

# Tire Force Estimation using a Proportional Integral Observer

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**Abstract.** This paper addresses a method for detecting critical stability situations in the lateral vehicle dynamics by estimating the non-linear part of the tire forces. These forces indicate the road holding performance of the vehicle. The estimation method is based on a robust fault detection and estimation approach which minimize the disturbance and uncertainties to residual sensitivity. It consists in the design of a *Proportional Integral Observer (PIO)*, while minimizing the well known  $H_\infty$  norm for the worst case uncertainties and disturbance attenuation, and combining a transient response specification. This multi-objective problem is formulated as a *Linear Matrix Inequalities (LMI)* feasibility problem where a cost function subject to *LMI* constraints is minimized. This approach is employed to generate a set of switched robust observers for uncertain switched systems, where the convergence of the observer is ensured using a *Multiple Lyapunov Function (MLF)*. Whilst the forces to be estimated can not be physically measured, a simulation scenario with *CarSim<sup>TM</sup>* is presented to illustrate the developed method.

**Keywords:** Proportional Integral Observer, Robust fault detection, Estimation, switched systems,  $H_\infty$ , Linear Matrix Inequalities, Multiple Lyapunov Function.

## 1. Introduction

Fully autonomous vehicles are unlikely to be commercially available before 2020. *Advanced Driver-Assistance Systems (ADAS)* will play a crucial role in preparing regulators, consumers, and corporations for the medium-term reality of cars taking over control from drivers.

The market introduction of *ADAS* has shown that the primary challenges preventing faster market penetration are pricing, consumer understanding and safety/security issues. The technological challenges are not insignificant, and will likely drive the delay between conditionally autonomous cars which allow the driver to cede control in certain situations (Level 3 according to the *National Highway Traffic Safety Administration (NHTSA)*) and fully autonomous cars, which require no driver intervention for the entire trip (Level 4 *NHTSA*). Tech players and startups will likely play an important role in achieving this level of technical complexity. Regulation and consumer acceptance may represent additional hurdles for



autonomous vehicles. A progressive scenario would see fully autonomous cars accounting for up to 15 % of passenger vehicles sold worldwide in 2030 [6].

Many safety mechanisms rely on the knowledge of vehicle dynamics states and parameters, that can be measured. Tire forces that act during maneuvers are nonlinear with respect to wheel slip and wheel slip angles and cause a wide range of stability and handling properties. The force depend on external factors, such as road properties. However, some of these parameters can not be directly measured because the expensive cost of the sensors, physical restrictions or simply they are correlated with other vehicle data. Example of this are the tire forces, involved in lateral vehicle dynamics. The tire force can be used as an indicator for road holding of the vehicle. It is well known that the lateral tire force has a nonlinear relationship with respect to the slip angle. During normal driving situations, the slip angle is low and the tire force can be approximated to a linear function of the slip angle. Conversely, in critical situations, where the tire is approaching its adherence limits, the lateral force is highly nonlinear. Therefore, by using the difference between the linear tire model and the actual tire force it is possible to discriminate the vehicle situation as normal or critical. Since measuring the actual tire force is difficult in practice, this paper deals with the estimation of the nonlinear part of the tire force, which is the difference between the actual tire force and the linear model of the force.

To maintain stability during emergencies follow rules based on intensive tests, rather than analytic control laws based on actual forces and accurate state estimates. The model of tire-road contact forces is complex because a wide variety of parameters including environmental factors and pneumatic properties that impact the tire-road contact interface. Many tire-force models have been proposed.

From the theoretical aspects, different longitudinal tire/road friction models can be classified into two types: empirical and analytical models. Empirical models are based on curve-fitting techniques. Empirical models can accurately represent the steady-state characteristics of the tire/road friction phenomena. But, they cannot describe several significant dynamic behaviors such as hysteresis. Analytical models are popular recently; they use differential equations to describe tire/road friction properties. The obtained results were based in Magic Formula proposed by Pacejka; however, there are options such as Piecewise Linear Model, Burckhardt Model [2], Rill Model [13], Dahl Model [3], and LuGre Model [5] that provide different results. Intensive testing is needed to determine the parameters of analytical models. It is very difficult to determine all of them in real-time for every tire, tire pressure and wear state.

Some related work concerning the nonlinear lateral force estimation can be found in the literature. In [11] an *Extended Kalman Filter (EKF)* designed using a 9 *DoF* vehicle model and an analytic tire force model is used to simulate the vehicle motion (i.e. maneuvers combining steering, braking, and varying road conditions), and a 5 *DoF* was proposed to estimate the nonlinear tire force. No prior information of the tire force characteristics or external factors is needed for the nonlinear estimation. Some simulations validated the effectiveness of the proposal that suggest the *EKF* could be implemented in real time. In [12] an *Extended Kalman Bucy Filter* and Bayesian hypothesis selection were also designed using a 8 *DoF* longitudinal-lateral vehicle model to estimate morion, tire forces, and road coefficient of friction of vehicles. In both cases simulation results are very promising; however, the approaches were just validated in a mild scenario, where the force remains close to the linear zone.

In [16] a comparison of the lateral force estimated by four observers one based on a linear vehicle model and three based on non linear model (an extended Luenberger observer, *EKF* and Sliding-Mode Observer), was carried out. *EKF* using a yaw rate sensor only gives less accurate estimations than the other two nonlinear observers; however, all non-linear observers have similar results. All observers had satisfactory performance when lateral acceleration was low. However, at larger acceleration values, some assumptions made for models are not totally righth (i.e. linearity), yielding worse estimations; better models are needed.

