation results. Finally, the conclusion of this paper is presented in section 8.

2. Spacecraft attitude model

The dynamic and the kinematic of the satellite are governed by the following equation system as [4-5],

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{t}) + \mathbf{B} \mathbf{u} \quad (1.1)$$

$$\mathbf{y} = \mathbf{H} \mathbf{x} \quad (1.2)$$

where,

$$\mathbf{x}(\mathbf{t}) = [q_1, q_2, q_3, q_4, \omega_x, \omega_y, \omega_z]^T \quad (2)$$

$$\mathbf{f}(\mathbf{x}, \mathbf{h}) = \begin{bmatrix} 0.5(\omega_{oz}q_2 - \omega_{oy}q_3 + \omega_{ox}q_4) \\ 0.5(-\omega_{oz}q_1 + \omega_{ox}q_3 + \omega_{oy}q_4) \\ 0.5(\omega_{oy}q_1 - \omega_{ox}q_2 + \omega_{oz}q_4) \\ 0.5(-\omega_{ox}q_1 - \omega_{oy}q_2 - \omega_{oz}q_3) \\ \mathbf{I}_x^{-1}(\mathbf{N}_x - (\mathbf{I}_z - \mathbf{I}_y)\omega_y\omega_z - \omega_y\mathbf{h}_z + \omega_z\mathbf{h}_y) \\ \mathbf{I}_y^{-1}(\mathbf{N}_y - (\mathbf{I}_x - \mathbf{I}_z)\omega_x\omega_z + \omega_x\mathbf{h}_z - \omega_z\mathbf{h}_x) \\ \mathbf{I}_z^{-1}(\mathbf{N}_z - (\mathbf{I}_y - \mathbf{I}_x)\omega_x\omega_y - \omega_x\mathbf{h}_y + \omega_y\mathbf{h}_x) \end{bmatrix} \quad (3)$$

$\mathbf{I} = [\mathbf{I}_x \quad \mathbf{I}_y \quad \mathbf{I}_z]$ moment of inertia of spacecraft;

$\omega_B^T = [\omega_x \quad \omega_y \quad \omega_z]^T$ angular velocity vector in the inertial frame;

$\mathbf{q} = [q_1 q_2 q_3 q_4]$ quaternion;

$\omega_S^0 = [\omega_{ox} \quad \omega_{oy} \quad \omega_{oz}]^T$ angular velocity vector in the orbital reference frame;

$\mathbf{h} = [h_x \quad h_y \quad h_z]^T$ angular momentum vector;

$\mathbf{N}_{ext} = [N_x \quad N_y \quad N_z]^T$ external disturbance torque vector;

$\mathbf{B} = [\mathbf{0}^{4 \times 3} \quad \mathbf{I}^{3 \times 3}]^T$ control matrix;

$\mathbf{U} = -\dot{\mathbf{h}}$ control input torque;

It assumed that the quaternion measurements are directly available $\mathbf{H} = [\mathbf{I}^{4 \times 4} \quad \mathbf{0}^{4 \times 3}]$.

3. Observability test

The observability concept is related to dynamic systems. It has a fundamental importance when studying attitude determination and estimation algorithms that we will presented in the following sections. For that reason we introduce this concept in what follows.

The nonlinear observability is tied to the Lie derivative. The observability test of our system is described as follows.

Firstly, we compute the matrix $\mathbf{G}$ of Lie derivative,

$$\mathbf{G} = \begin{bmatrix} L_f^0(h_1) & \dots & L_f^0(h_p) \\ \vdots & \ddots & \vdots \\ L_f^{n-1}(h_1) & \dots & L_f^{n-1}(h_p) \end{bmatrix} \quad (4)$$

Where, $\mathbf{h}(\mathbf{x}) = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$ is the measurement equation, L_f is the Lie derivative, p is the size of the measurement equation, and n is the size of the state vector.

Then, the gradients operator on matrix $\mathbf{G}$ is calculated as follow,

$$\mathbf{dG} = \begin{bmatrix} dL_f^0(h_1) & \dots & dL_f^0(h_p) \\ \vdots & \ddots & \vdots \\ dL_f^{n-1}(h_1) & \dots & dL_f^{n-1}(h_p) \end{bmatrix} \quad (5)$$

The system is observable if the matrix $\mathbf{dG}$ has rank n (seven).

