A viscous blast-wave model for heavy-ion collisions

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Abstract. We present a generalization of the blast-wave model by incorporating viscous effects in the fluid velocity profile as well as in the Cooper-Frye freeze-out. We apply this model to study the identified particles spectra and anisotropic flow at the Large Hadron Collider (LHC). We show that this improved viscous blast-wave model leads to good description of the transverse momentum distribution of particle multiplicities and elliptic as well as triangular flow. Within this model, we estimate the shear viscosity to entropy density ratio $\eta/s \simeq 0.24$ at the LHC.

1. Introduction

The quark-gluon plasma (QGP) formed in relativistic heavy-ion collisions exhibit strong collective behaviour and hence can be studied within the framework of relativistic hydrodynamics. The hydrodynamical modelling of heavy-ion collisions suggests that the QGP behaves like a nearly perfect fluid having an extremely small shear viscosity to entropy density ratio η/s [1]. Apart from hydrodynamics, the collective behaviour of QGP can also be studied within the so-called blast-wave model [2, 3, 4, 5, 6]. Here we generalize the blast-wave model to include viscous effects by using a viscosity-based survival scale for initial geometrical anisotropies, formed in relativistic heavy-ion collisions, to parametrize the radial flow velocity. We employ this viscous blast-wave model to obtain the transverse momentum dependence of particle yields and flow harmonics for the Large Hadron Collider (LHC). We fix the model parameters by fitting the transverse momentum distribution of identified particle spectra. Subsequently, we demonstrate that this leads to reasonably good agreement with transverse momentum dependence of elliptic and triangular flow for various centralities. Within the present model, we estimate the shear viscosity to entropy density ratio $\eta/s \simeq 0.24$ at the LHC.

2. The model

We work in the Milne co-ordinate system where, $\tau = \sqrt{t^2 - z^2}$, $\eta_s = \tanh^{-1}(z/t)$, $r = \sqrt{x^2 + y^2}$, $\varphi = \mathrm{atan2}(y, x)$. The most important feature of the blast-wave model is the Hubble like parametrization of transverse velocity, $u^r \sim r$, which is found to be in agreement with hydro results [7]. In addition, we include angular anisotropy in the radial fluid velocity profile in the transverse plane. The hydrodynamic fields are parametrized as [8]

$$T = T_f, \quad u^r = u_0 \frac{r}{R}\left[1 + 2\sum_{n=1}^{\infty} u_n \cos[n(\varphi - \psi_n)]\right], \quad u^\varphi = u^{\eta_s} = 0, \quad u^\tau = \sqrt{1 + (u^r)^2}, \quad (1)$$

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where R is the transverse radius of the fireball at freeze-out. The condition $u^\mu u_\mu = 1$ leads to the expression for u^τ. In order to determine u_n, we use the fact that the initial geometrical anisotropies eventually converts to anisotropies in the radial fluid velocity. The participant anisotropies, ε_n, is defined in terms of the Fourier expansion for a single-particle distribution as

$$f(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} \varepsilon_n \cos[n(\varphi - \psi_n)]\right], \tag{2}$$

where ψ_n is the angle between the x axis and the major axis of the participant distribution. Next, we determine the conversion efficiency of the initial geometrical eccentricity to final anisotropy in the radial fluid velocity, i.e., u_n/ε_n.

We start from the dispersion relation for sound in a viscous medium, $\omega = c_s k + ik^2 \frac{1}{T}\left(\frac{2}{3}\frac{\eta}{s}\right)$, where c_s is the speed of sound in the medium, η is the coefficient of shear viscosity and $s = (\epsilon + P)/T$ is the entropy density. Using a Fourier ansatz, one finds that the amplitudes of the stress tensor harmonics, with momentum k, are attenuated by a factor [9]

$$\delta T^{\mu\nu}(t, k) = \exp\left[-\left(\frac{2}{3}\frac{\eta}{s}\right)\frac{k^2 t}{T}\right]\delta T^{\mu\nu}(0, k), \tag{3}$$

where the oscillatory pre-factor has been ignored. Each harmonics is a damped oscillator and form standing waves on the fireball circumference throughout the evolution, i.e., $k = n/R$. Note that the presence of momentum squared in the exponent results in enhanced effect of viscosity for the higher harmonics. Therefore, at freeze-out time t_f, the wave amplitude reaction is [10]

$$\frac{\delta T^{\mu\nu}|_{t=t_f}}{\delta T^{\mu\nu}|_{t=0}} = \exp\left[-n^2\left(\frac{2}{3}\frac{\eta}{s}\right)\frac{t_f}{R^2 T_f}\right], \tag{4}$$