Employing the bicycle model dynamical equations, in [17] the tire forces are estimated using algebraic techniques for filtering and estimating derivatives of noisy signals. The drawback of this approach is that the estimation is highly dependent on an accurate identification of the vehicle parameters

and on the quality of the yaw and lateral acceleration measurements.

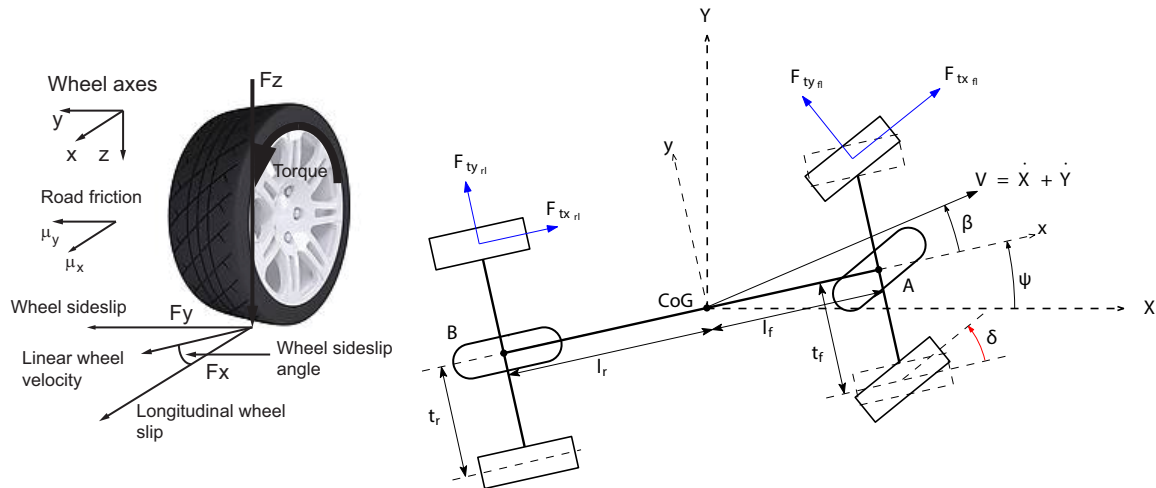
The *Proportional Integral Observer (PIO)*, [1], was introduced by [7] for *SISO* systems, and extended for *MIMO* systems [18]. The synthesis with robustness performances toward parametric uncertainties and unknown inputs was proposed by [14]. Later, they were used for fault estimation [15] [9].

A hybrid process consists of several modes of behavior that has a different dynamic. One class of hybrid system is switched system. Transition between these modes might be state-dependent, time-dependent, input-dependent, event-dependent or a combination of these conditions, [4]. The transitions between modes is defined by a switching signal [8]. A design of *Switched Proportional Integral Observer (SPIO)* for fault estimation in discrete time is proposed to estimate the non-linear tire force. The integral part of this observer provides an additional degree of freedom in comparison to a traditional observer that is used to improve the accuracy of the steady-state fault estimation and the robustness to disturbance inputs.

The outline of this paper is as follows. A lateral vehicle model as a switched system is developed in section 2. In section 3, the method for the synthesis of fault estimation *SPIO* is presented. *CarSim<sup>TM</sup>* simulations are carried out to validate of the *SPIO* design in section 4. Finally, section 5 concludes the research and shows the possible future work. Table 1 the used variables of this research paper.

## 2. Switched vehicle model

The well known bicycle model, Fig. 1, is used to model the lateral vehicle dynamics. The reference point *Center of Gravity (CoG)* is chosen to represent the center of gravity for the vehicle body, where the vehicle velocity is defined. Symbols A and B denote the positions of the front and rear tire/road interfaces. The heading angle  $\psi$  is the angle from the guideline to the longitudinal axis of vehicle body AB. The side-slip angle  $\beta$  is the angle from the longitudinal axis of the vehicle body to the direction of the vehicle velocity,  $\delta$  is the steering angle,  $\dot{\psi}$  is the yaw rate. The involved forces in this model are the lateral tire forces at the front  $F_{ty_f}$  and the rear  $F_{ty_r}$  of the vehicle, the load transfer forces  $F_d$  and the wind forces  $F_w$ .



**Figure 1.** Bicycle Model

Applying the second law of Newton to translational and rotational movements yields:

$$mv(\dot{\beta} + \dot{\psi}) = F_{ty_f} + F_{ty_r} + F_{dy} + F_w \quad (1a)$$

$$I_z \ddot{\psi} = l_f F_{ty_f} - l_r F_{ty_r} + M_{dz} \quad (1b)$$

where  $m$ ,  $I_z$ ,  $l_f$  and  $l_r$  are the vehicle parameters, Table 1.