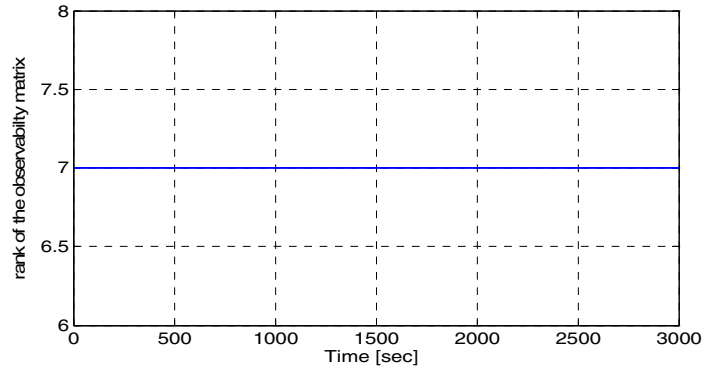


Figure 1. Rank of the observability matrix $\mathbf{dG}$.

Figure 1 shows that the rank of the matrix $\mathbf{dG}$ is equal to seven. Therefore, our system is observable.

4. Attitude determination algorithms

The objective of the attitude determination algorithms is to calculate the space orientation of the satellite based on sensors that can give measurements of known quantity (magnetic field, direction of the sun, direction of the star, etc) in the form of three dimensional vectors [6].

The three known algorithms are: Least Squares, Quest method and Optimized TRIAD.

4.1. Least Squares

Consider the following nonlinear system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{t}) + \mathbf{B} \mathbf{u}(\mathbf{t}) \quad (6.1)$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \quad (6.2)$$

Now define $\boldsymbol{\epsilon}_y$ as the difference between the noisy measurements and the vector $\mathbf{H}\hat{\mathbf{x}}$ [7]

$$\boldsymbol{\epsilon}_y = \mathbf{y} - \mathbf{H}\hat{\mathbf{x}} \quad (7)$$

$\hat{\mathbf{x}}$ represents the state estimates.

$\boldsymbol{\epsilon}_y$ is called the measurement residual. The most probable value of the vector $\mathbf{x}$ is the vector $\hat{\mathbf{x}}$ that minimizes the sum of squares between the observed values $\mathbf{y}$ and the vector $\mathbf{H}\hat{\mathbf{x}}$. So we will try to compute the $\hat{\mathbf{x}}$ that minimizes the cost function J , where J is given as [7],

$$J = \boldsymbol{\epsilon}_y^T \boldsymbol{\epsilon}_y \quad (8)$$

The algorithm of LS is written in the following form [7]

$$\mathbf{K}_k = \mathbf{P}_{k-1} \mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_{k-1} \mathbf{H}_k^T + \mathbf{R}]^{-1} \quad (9)$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k-1} + \mathbf{K}_{k+1} [\mathbf{y}_{k+1} - \mathbf{H}_k \hat{\mathbf{x}}_{k-1}] \quad (10)$$

$$\mathbf{P}_k = [\mathbf{I} - \mathbf{K}_k \mathbf{H}_k] \mathbf{P}_{k-1} [\mathbf{I} - \mathbf{K}_k \mathbf{H}_k]^T + \mathbf{K}_k \mathbf{R} \mathbf{K}_k^T \quad (11)$$

4.2. Quest-Method

The principle of the Quest algorithm can be done by solving Wahba's problem, this latter can be described as follows.

Considering n vectors $\mathbf{b}_i$, $i = 1 \dots n$, where n corresponds to the number of independent sensors installed in the satellite body and $\mathbf{b}_i$ is the observation vectors. For each observed vector, a reference vector $\mathbf{r}_i$ is necessary to be used in the attitude computation [6]. This description can be described as follows,

$$\mathbf{b}_i = \mathbf{A} \mathbf{r}_i + \mathbf{n}_i \quad (12)$$

In mathematical terms, the Wahba's problem consists of finding the matrix $\mathbf{A}$ which minimizes the following cost function [6]

$$L(\mathbf{A}) = \frac{1}{2} \sum_{i=1}^n a_i |\mathbf{b}_i - \mathbf{A} \mathbf{r}_i|^2 \quad (13)$$

This problem is formulated as an eigenvector problem. The Quest algorithm estimates the optimal eigenvalue and eigenvector for the Wahba problem.