where T_f is the freeze-out temperature. The above equation indicates a viscosity-based survival scale for anisotropic structures formed by point like perturbations. The final radial fluid velocity is due to initial geometrical perturbations and, in the absence of viscosity, the conversion efficiency remains the same for all harmonics. Therefore, for a viscous medium, the conversion efficiency must be proportional to the wave amplitude reaction,

$$\frac{u_n}{\varepsilon_n} = \alpha_0 \exp\left[-n^2\left(\frac{2}{3}\frac{\eta}{s}\right)\frac{t_f}{R^2 T_f}\right], \tag{5}$$

where α_0 is the constant of proportionality. The above equation, along with Eq. (1), leads to the fluid velocity profile which is used in the freeze-out prescription to obtain particle spectra and anisotropic flow coefficients.

3. Particle spectra and anisotropic flow

Hadron spectra can be obtained using the Cooper-Frye prescription for particle production [11]

$$\frac{dN}{d^2p_T dy} = \frac{1}{(2\pi)^3}\int p_\mu d\Sigma^\mu f(x, p), \tag{6}$$

where $d\Sigma_\mu$ is the freeze-out hyper-surface and $f(x, p)$ is the distribution function of the particles. For a system close to equilibrium, $f = f_0 + \delta f$ where $\delta f \ll f_0$. For the equilibrium distribution function, f_0, we use Fermi-Dirac statistics $(a = +1)$ for baryons and Bose-Einstein statistics $(a = -1)$ for mesons. For the non-equilibrium part, we employ the Grad's 14-moment approximation with viscous corrections up to first-order [12]

$$\delta f_1 = \frac{f_0 \tilde{f}_0}{T^3}\left(\frac{\eta}{s}\right) p^\alpha p^\beta \nabla_{\langle\alpha} u_{\beta\rangle}. \tag{7}$$

where $\tilde{f}_0 = 1 - af_0$ and the angular brackets denote traceless symmetric projection orthogonal to u^μ. In the case of blast-wave model, the form of $p^\alpha p^\beta \nabla_{\langle\alpha} u_{\beta\rangle}$ is explicitly calculated in Refs. [5, 8].

The anisotropic flow is defined as

$$v_n(p_T) \equiv \frac{\displaystyle\int_{-\pi}^{\pi} d\phi \, \cos[n(\phi - \Psi_n)] \, \frac{dN}{dy\, p_t\, dp_T\, d\phi}}{\displaystyle\int_{-\pi}^{\pi} d\phi \, \frac{dN}{dy\, p_t\, dp_T\, d\phi}}, \tag{8}$$

where Ψ_n is the event-plane angle for the n-th harmonic. Up to first order in viscosity [5],

$$v_n(p_T) = v_n^{(0)}(p_T) \left(1 - \frac{\displaystyle\int d\phi \, \frac{dN^{(1)}}{dy\, p_t\, dp_T\, d\phi}}{\displaystyle\int d\phi \, \frac{dN^{(0)}}{dy\, p_t\, dp_T\, d\phi}} \right) + \frac{\displaystyle\int d\phi \, \cos[n(\phi - \Psi_n)] \, \frac{dN^{(1)}}{dy\, p_t\, dp_T\, d\phi}}{\displaystyle\int d\phi \, \frac{dN^{(0)}}{dy\, p_t\, dp_T\, d\phi}}, \tag{9}$$

where the superscript '(0)' and '(1)' denote quantities calculated using the ideal distribution function and first-order viscous correction, Eq. (7), respectively.

4. Initial conditions and numerical results

We consider two identical relativistic nuclei with mass number A colliding with impact parameter b. The transverse expansion of the fireball is obtained by using the unperturbed radial velocity,

$$u^r \equiv \frac{dr}{d\tau} = u_0 \frac{r}{R} \quad \Rightarrow \quad \int_{r_0}^{R} \frac{dr}{r} = \int_0^{\tau_f} \frac{u_0}{R}\, d\tau \quad \Rightarrow \quad R = r_0 \exp\left(\frac{u_0\, \tau_f}{R}\right), \tag{10}$$

where $r_0 = \frac{1}{2}\left(b^2 - 2\,bR_0\sqrt{2 + b/R_0} + 4R_0^2\right)^{1/2}$ and $R_0 = 1.25A^{1/3}$ fm is the radius of each colliding nuclei [8]. In order to determine the freeze-out times for non-central collisions, we employ the analytical result of Bjorken expansion $\epsilon \propto \tau^{-4/3}$. Assuming the initial thermalization time and the freeze-out energy density to be same for all collisions, we get