**Table 1.** D-Class Sedan parameters for bicycle model

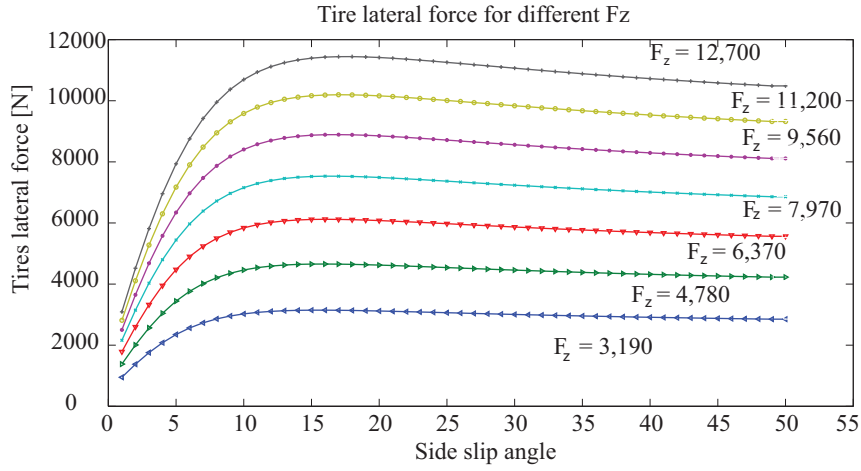
| Symbol         | Variable                           | Value     | Unit                 |
|----------------|------------------------------------|-----------|----------------------|
| $c_{yf}$       | Front linear cornering coefficient | 66,257.00 | [N/rad]              |
| $c_{yr}$       | Rear linear cornering coefficient  | 46,545.00 | [N/rad]              |
| $d$            | Disturbance vector                 |           | $\mathbb{R}^{n_d}$   |
| $e_x, e_f$     | State and fault estimation error   |           |                      |
| $f$            | Faults vector                      |           | $\mathbb{R}^{n_f}$   |
| $F_d$          | Load transfer force                |           | N                    |
| $F_{tyf}$      | Lateral force at the front         |           | N                    |
| $F_{tyr}$      | Lateral force at the rear          |           | N                    |
| $F_w$          | Wind force                         |           | N                    |
| $L_{I,\alpha}$ | Integral gain                      |           |                      |
| $L_{P,\alpha}$ | Proportional gain                  |           |                      |
| $I_z$          | Inertia around z-axis              | 2,315.30  | [kg.m <sup>2</sup> ] |
| $l_f$          | Distance from CoG to front axle    | 1.11      | [m]                  |
| $l_r$          | Distance from CoG to rear axle     | 1.67      | [m]                  |
| $m$            | Vehicle total mass                 | 1,530.00  | [kg]                 |
| $m_s$          | Sprung mass                        |           | [kg]                 |
| $m_{us}$       | Unsprung mass                      |           | [kg]                 |
| $r$            | Residual error ( $y - \hat{y}$ )   |           |                      |
| $x$            | State vector                       |           | $\mathbb{R}_x^n$     |
| $y$            | Measurement output vector          |           | $\mathbb{R}_y^n$     |
| $\alpha$       | Switching signal                   |           |                      |
| $\delta$       | Wheel steer angle                  | -         | [rad]                |
| $\beta$        | Side slip angle                    | -         | [rad]                |
| $\Gamma_{ij}$  | Non-linear part of the tire force  |           | N                    |
| $v$            | Longitudinal speed of the vehicle  | -         | [m/s]                |
| $\psi$         | Yaw rate                           | -         | [rad/s]              |
| $n_d$          | Number of disturbances             |           |                      |
| $n_f$          | Number of faults                   |           |                      |
| $n_x$          | Number of states                   |           |                      |
| $n_y$          | Number of outputs                  |           |                      |
| $N$            | Number of subsystems               |           |                      |

The lateral tire forces  $F_{tyij}$  are function of the wheel load, slip angle and camber. Fig 2 shows the lateral force versus slip-angle curves for the tire considered in this paper. As observed, the tire characteristic is highly nonlinear.

One of the most used models for tire forces was proposed by Pacejka [10], often called the *magic formula*. In this model, lateral tire force is computed as:

$$F_{tyij} = D_y \sin \left[ C_y \tan^{-1} \left( B_y \beta_{yij} - E_y (B_y \beta_{yij} - \tan^{-1} (B_y \beta_{yij})) \right) \right] \quad (2)$$

where the parameters are function of the wheel load.  $D_y$  represents the maximum value of the characteristic,  $B_y$  and  $C_y$  are the slope and the shape of the curve and  $E_y$  adjusts the x-coordinate (side-slip angle). This model can also be described as the sum of a linear function  $\bar{F}_{tyij}$  for small slip angles



**Figure 2.** Lateral tire force for different normal efforts

and a non-linear part  $\Gamma_{ij}$ :

$$F_{tyij} = \bar{F}_{tyij} + \Gamma_{ij} \quad (3)$$

where

$$\bar{F}_{tyf} = c_{yf}\beta_{yf} = c_{yf}\left(\delta - \beta - \frac{l_f\psi}{v(t)}\right) \quad \bar{F}_{tyr} = c_{yr}\beta_{yr} = -c_{yr}\left(\beta - \frac{l_r\psi}{v(t)}\right) \quad (4)$$

The state space representation of the linear vehicle model is:

$$\begin{bmatrix} \dot{\beta}(t) \\ \dot{\psi}(t) \end{bmatrix} = \begin{bmatrix} -\frac{c_r+c_f}{mv(t)} & \frac{c_rl_r-c_fl_f}{mv^2(t)} - 1 \\ \frac{c_rl_r-c_fl_f}{I_z} & -\frac{c_rl_r^2+c_fl_f^2}{I_zv(t)} \end{bmatrix} \begin{bmatrix} \beta(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} \frac{c_f}{mv(t)} \\ \frac{c_rl_f}{I_z} \end{bmatrix} u_L(t) + \begin{bmatrix} \frac{1}{mv} & \frac{1}{mv} \\ \frac{l_r}{I_z} & -\frac{l_f}{I_z} \end{bmatrix} \begin{bmatrix} \Gamma_{yf} \\ \Gamma_{yr} \end{bmatrix} + \begin{bmatrix} \frac{1}{mv} \\ \frac{l_w}{I_z} \end{bmatrix} F_w \quad (5)$$