It is equivalent to a research problem of the maximum eigenvalue $\lambda_{\max}$ of the matrix $\mathbf{K}$

$$\mathbf{K} = \begin{bmatrix} \mathbf{S} - \sigma \mathbf{I} & \mathbf{z} \\ \mathbf{z}^T & \sigma \end{bmatrix} \quad (14)$$

where, σ , $\mathbf{S}$, and $\mathbf{Z}$ are defined as follows,

$$\mathbf{B} = \sum_{i=1}^n a_i \mathbf{b}_i \mathbf{r}_i^T \quad \sigma = \text{trace } \mathbf{B} \quad \mathbf{S} = \mathbf{B} + \mathbf{B}^T \quad \mathbf{Z} = \sum_{i=1}^n a_i \mathbf{b}_i \times \mathbf{r}_i \quad (15)$$

The largest eigenvalue is obtained by solving numerically the equation [6],

$$\det(\mathbf{K} - \lambda \mathbf{I}_{4 \times 4}) = 0 \quad (16)$$

The optimal value λ_{opt} is given as,

$$\lambda_{\text{opt}} \approx \sum_{i=1}^n a_i - L(\mathbf{A}) \quad (17)$$

The values of $L(\mathbf{A})$ are quite smalls, an approximate solution is given as,

$$\lambda_{\text{opt}} \approx \sum_{i=1}^n a_i \quad (18)$$

Using this eigenvalue, the respective eigenvector is calculated as follows,

$$\bar{\mathbf{q}} = \frac{1}{\sqrt{1 + \mathbf{p}^T \mathbf{p}}} \begin{bmatrix} \mathbf{p} \\ 1 \end{bmatrix} \quad \mathbf{p} = [(\lambda_{\text{opt}} \mathbf{I} - \mathbf{S})^{-1} \mathbf{Z}] \quad (19)$$

4.3. Optimized TRIAD

The optimized TRIAD is a linear combination between two TRIAD algorithms, called TRIAD-I and TRIAD-II. The first algorithm generates the matrix $\mathbf{A}_1$, and the second generates the matrix $\mathbf{A}_2$.

It consists of the construction of two vectors triads using two pairs of vector measurements: two in the orbital reference frame, noted $\mathbf{v}_1$ and $\mathbf{v}_2$, and two in the body reference frame, noted $\mathbf{w}_1$ and $\mathbf{w}_2$. Let $\mathbf{n}_1$ and $\mathbf{n}_2$ be the measured noises of $\mathbf{w}_1$ and $\mathbf{w}_2$, respectively [8].

The optimized TRIAD algorithm requires the computation of the matrices $\mathbf{r}_i$ in body coordinate, and the corresponding column matrices $\mathbf{s}_i$ in the reference system.

For the TRIAD-I algorithm, we define

$$\mathbf{r}_1 = \mathbf{w}_1 / |\mathbf{w}_1| \quad \mathbf{r}_2 = (\mathbf{r}_1 \times \mathbf{w}_2) / |\mathbf{r}_1 \times \mathbf{w}_2| \quad \mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2 \quad (20)$$

$$\mathbf{s}_1 = \mathbf{v}_1 / |\mathbf{v}_1| \quad \mathbf{s}_2 = (\mathbf{s}_1 \times \mathbf{v}_2) / |\mathbf{s}_1 \times \mathbf{v}_2| \quad \mathbf{s}_3 = \mathbf{s}_1 \times \mathbf{s}_2 \quad (21)$$

The matrix $\mathbf{A}_1$ is expressed as,

$$\mathbf{A}_1 = \mathbf{r}_1 \cdot \mathbf{s}_1^T + \mathbf{r}_2 \cdot \mathbf{s}_2^T + \mathbf{r}_3 \cdot \mathbf{s}_3^T \quad (22)$$

For the TRIAD-II algorithm, we define

$$\mathbf{r}_5 = \mathbf{w}_2 / |\mathbf{w}_2| \quad \mathbf{r}_2 = (\mathbf{r}_1 \times \mathbf{w}_2) / |\mathbf{r}_1 \times \mathbf{w}_2| \quad \mathbf{r}_4 = \mathbf{r}_5 \times \mathbf{r}_2 \quad (23)$$

$$\mathbf{s}_5 = \mathbf{v}_2 / |\mathbf{v}_2| \quad \mathbf{s}_2 = (\mathbf{s}_1 \times \mathbf{v}_2) / |\mathbf{s}_1 \times \mathbf{v}_2| \quad \mathbf{s}_4 = \mathbf{s}_5 \times \mathbf{s}_2 \quad (24)$$

And the matrix $\mathbf{A}_2$ is expressed as,

$$\mathbf{A}_2 = \mathbf{r}_5 \cdot \mathbf{s}_5^T + \mathbf{r}_2 \cdot \mathbf{s}_2^T + \mathbf{r}_4 \cdot \mathbf{s}_4^T \quad (25)$$

Then, the optimization attitude matrix $\hat{\mathbf{A}}$ is calculated as,

$$\hat{\mathbf{A}} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \mathbf{A}_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \mathbf{A}_2 \quad (26)$$

Finally, the attitude matrix of optimized TRIAD is expressed as,

$$\mathbf{A} = 0.5 \left[\hat{\mathbf{A}}' + \left(\hat{\mathbf{A}}'^{-1} \right)^T \right] \quad (27)$$

5. Attitude estimation algorithms

The attitude estimation theory is very useful in the space field, the attitude estimation algorithms estimate the full elements of the state vector (attitude and angular rate) using the dynamic and kinematic models to propagate the estimated state vector.