$$\tau_f = \tau_{f0} \left(\frac{\epsilon_i}{\epsilon_{i0}}\right)^{3/4}, \tag{11}$$

where τ_{f0} is the freeze-out time for most central collisions which is fitted to match the transverse momentum distribution of particle multiplicities. The ratio ϵ_i/ϵ_{i0} is the initial central energy density for all centrality, scaled by its corresponding value in most central collisions. While the Bjorken estimate for the absolute values of the freeze-out times is rather crude, we have used the above equation to fix the freeze-out time for non-central collisions relative to that of the most central ones. In the present case, it seems to be a reliable approximation as is evident from Fig. 1. Therefore, in order to fit the transverse momentum spectra, the parameters that needs to be fixed within the viscous-blast wave model are the freeze-out temperature T_f, the freeze-out time for central collision τ_{f0} and the maximum radial flow velocity u_0. An interplay between α_0 in Eq. (5) and η/s is important to reproduce the transverse momentum dependence of flow harmonics.

Figure 1 shows the transverse momentum distribution of pions, kaons, and protons spectra for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in (a): $0 - 5\%$ and (b): $20 - 30\%$ centrality. We observe that, for a freeze-out temperature of 120 MeV, the spectra for π^+ and K^+ from the viscous blast-wave model are in good overall agreement with the ALICE data [13]. On the other hand, the freeze-out temperature for protons is considered to be 135 MeV in order to

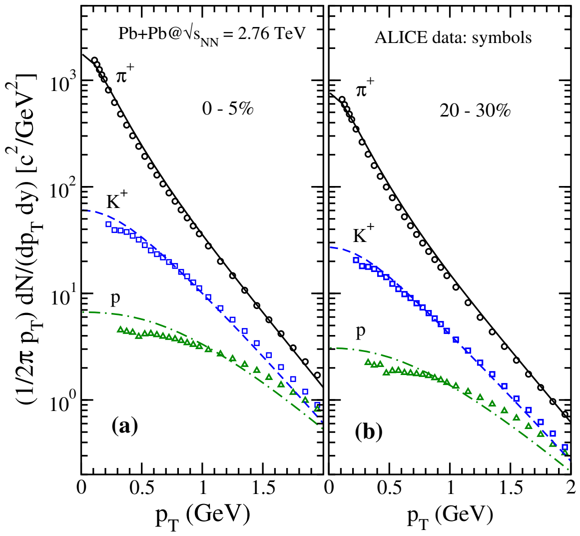

Figure 1. Transverse momentum distribution of π^+, K^+ and p multiplicities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in two centrality ranges, (a): $0-5\%$ and (b): $20-30\%$. The symbols represent ALICE data [13] at mid-rapidity and the lines correspond to viscous blast-wave results.

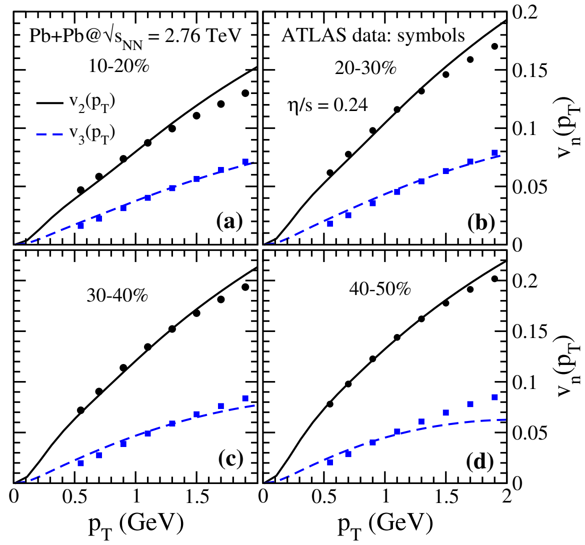

Figure 2. Transverse momentum dependence of the anisotropic flow coefficients $v_n(p_T)$ of charged hadrons, for $n = 2$ and 3, calculated at various centralities in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in the viscous blast-wave model (lines) with $\eta/s = 0.24$ as compared to the ATLAS data [14] (symbols).

fit the experimental data. Figure 2 shows our results for the $v_n(p_T)$, for various centralities, in comparison with the ATLAS data [14]. For $\alpha_0 = 0.4$ and $\eta/s = 0.24$, we find fairly good agreement with the experimental data for elliptic flow, $v_2(p_T)$, and triangular flow, $v_3(p_T)$, at the LHC. We have used the root-mean square values of initial eccentricities, ε_n, obtained from the Monte-Carlo Glauber model [15].

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