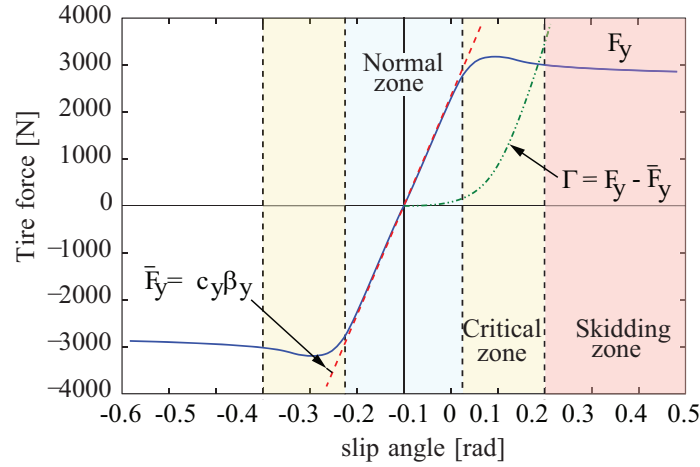
with  $u_L(t)$  is the steering wheel angle, it is assumed equal to the steering angle at the front wheels, if the transmission is considered as a linear constant function  $u_L(t) = \delta(t)$ , and the yaw rate as the measured output.

The lateral tire force  $F_{tyij}$  can be divided in three operating regions according to the magnitude of the tire slip angle, Fig. 3:

- Normal zone, where  $F_{tyf} = \bar{F}_{tyf}$  and  $\Gamma_{ij} \cong 0$ . In this region the road holding performance is satisfied.
- Critical zone, where  $\Gamma_{ij} \neq 0$ ; the vehicle starts losing its road holding, approaching risk zone,
- Skidding zone, where  $F_{tyf}$  becomes constant. In this condition the tire loses its grip, compromising the stability and handling of the vehicle.

The problem of detecting critical driving situation can be traduced into the detection and estimation of the non-linear tire forces ( $\Gamma_f$ ,  $\Gamma_r$ ), by considering them as faults. A dedicated residual generator scheme is adopted. Two observers are designed to estimate  $\Gamma_f$  and  $\Gamma_r$ . The state-space representation used to synthesize the residual that estimates  $\Gamma_f$  is given by:

$$\begin{bmatrix} \dot{\beta}(t) \\ \dot{\psi}(t) \end{bmatrix} = \begin{bmatrix} -\frac{c_r+c_f}{mv(t)} & \frac{c_rl_r-c_fl_f}{mv^2(t)} - 1 \\ \frac{c_rl_r-c_fl_f}{I_z} & -\frac{c_rl_r^2+c_fl_f^2}{I_zv(t)} \end{bmatrix} \begin{bmatrix} \beta(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} \frac{c_f}{mv(t)} \\ \frac{c_rl_f}{I_z} \end{bmatrix} u_L(t) + \begin{bmatrix} \frac{1}{mv} & \frac{1}{mv} \\ \frac{l_w}{I_z} & \frac{l_r}{I_z} \end{bmatrix} \begin{bmatrix} F_w \\ \Gamma_{yr} \end{bmatrix} + \begin{bmatrix} \frac{1}{mv} \\ -\frac{l_f}{I_z} \end{bmatrix} \Gamma_{yf} \quad (6)$$



**Figure 3.** Lateral tire force and working regions

where  $[F_w \ \Gamma_{yr}]^T$  is considered as a perturbation signal to be rejected. Conversely, the representation used to synthesize the residual for  $\Gamma_r$  is:

$$\begin{bmatrix} \dot{\beta}(t) \\ \dot{\psi}(t) \end{bmatrix} = \begin{bmatrix} -\frac{c_r+c_f}{mv(t)} & \frac{c_r l_r - c_f l_f}{mv^2(t)} - 1 \\ \frac{c_r l_r - c_f l_f}{I_z} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v(t)} \end{bmatrix} \begin{bmatrix} \beta(t) \\ \psi(t) \end{bmatrix} + \begin{bmatrix} \frac{c_f}{mv(t)} \\ \frac{c_r l_f}{I_z} \end{bmatrix} u_L(t) + \begin{bmatrix} \frac{1}{mv} & \frac{1}{mv} \\ \frac{l_w}{I_z} & \frac{l_r}{I_z} \end{bmatrix} \begin{bmatrix} F_w \\ \Gamma_{yf} \end{bmatrix} + \begin{bmatrix} \frac{1}{mv} \\ -\frac{l_f}{I_z} \end{bmatrix} \Gamma_{yr} \quad (7)$$

with the perturbation signal  $[F_w \ \Gamma_{yf}]^T$ .