The two known algorithms are: Extended Kalman Filter and Unscented Kalman Filter.

5.1. Extended Kalman Filter

The Extended Kalman filter (EKF) [9] is a recursive state estimator. Its cost function is given as [7],

$$J = \lim_{n \rightarrow \infty} \sum_{k=0}^N \mathbf{E}(\|\mathbf{x}_k - \hat{\mathbf{x}}_k\|) \quad (28)$$

where, $\mathbf{E}$: the expectation; $\mathbf{x}_k$: true state vector; $\hat{\mathbf{x}}_k$: estimated state vector.

The EKF algorithm consists of two parts: propagation and correction cycles, it is written in the following form [10-11].

5.1.1. Propagation cycle.

$$\hat{\mathbf{x}}_{k+1/k} = \mathbf{f}(\hat{\mathbf{x}}_{k/k}, \mathbf{u}_k) \quad (29)$$

$$\mathbf{P}_{k+1} = \Phi_k \mathbf{P}_k \Phi_k^T + \mathbf{Q}_k \quad (30)$$

5.1.2. Correction cycle.

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1} \mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_{k+1} \mathbf{H}_k^T + \mathbf{R}]^{-1} \quad (31)$$

$$\mathbf{P}_{k+1/k+1} = [\mathbf{I} - \mathbf{K}_{k+1} \mathbf{H}_k] \mathbf{P}_{k+1/k} \quad (32)$$

$$\hat{\mathbf{x}}_{k+1/k+1} = \hat{\mathbf{x}}_{k+1/k} + \mathbf{K}_{k+1} [\mathbf{y}_{k+1} - \mathbf{H}_k \hat{\mathbf{x}}_{k+1/k}] \quad (33)$$

where,

$\mathbf{P}$: Covariance matrix (7x7);

Φ : State transformation matrix (7x7);

$\mathbf{Q}$: Process noise covariance matrix (7x7);

$\mathbf{K}$: Kalman gain matrix (7x4);

$\mathbf{R}$: Measurement noise covariance matrix (4x4).

5.2. Unscented Kalman Filter

The Unscented Kalman Filter was proposed by Julier and Uhlmann [12]. It is an estimator which applies to the nonlinear systems without passing by the linearization required by the EKF.

The principle of the UKF is to perform a deterministic sampling of the state vector using a nonlinear transformation called "Unscented transformation". The state vector is represented by a set of samples (sigma points), these samples are stored in a matrix of size $n \times (2n + 1)$. The algorithm of this filter is described as follows [13],

5.2.1. Calculation of the sigma point. The columns of the sigma point matrix are calculated as follows

$$\mathbf{x}_{k-1} = \left[\hat{\mathbf{x}}_{k-1} \quad \hat{\mathbf{x}}_{k-1} + \sqrt{(n + \lambda)(\hat{\mathbf{P}}_{k-1})_i} \quad \hat{\mathbf{x}}_{k-1} - \sqrt{(n + \lambda)(\hat{\mathbf{P}}_{k-1})_i} \right] \quad (34)$$

The square root of the $(n + \lambda)\hat{\mathbf{P}}_{k-1}$ matrix is computed using Cholesky decomposition, and the parameter λ is defined as,

$$\lambda = \alpha^2(n + k) - n \quad (35)$$

where, n is the dimension of the state vector.

5.2.2. Propagation. If the $\mathbf{X}_{k-1}$ matrix is calculated, we propagate each column of this matrix using the mathematical model.

$$\mathbf{x}_{k/k-1}^i = \mathbf{f}(\mathbf{x}_{k-1}^i) \quad (36)$$

The mean and covariance of the state are calculated as follows

$$\hat{\mathbf{x}}_{k/k-1} = \sum_{i=0}^{2n} W_i^{(m)} \mathbf{x}_{k/k-1}^i \quad (37)$$

$$\hat{\mathbf{P}}_{k/k-1} = \sum_{i=0}^{2n} W_i^{(c)} (\mathbf{x}_{k/k-1}^i - \hat{\mathbf{x}}_{k/k-1})(\mathbf{x}_{k/k-1}^i - \hat{\mathbf{x}}_{k/k-1})^T + \mathbf{Q}_k \quad (38)$$

where,

$$W_0^{(m)} = \frac{\lambda}{n + \lambda} \quad W_0^{(c)} = \frac{\lambda}{n + \lambda} + (1 - \alpha^2 + \beta) \quad W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n + \lambda)}, i = 1, 2 \dots n \quad (39)$$