Clearly, the bicycle model is non-linear because of the terms  $v^{-1}(t)$  and  $v^{-2}(t)$ . This model is linearized around multiple velocities  $v_\alpha$ . A switching rule dependent on the measured velocity is deduced, and the switching signal is then calculated as the integer part of  $v(t)/\Delta$ , where  $\Delta$  is the half distance between two successive operating points. The non-linear bicycle model is transformed into a switched uncertain model, using a Taylor expansion around  $v_\alpha$ :

$$\frac{1}{v}|_{v=v_\alpha} = \frac{1}{v_\alpha} - \frac{1}{v_\alpha^2}(v - v_\alpha) + \mathcal{O}\left(\frac{1}{v^2}\right) \quad \frac{1}{v^2}|_{v=v_\alpha} = \frac{1}{v_\alpha^2} - \frac{2}{v_\alpha^3}(v - v_\alpha) + \mathcal{O}\left(\frac{1}{v^3}\right) \quad (8)$$

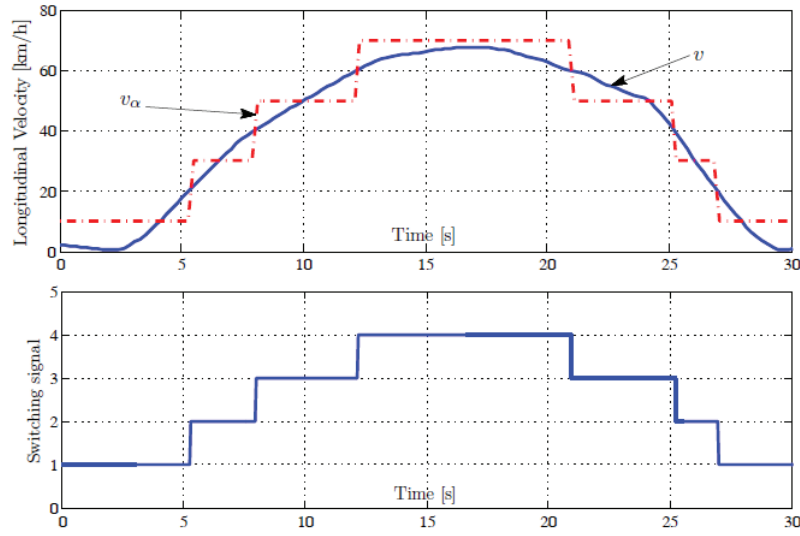
For velocities  $v > 10$  km/h,  $\mathcal{O}(\frac{1}{v^3}) \ll \mathcal{O}(\frac{1}{v^2}) \ll 1$ , then:

$$\begin{aligned} A_\alpha &= \underbrace{A_0 + \frac{1}{v_\alpha} A_1 + \frac{1}{v_\alpha^2} A_2}_{A_\alpha} + \underbrace{\left(-\frac{1}{v_\alpha^2} A_1 - \frac{2}{v_\alpha^3} A_2\right)}_{H_{A,\alpha}} \underbrace{(v - v_\alpha)}_{\Delta_{x,\alpha}} & B_\alpha &= \underbrace{B_0 + \frac{1}{v_\alpha} B_1}_{B_\alpha} + \underbrace{-\frac{1}{v_\alpha^2} B_1}_{H_{B,\alpha}} \underbrace{(v - v_\alpha)}_{\Delta_{u,\alpha}} \\ E_{d\alpha} &= \underbrace{E_{d0} + \frac{1}{v_\alpha} E_{d1}}_{E_{d\alpha}} + \underbrace{-\frac{1}{v_\alpha^2} E_{d1}}_{H_{C,\alpha}} \underbrace{(v - v_\alpha)}_{\Delta_{d,\alpha}} & E_{f\alpha} &= \underbrace{E_{f0} + \frac{1}{v_\alpha} E_{f1}}_{E_{f\alpha}} + \underbrace{-\frac{1}{v_\alpha^2} E_{f1}}_{H_{D,\alpha}} \underbrace{(v - v_\alpha)}_{\Delta_{f,\alpha}} \end{aligned} \quad (9)$$

Speeds higher than 40 km/h ( $v_{min} > 10$  m/s) are considered. In this case, the matrices  $H_{A,\alpha}, H_{B,\alpha}, H_{C,\alpha}, H_{D,\alpha}$  can be neglected. The transformed linear switched system is discretized with a sampling time of 20 ms:

$$x_{k+1} = A_\alpha x_k + B_\alpha u_k + E_{d\alpha} d_k + E_{f\alpha} f_k \quad (10)$$

where  $d_k$  is the disturbance signal and  $f_k$  denotes the non-linear tire force to be estimated. An example of a velocity profile and the deduced switching signal is illustrated in Fig. 4



**Figure 4.** Velocity profile and the corresponding switching signal

### 3. Observer Design

Consider this discrete-time switched linear system subject to faults and disturbances:

$$\begin{cases} x_{k+1} &= A_{\alpha(k)}x_k + B_{\alpha(k)}u_k + E_{d,\alpha(k)}d_k + E_{f,\alpha(k)}f_k \\ y_k &= C_{\alpha(k)}x_k + D_{\alpha(k)}u_k + F_{d,\alpha(k)}d_k + F_{f,\alpha(k)}f_k \end{cases} \quad (11)$$

where  $\alpha(k) : [0, \infty) \rightarrow \mathcal{S} = \{1, 2, \dots, N\}$  is the switching signal. The fault detection problem can be solved by *LMI* optimization. The *SPIO* for the estimation of the faults has the following form:

$$\begin{cases} \hat{x} &= A_{\alpha}\hat{x} + B_{\alpha}u + L_{P,\alpha}(y - \hat{y}) + E_{f,\alpha}\hat{f} \\ \hat{f} &= L_{I,\alpha}(y - \hat{y}) \\ \hat{y} &= C_{\alpha}\hat{x} + D_{\alpha}u + F_{f,\alpha}\hat{f} \end{cases} \quad (12)$$

where  $L_{P,\alpha}$  and  $L_{I,\alpha}$  are the Proportional and Integral gains. The state and fault estimation errors,  $e_x = x - \hat{x}$  and  $e_f = f - \hat{f}$ , respectively, are:

$$\begin{aligned} e_{x,k+1} &= (A_{\alpha} - L_{P,\alpha}C_{\alpha})e_{x,k} + (E_{f,\alpha} - L_{P,\alpha}F_{f,\alpha})e_{f,k} + (E_{d,\alpha} - L_{P,\alpha}F_{d,\alpha})d_k \\ e_{f,k+1} &= -L_{I,\alpha}C_{\alpha}e_{x,k} - L_{I,\alpha}F_{f,\alpha}e_{f,k} - L_{I,\alpha}F_{d,\alpha}d_k \end{aligned} \quad (13)$$