5.2.3. Update. The sigma points for the measurements are calculated as follows,

$$\mathbf{y}_{k/k-1}^i = \mathbf{h}(\mathbf{x}_{k-1}^i) \quad (40)$$

The mean and covariance of the sigma points for the measurements $\mathbf{y}_{k/k-1}^i$

$$\hat{\mathbf{y}}_{k/k-1} = \sum_{i=0}^{2n} W_i^{(m)} \mathbf{y}_{k/k-1}^i \quad (41)$$

$$\hat{\mathbf{P}}_{yy,k} = \sum_{i=0}^{2n} W_i^{(c)} (\mathbf{y}_{k/k-1}^i - \hat{\mathbf{y}}_{k/k-1})(\mathbf{y}_{k/k-1}^i - \hat{\mathbf{y}}_{k/k-1})^T + \mathbf{R}_k \quad (42)$$

The covariance between state and measurement

$$\hat{\mathbf{P}}_{xy,k} = \sum_{i=0}^{2n} W_i^{(c)} (\mathbf{x}_{k/k-1}^i - \hat{\mathbf{x}}_{k/k-1})(\mathbf{y}_{k/k-1}^i - \hat{\mathbf{y}}_{k/k-1})^T \quad (43)$$

Finally, the gain, the state vector, and the covariance matrix are calculated as follows,

$$\mathbf{K}_k = \hat{\mathbf{P}}_{xy,k} \hat{\mathbf{P}}_{yy,k}^{-1} \quad \hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k/k-1} + \mathbf{K}_k [\mathbf{y}_k - \hat{\mathbf{y}}_{k/k-1}] \quad \hat{\mathbf{P}}_k = \hat{\mathbf{P}}_{k/k-1} - \mathbf{K}_k \hat{\mathbf{P}}_{yy,k} \mathbf{K}_k^T \quad (44)$$

6. Nonlinear observers

An observer is a dynamic system that can be called “software sensor”. It estimates the state vector using the system input, the system output, and knowledge of the mathematic model.

Among the most robust observers, we have the sliding mode observers. The sliding mode observers are a nonlinear state observers based on the theory of variable structure systems [14].

Consider the following nonlinear system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{t}) + \mathbf{B} \mathbf{u}(\mathbf{t}) \quad (45.1)$$

$$\mathbf{y} = \mathbf{C} \mathbf{x} + \mathbf{v} \quad (45.2)$$

where, $\mathbf{v}$ presents the measurement noise.

The Sliding Mode Observer is chosen as the following form [14]

$$\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}, \mathbf{t}) + \mathbf{B} \mathbf{u}(\mathbf{t}) + \mathbf{K} \text{sat}\left(\frac{\mathbf{S}}{\Phi}\right) \quad (46.1)$$

$$\hat{\mathbf{y}} = \mathbf{C} \hat{\mathbf{x}} \quad (46.2)$$

where, $\mathbf{S}$ is the sliding surface function, it is defined as,

$$\mathbf{S} = \mathbf{y} - \mathbf{C} \hat{\mathbf{x}} \quad (47)$$

$\hat{\mathbf{x}}$ represents the state estimates;

$\mathbf{f}(\hat{\mathbf{x}}, \mathbf{t})$ is the model of the nonlinear system;

$\mathbf{K} = \begin{bmatrix} \mathbf{K}_\omega \\ \mathbf{K}_q \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} \mathbf{H}_\omega \\ \mathbf{H}_q \end{bmatrix}$, $\Phi = \begin{bmatrix} \Phi_\omega \\ \Phi_q \end{bmatrix}$ are gain matrices to be determined.

To accelerate the estimation convergence, the linear correction term of Luenberger $\mathbf{H}(\mathbf{y} - \hat{\mathbf{y}})$ is included in the observer.

$$\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}, \mathbf{t}) + \mathbf{B} \mathbf{u}(\mathbf{t}) + \mathbf{H}(\mathbf{y} - \hat{\mathbf{y}}) + \mathbf{K} \text{sat}\left(\frac{\mathbf{S}}{\Phi}\right) \quad (48.1)$$

$$\hat{\mathbf{y}} = \mathbf{C} \hat{\mathbf{x}} \quad (48.2)$$

where, $\mathbf{H} = \begin{bmatrix} \mathbf{H}_\omega \\ \mathbf{H}_q \end{bmatrix}$ is gain matrix of Luenberger.