This can be written in the matrix form:

$$\begin{bmatrix} e_{x,k+1} \\ e_{f,k+1} \end{bmatrix} = \left( \begin{bmatrix} A_{\alpha} & E_{f,\alpha} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} L_{P,\alpha} \\ L_{I,\alpha} \end{bmatrix} \begin{bmatrix} C_{\alpha} & F_{f,\alpha} \end{bmatrix} \right) \begin{bmatrix} e_{x,k} \\ e_{f,k} \end{bmatrix} + \begin{bmatrix} E_{d,\alpha} - L_{P,\alpha}F_{d,\alpha} \\ -L_{I,\alpha}F_{d,\alpha} \end{bmatrix} d_k \quad (14)$$

or equivalently:

$$\tilde{x}_{k+1} = (\tilde{A}_{\alpha} - L_{\alpha}\tilde{C}_{\alpha})\tilde{x}_k + (\tilde{E}_{d,\alpha} - L_{\alpha}F_{d,\alpha})d_k \quad (15)$$

where  $\tilde{x} = [e_x^T \ e_f^T]^T$  and

$$\tilde{A}_{\alpha} = \begin{bmatrix} A_{\alpha} & E_{f,\alpha} \\ 0 & 0 \end{bmatrix}, \tilde{B}_{\alpha} = \begin{bmatrix} B_{\alpha} \\ 0 \end{bmatrix}, \tilde{E}_{d,\alpha} = \begin{bmatrix} E_{d,\alpha} \\ 0 \end{bmatrix}, \tilde{C}_{\alpha} = [C_{\alpha} \ F_{f,\alpha}], L_{\alpha} = \begin{bmatrix} L_{P,\alpha} \\ L_{I,\alpha} \end{bmatrix} \quad (16)$$

To ensure the robustness of the observer, eqn (12), to unknown inputs and perturbation, a residual signal to be minimized is  $r = y - \hat{y}$ , where  $A_\alpha^a = \tilde{A}_\alpha - L_\alpha \tilde{C}_\alpha$ ,  $E_{d,\alpha}^a = \tilde{E}_{d,\alpha} - L_\alpha F_{d,\alpha}$ ; then, a switched residual generator is:

$$\begin{cases} \tilde{x}_{k+1} &= A_\alpha^a \tilde{x}_k + E_{d,\alpha}^a d_k \\ r_k &= \tilde{C}_\alpha \tilde{x}_k + F_{d,\alpha} d_k \end{cases} \quad (17)$$

Defining the sensitivity functions of disturbance to the residual as:

$$T_{rd_\alpha}(z) = \tilde{C}_\alpha(zI - A_\alpha^a)^{-1} E_{d,\alpha}^a + F_{d,\alpha} \quad (18)$$

The goal of the *SPIO* in the  $\mathcal{H}_\infty$  formulation is:

$$\|T_{rd_\alpha}\|_\infty < \gamma_\alpha \quad (19)$$

The problem is: *Find the gain matrices  $L_\alpha$  that minimize  $\gamma_\alpha$  such that the *SPIO* is stable.* The *SPIO* poles can be arbitrary assigned using an  $H_\infty$ /pole placement technique. Consider the switched system, eqn. (11), if there exists matrices  $P_i > 0$ ,  $P_j > 0$ ,  $U_i$  and  $U_j$  and positive scalars  $\gamma_i$  and  $\xi_i \forall (i, j) \in \mathcal{I} \times \mathcal{I}$  such that the following *LMIs* are satisfied:

$$\begin{bmatrix} P_j - 2P_i & P_i \tilde{A}_i + U_i \tilde{C}_i & P_i \tilde{E}_{d,i} + U_i F_{d,i} & 0 \\ \star & -P_i & 0 & \tilde{C}_i^T \\ \star & \star & -\gamma_i^2 I & F_{d,i}^T \\ \star & \star & \star & -I \end{bmatrix} < 0 \quad (20a)$$

$$\begin{bmatrix} \xi_i(P_j - 2P_i) & P_i \tilde{A}_i + U_i \tilde{C}_i \\ \star & \xi_i(P_j - 2P_i) \end{bmatrix} \quad (20b)$$

$$P_\alpha > 0 \quad (20c)$$

then, there exist a *SPIO*, eqn (12), such that the  $\mathcal{H}_\infty$  and design objectives are satisfied. The *SPIO* gains are computed as:

$$L_i = P_i^{-1} U_i \quad L_{P_i} = [I_p \quad 0_{p \times n_f}] L_i \quad L_{I_i} = [0_{n_f \times p} \quad I_{n_f}] L_i \quad (21)$$