7. Simulation results

In this section, we present the simulation results obtained for the different algorithms exposed previously. The satellite is placed at an average altitude of 686 km on a circular orbit inclined 98° to the equator. The inertia matrix is $\mathbf{I} = [12 \ 0 \ 0; 0 \ 14 \ 0; 0 \ 0 \ 10]$ Kg.m². And the initial conditions are $[5 \ 10 \ -10]$ deg and $[0 \ -0.06 \ 0]$ rad/sec for the Euler angles and the attitude rate respectively.

The presented algorithms are simulated using the same simulation method, and they are initialized in the same way, using the same initialization values. The initial parameters are detailed in what follows.

Table 1. Extended Kalman Filter parameters.

Parameter	Value
$\mathbf{X}_0$	$[-0.22 \ -0.16 \ 0.21 \ 0.93 \ 0 \ -0.01 \ 0]$
$\mathbf{P}_0$	$\begin{bmatrix} (0.1)^2 \mathbf{I}_{4 \times 4} & \mathbf{0}_{4 \times 3} \\ \mathbf{0}_{3 \times 4} & 10^{-14} \mathbf{I}_{3 \times 3} \end{bmatrix}$
Measurement	$(10^{-6})^2$
Process Noise	$(10^{-5})^2$

Table 2. Unscented Kalman Filter parameters.

Parameter	Value
$\mathbf{X}_0$	$[-0.22 \ -0.16 \ 0.21 \ 0.93 \ 0 \ -0.01 \ 0]$
$\mathbf{P}_0$	$\begin{bmatrix} (0.1)^2 \mathbf{I}_{4 \times 4} & \mathbf{0}_{4 \times 3} \\ \mathbf{0}_{3 \times 4} & 10^{-14} \mathbf{I}_{3 \times 3} \end{bmatrix}$
Measurement	$(10^{-6})^2$
Process Noise	$(10^{-5})^2$
α	10^{-4}
β	2
k	0

Table 3. Sliding Mode Observer parameters.

Parameter	Value
$\mathbf{K}_\omega$	$\text{diag}([0.077 \ 0.009 \ 0.009])$
$\mathbf{K}_q$	$\text{diag}([0.035 \ 0.099 \ 0.0987 \ 0.098])$
$\mathbf{H}_\omega$	$\text{diag}([0.05 \ 0.09 \ 0.098])$
$\mathbf{H}_q$	$\text{diag}([0.28 \ 0.28 \ 0.28 \ 0.28])$
Φ_ω	$\text{diag}([0.57 \ 0.03 \ 0.03])$
Φ_q	$\text{diag}([0.52 \ 0.027 \ 0.03 \ 0.03])$
$\mathbf{I}_{\text{SMO}}[\text{Kg.m}^2]$	$[14 \ 13 \ 12]$

Due to a lack of obvious physical interpretation of the quaternion, the Euler angles will normally be used to present the attitude during simulation tests.

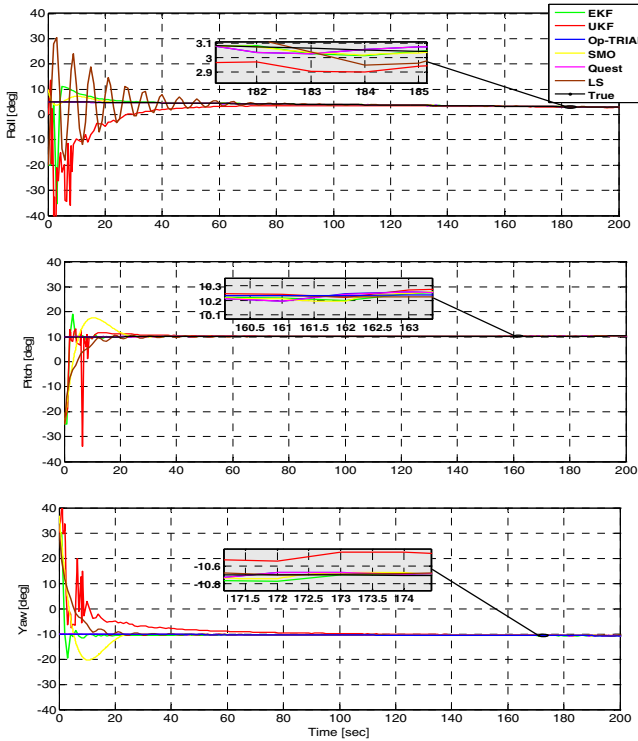


Figure 2. Actual and estimated attitude.