Consider the switched Lyapunov function candidate:

$$V_k = x_k^{aT} P_{\alpha(k)} x_k^a > 0 \quad (22)$$

where  $P_{\alpha(k)} > 0$  is a positive definite matrix. If such Lyapunov function exists and  $\Delta V = V_{\alpha(k+1)} - V_{\alpha(k)}$  is negative definite along system trajectories of eqn (17), then the origin of the system is globally asymptotically stable. The  $\mathcal{H}_\infty$  feasibility problem formulated in eqn (19) with the sufficient Lyapunov-stability condition can be expressed with the following inequality:

$$\mathbb{V}_k = \Delta V_k + r_k^T r_k - \gamma_i^2 w_k^T w_k < 0 \quad (23)$$

The *SPIO* for the fault estimation is implemented using the following structure:

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hat{f}_{k+1} \end{bmatrix} = \begin{bmatrix} A_\alpha - L_{P,\alpha} C_\alpha & E_{f,\alpha} - L_{P,\alpha} F_{f,\alpha} \\ -L_{I,\alpha} C_\alpha & -L_{I,\alpha} F_{f,\alpha} \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hat{f}_k \end{bmatrix} + \begin{bmatrix} B_\alpha - L_{P,\alpha} D_\alpha & L_{P,\alpha} \\ -L_{I,\alpha} D_\alpha & L_{I,\alpha} \end{bmatrix} \begin{bmatrix} u_k \\ y_k \end{bmatrix} \quad (24)$$

In this study, a non-conventional modelling of vehicle dynamics has been used. Since the velocity is a continuous varying parameter, the *LPV* method is more commonly used. In fact, within the *LPV* framework, the construction of the gain matrices and the stability analysis at each vertex will consider a common Lyapunov function, i.e. a unique  $P$  matrix. However, within the result of switching modeling, a switched Lyapunov function is considered, and thus the *LPV* result can be considered as one case of a general solution of the switched observer.



#### 4. Results

The synthesis of *SPIO* is given as the solution of the *LMI* optimization problem. The non-linear tire forces to be estimated are computed as:

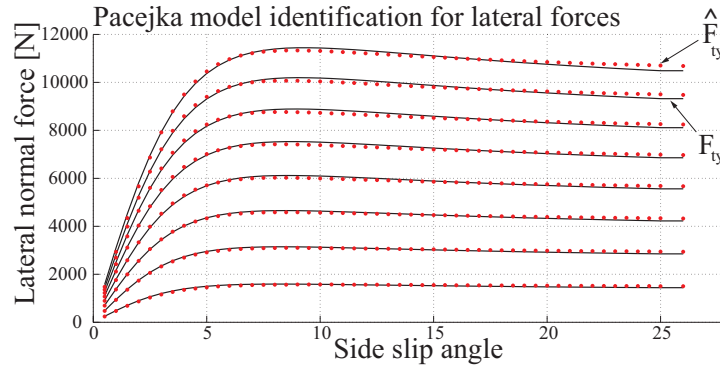
$$\Gamma_i = \sum_{j=r}^{j=l} F_{tyij} - \sum_{j=r}^{j=l} c_{yij} \beta_{yi} \quad (25)$$

and it is considered that the force is the same in the left and right side. As Fig. 2 shows,  $c_{yi}$  denotes the slope of the curve  $F_y(\beta)$  in the linear zone.

In order to obtain  $c_{yi}$  for each tire, the parameters of the Pacejka model, eqn. (26), were identified using the data from the tires curves of Fig. 2. As described in eqn (2), the lateral tire forces according to the Pacejka model depend on  $C_y$ , which is constant.  $B_y$ ,  $D_y$  and  $E_y$  are functions of  $F_{z_i}$ :

$$D_y(F_{z_{ij}}) = a_1 F_{z_{ij}}^2 + a_2 F_{z_{ij}} \quad B_y(F_{z_{ij}}) = a_3 \frac{\sin(a_4 \tan^{-1}(a_5 F_{z_{ij}}))}{C_y D_y(F_{z_{ij}})} \quad E_y(F_{z_{ij}}) = a_6 F_{z_{ij}}^2 + a_7 F_{z_{ij}} + a_8 \quad (26)$$

The non-linear least squares algorithm was used to fit the coefficients to the experimental tire data vectors  $(F_{tyij}, F_{z_i}, \beta_{yij})$ . Once these coefficients are estimated, the identification of the model is validated with *CarSim<sup>TM</sup>* data, Fig. 5, using the *Error to Signal Ratio (ESR)* as performance index. An *ESR* of 0.245 % was obtained, thus this model accurately represents the tire lateral force.



**Figure 5.** Validation of Pacejka model for lateral pneumatic efforts

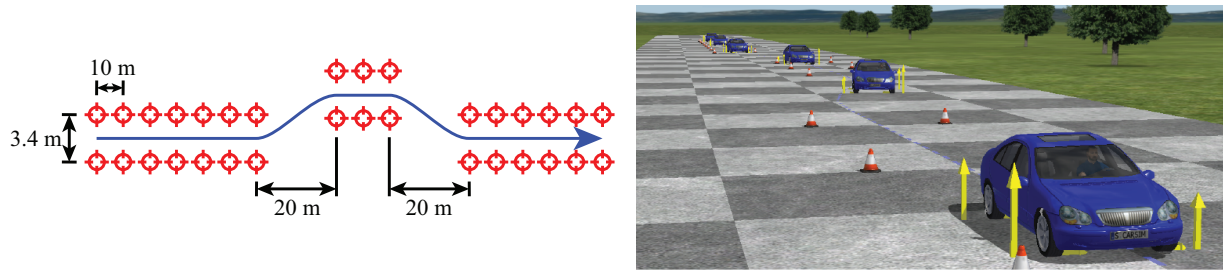
On the other hand, the vertical force on the tire  $(i, j)$  is calculated as the vehicle weight divided by 2 and weighted by the front-rear load distribution:

$$F_{z_{rj}} = \frac{1}{2}(m_s + m_{us})g \frac{l_r}{l_r + l_f} \quad F_{z_{fj}} = \frac{1}{2}(m_s + m_{us})g \frac{l_f}{l_r + l_f} \quad (27)$$