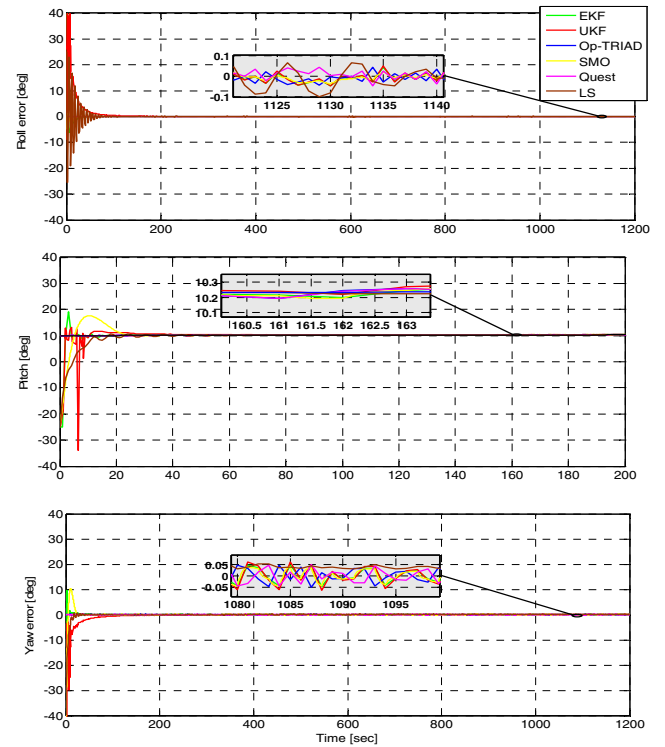


Figure 3. Errors of the estimated attitude.

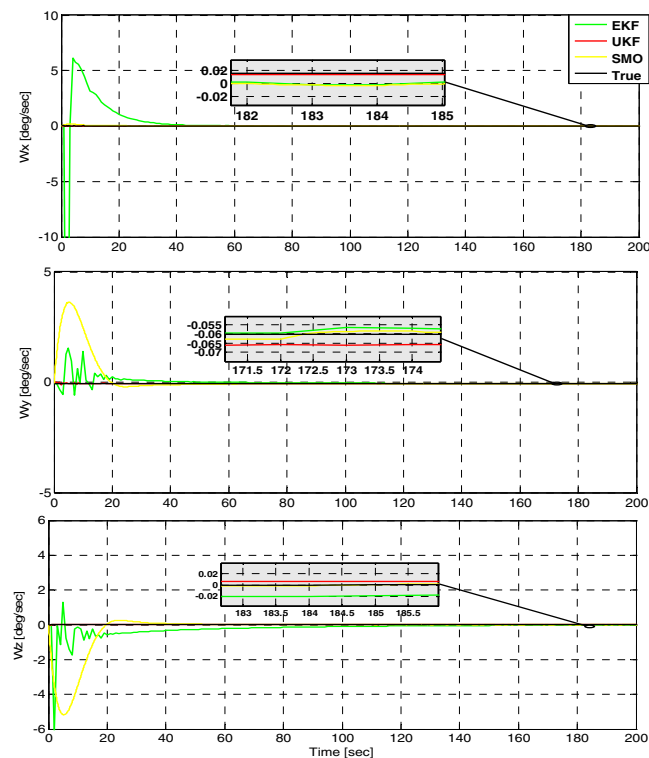


Figure 4. Estimated angular rates.

For more detailed analysis, the root mean square (RMS) values of error results for the time period 35000-36000 sec, the amount of memory and the convergence time are computed; they are presented in the Table 5. In addition, the estimated computation time along the 3000 sec is calculated with two ways. In the fourth column, it is computed using a personal computer with i54590S CPU 3.00 GHz, 4 GByte of RAM, and using On-board Computer (OBC 750) in the last column, this OBC is capable of 500 Dhrystone 2 MIPS and has 512 MByte of RAM.

Table 4. Performance evaluation of the different algorithms.

	RMS error [deg]	Amount of memory [M Bytes]	Convergence time [sec]	Computation time for PC [sec]	Computation time for OBC 750 [sec]
Least Squares	Roll $\begin{bmatrix} 0.0726 \end{bmatrix}$	0.765	60	0.56	12.30
	Pitch $\begin{bmatrix} 0.0563 \end{bmatrix}$				
	Yaw $\begin{bmatrix} 0.0250 \end{bmatrix}$				
Quest Method	Roll $\begin{bmatrix} 0.0402 \end{bmatrix}$	0.6484	/	0.31	6.81
	Pitch $\begin{bmatrix} 0.0278 \end{bmatrix}$				
	Yaw $\begin{bmatrix} 0.0271 \end{bmatrix}$				
Optimized TRIAD	Roll $\begin{bmatrix} 0.0415 \end{bmatrix}$	0.8867	/	0.49	10.76
	Pitch $\begin{bmatrix} 0.0332 \end{bmatrix}$				
	Yaw $\begin{bmatrix} 0.0312 \end{bmatrix}$				
EKF	Roll $\begin{bmatrix} 0.0294 \end{bmatrix}$	0.9492	100	0.74	16.25
	Pitch $\begin{bmatrix} 0.0294 \end{bmatrix}$				
	Yaw $\begin{bmatrix} 0.0294 \end{bmatrix}$				
UKF	Roll $\begin{bmatrix} 0.0297 \end{bmatrix}$	10.218	200	2.72	59.75
	Pitch $\begin{bmatrix} 0.0316 \end{bmatrix}$				
	Yaw $\begin{bmatrix} 0.0295 \end{bmatrix}$				
SMO	Roll $\begin{bmatrix} 0.0245 \end{bmatrix}$	1.4063	30	0.63	13.84
	Pitch $\begin{bmatrix} 0.0192 \end{bmatrix}$				
	Yaw $\begin{bmatrix} 0.0195 \end{bmatrix}$				

The figures 2-4 present the attitude, the errors of the estimated attitude and the angular rates respectively of the different algorithms presented in this paper. These figures show that the attitude determination algorithms (Least Squares, Quest method and optimized TRIAD) are able to estimate only the attitude. However, the attitude estimation algorithms (EKF and UKF) and the sliding mode observer estimate the attitude and the angular rates.

The table 4 presents a comparison between the different algorithms in terms of accuracy, amount of memory convergence time, and computation time. From this table, it is seen that the attitude determination algorithms are characterized by fast computation time (0.56 sec, 0.31 sec, and 0.49 sec) for personal computer and (12.30 sec, 6.81 sec, and 10.76 sec) for OBC750, and they require less amount of memory but they provide less accuracy in comparison with the attitude estimation algorithms and the Sliding mode observer. In addition, the Quest and Optimized TRIAD algorithms have not a convergence time because they do not need the initial parameters. The attitude estimation algorithms have a good performance in term of accuracy but they are more expensive in computation time, 16.25 sec for the EKF and 59.75 for the UKF. the computation time of this latter is

approximately nine times the computation time of Quest-Method, it is seen that this filter has the slowest convergence. In addition, its requirement of memory is very largest (10.21 M Bytes) due to calculation of the sigma points.

The table 4 results indicate also that the SMO has a good performance in terms of accuracy, computation time and convergence time (it is characterized by accuracy two times better than the Optimized TRIAD and the Quest-Method).

8. Conclusion

The primary focus of this paper is to present a performance analysis of the attitude determination algorithms, attitude estimation algorithms, and nonlinear observer for application on low earth orbit satellites. The selected algorithms were compared to find the required accuracy, computation time, convergence time, and amount of memory. The software implementation of these algorithms was executed in a personal computer and in the On-board Computer 750 that was developed in SSTL to function as the primary flight computer for applications requiring processing On-board. The presented study can be a help tool for the ADCS designer.

The results of this work indicate that each of the attitude determination, attitude estimation, and nonlinear observer algorithms is able to determine the attitude. However, there are some differences between these algorithms.

Attitude determination algorithms are deterministic methods. They are characterized by low computational efforts, less accuracy and amount of memory in comparison with attitude estimation algorithms. This is due to its independence of dynamic equation. However, this latter is used by attitude estimation algorithms, the use of this equation increases the accuracy of these algorithms, but they have a high time consuming (the execution time of the UKF is approximately nine times the execution time of the Quest-Method) because they need much calculation (calculation of the covariance matrix) and its structure uses the dynamic models. In addition, the memory requirement of these algorithms is very largest.

On the other hand, the nonlinear observers have high performances in terms of time consuming and accuracy, it is characterized by accuracy two times better than the Optimized TRIAD and the Quest-Method.

From all the above, we can give a solution to the problem posed in the beginning of this paper, we should use the nonlinear observer to extract the attitude of spacecraft because its accuracy is more high than the others algorithms. In addition, it is helpful for micro satellite not equipped with gyroscope (which is costly and heavy).

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