The normal force  $F_{z_{ij}}$ , eqn (27), was used to obtain the corresponding coefficients  $c_{y_i}$ , which are computed as  $c_{y_i} = B_y C_y D_y$ , [10]. A gyroscope gives the yaw rate as the measured output, the output state matrices are:

$$C_\alpha = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D_\alpha = 0, \quad F_{d,\alpha} = 0, \quad F_{f,\alpha} = 0 \quad (28)$$

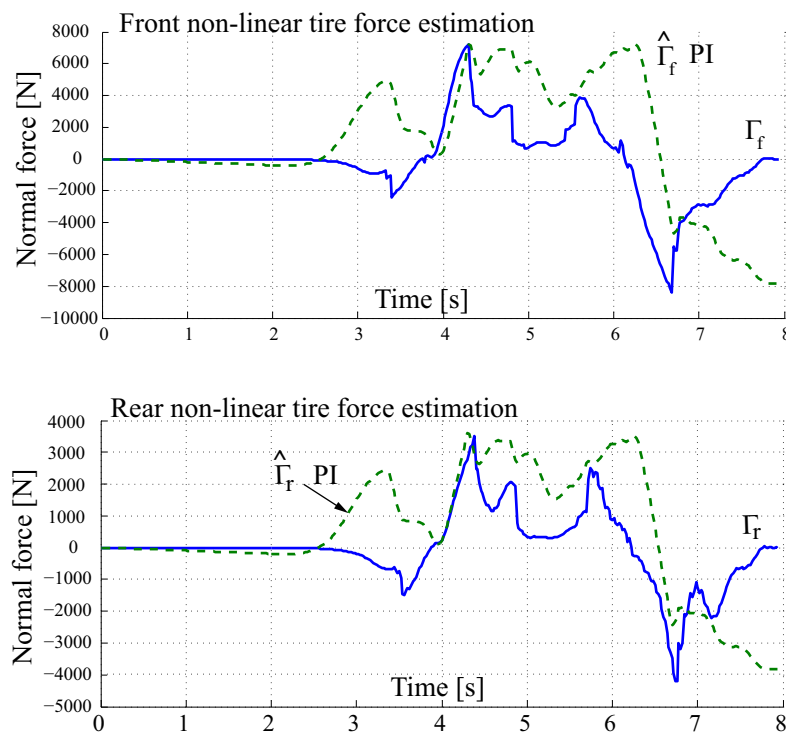
The driving scenario was the *Double Lane Change (DLC)* test, at high velocities ( $> 70 \text{ km/h}$ ). This test starts with the vehicle driving in straight line at a constant target speed. After 60 m of longitudinal displacement the driver performs an avoidance steering manoeuvre to have a lateral displacement of 3.4



**Figure 6.** Double Lane Change test

m widthin the next 20 m of longitudinal displacement; and finally, after another 20 m the driver performs a steering manoeuvre in the opposite sense to return to the original lane, Fig. 6.

Figure 7 shows a comparison of the nonlinear lateral force for the front and rear tires (solid blue) with the estimated ones with the *SPIO* (dashed green). It can be observed that at the beginning of the test, when the vehicle runs in straight line and thus the side-slip angle is close to zero, the tires operate in the linear region and the estimation is close to zero. When the vehicle perform the lane change maneuver, the side-slip increase and the tires operate in the nonlinear region. In this scenario, the estimation presents some sign error, however, it has similar dynamical characteristics of the actual nonlinear force. Therefore, even if the estimation is imperfect, this signal can be used for detection purposes, *i.e.* bounds in the nonlinear force estimation can be established to determine if the vehicle is entering in a critical condition.



**Figure 7.** Estimation of non-linear tire forces using *SPIO*

As shows the results in the different figures, the estimated signal roughly follows the actual signal, and this is due to the modeling of the tire dynamics. In fact, the tire stiffness coefficient is not constant, it depends on the tire normal force that varies due to load transfer. A robustness study on this parameter

could be performed to overcome these errors, that it is visible on the plots, but it was not studied in this paper.

## 5. Conclusions

An adaptation of fault estimation problem is used for critical driving situation detection. A signal is used as an indicator of the loss of lateral stability. This signal to be estimated as the non-linear part of the lateral force of the tires.

A model with uncertainties on the tire stiffness coefficients was obtained. This model is then used for the synthesis of *SPIO* to estimate the non-linear force of the tires.

A comparison of the estimated nonlinear normal force by *SPIO* is given with simulation results in *CarSim<sup>TM</sup>*. Results show that even if the nonlinear force estimation has error, this signal can be used to detect critical situations since the dynamical characteristics of the estimation are similar to the actual ones.

Possible future work should consist on better tire force modeling in order to take into consideration load transfers and the road properties, and extend the estimation of the zone for a coupling with existing driving assistance strategies (*ESP*, *ABS*).

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