The neutron skin thickness in nuclei with clustering at low densities

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Abstract. This study concentrates on searching for a dependable, fully microscopic theory to find out new behaviours and understand their consequences for theoretical pictures. The models for nuclear structure are tested, refined and developed by acquiring new data [1][2][3]. This data is useful for astrophysical calculations and predictions. In density functional theories, including the ETF theory, the equation of state (EOS) of symmetric nuclear matter (SNM), is an important measure. Empirically, we receive information about quantities relating to SNM, all these measures are thoroughly tested. In the absence of any unswerving knowledge below this density we shall take that energy still rises up to some density, neglecting possible small fluctuations, as the density is brought down. Our discussion at the moment is without the Coulomb forces applicable only for the hypothetical nuclear matter; they are added finally to correctly portray the actual picture in nuclei. Our approach in this study is macroscopic. This work concludes that the neutron skin thickness in nuclei is found to reduce significantly, for the reason of clustering.

Introduction

We start with the statement that the binding energy of SNM per nucleon in the limit of zero density must get near $u_v (\approx 16 \text{ MeV})$ at the saturation density $\rho_0 (\approx 0.16 \text{ fm}^{-3})$, where u_v is the volume term in the Weizsäcker mass formula. The previous statement is based upon the theory that nuclear matter in its ground state (T=0 MeV) at a given density will achieve the lowest possible energy. Our statement can be demonstrated in the following procedure. It was explained in [4] that an idealized α-matter picture of SNM in the neighbourhood of zero density provides energy per nucleon, $E/A \approx -7.3$ MeV, which is the energy of α-particle per nucleon with Coulomb interaction turned off. The previous conclusion is at variance with the mean field theories, results where $E(\rho) \rightarrow 0$ as the density $\rho \rightarrow 0$, for instance, in the Skyrme-Hartree-Fock calculations. Questions may arise, why α-particles? Why not heavier nuclei that have lower energies per nucleon compared to α-particle? For example, an ideal ^{40}Ca-matter will have lower E/A than an ideal α-matter. This statement of the argument can be expanded further, because, heavier nuclei will lead to lower E/A as the Coulomb interaction is turned off. We might consequently look at SNM as an ideal gas of chunks of NM or very heavy nuclei where we can neglect the surface effects. We consider the densities low enough so that

interactions amongst clusters are negligibly small and the idea of the perfect cluster - matter is valid. In that condition, we acquire the exact result $E(\rho) \to -u_v$ as $\rho \to 0$. Hence, we get the relation

$$E(\rho \to 0) \to E(\rho = \rho_0) = -u_v. \tag{1}$$

Extended Thomas-Fermi Theory

Incorporation of (1) into a theory is far from trivial. Clearly Hartree-Fock methods cannot take into account clustering at the nuclear surface since they do not satisfy (1). Although mean-field theories do contain some aspect of clustering, they are not related to surface properties. There are cluster models of nuclei and hypernuclei in terms of specific clusters, mostly α-particle clusters, but these are confined to light nuclei [5]. On the other hand, we have *ab initio* theories which give accurate or nearly exact results, but they are also confined to light nuclei [6]. In the future, it may become possible to extend *ab initio* theories to heavy nuclei with realistic Hamiltonians, but it would be difficult to entangle cluster properties, particularly its universal character, if any, at the nuclear surface. Recently, there has been a spurt of activity in developing microscopically based nuclear density-functional theories [7]–[9] for medium and heavy nuclei with reference to realistic two- and threenucleon or Skyrme interactions. The realistic interactions used are either the Argonne AV18 (two-body) [10] and Urbana IX (three-body) [11] or they are derived from chiral effective field theory. These studies have started with the motivation to search for a universal nuclear energy density functional - a new model for nuclear theory [12], which in principle is supposed to have all the many-body correlations. But none of these theories, because of constraints or choice of density functional, conform to identity (1); on the contrary, they give $E(\rho)\to 0$ as $\rho\to 0$. They do not include the clustering aspect in nuclei to the full extent. Besides, on the basis of fits to energy alone it will be difficult to see the impact of (1) unless one probes those properties of nuclei which are sensitive to surface region, such as the neutron skin thickness, symmetry energy at low densities, and the one-neutron separation energies for those pair of nuclei where separation energies are small.

To find a detailed structure of the EOS of SNM with GFMC/DMC methods at low densities is a formidable task and beyond the reach of present-day techniques and computing power. We thus resort to phenomenology and approximate methods. We use an extended version of the Thomas-Fermi model (ETF), which we believe is the only theory at present which can incorporate (1) easily and at the same time describe the properties of a large number of nuclei. The ETF theory provides quick and reliable estimates of energies and other physical quantities of interest with a global insight. This implementation not only explains the experimental large values of symmetry energy [13] at low densities, but also affects the binding energies of nuclei, one- and two-neutron separation, and β-decay energies in the right direction, although by a modest amount. But the influence of clustering on the neutron skin thickness and one-neutron separation energies for those pairs of nuclei with separation energies less than 5 MeV is significant. The latter improvement is crucial for describing the nuclei near the drip line.

Because of cluster formation, the neutron and proton densities at the nuclear surface would tend to equalize each other. This tendency to equalize results in reduced neutron skin thickness as compared to when clustering is absent. We do find considerable reduction in neutron skin thickness. It has bearings on the recently concluded parity-violating electron-scattering experiment on 208Pb and in turn on the neutron star radii [14].

Results and Discussion

We present results for the neutron skin thickness and other relevant quantities. Discussion follows the results as they are described. Finally, we present an overall critique of the results.

A quantity of interest is the neutron skin thickness [15], defined as the difference between the rms radii of neutrons and protons:

$$\delta R = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} \, . \tag{2}$$

In Skyrme-HF theories, δR is sensitive to the slope of the symmetry energy L at the saturation density, defined as

$$L = 3\rho_0 \left. \frac{\partial E_{sym}}{\partial \rho} \right|_{\rho_0} . \tag{3}$$

We expect the clustering to affect δR significantly as it is a direct surface phenomenon. In Table I we give results of our calculations for clustering and no clustering. Results for no-clustering are in reasonable agreement with Skyrme-HF [16], [17] and RMF [18] calculations, and experimental deductions [19], [20] but there is clear discrepancy between the clustering and the other results including the experiment. We find much lower values for δR. Does this imply that experimental deductions are implemented assuming no clustering? They are indeed model dependent. A recently completed parity-violating electron-scattering experiment at the Jefferson Laboratory will greatly help to clarify this inconsistency. Our L value for both cases of clustering and no clustering is the same: $L \approx 68$ MeV, well within the range of values extracted from isospin diffusion data.

From Table VI [2], we can see that increase in $\sigma(E)$ is not very large compared to the results for 367 spherical nuclei. On average it increases by 0.08 MeV, still within the reasonable limits from the viewpoint of microscopic-macroscopic theories. But $\sigma(R)$ increases by a factor of 2. We do not have any clear explanation for this discrepancy. We may attribute this increase to the absence of self-consistency and particularly, to deformation in our formulation. We also give the *rms* deviations $\sigma(S1)$, $\sigma(S2)$ and $\sigma(Q_\beta)$ for one-($S1$) and two-($S2$) neutrons separation, and β-decay energies, respectively.

Correct prediction of one- and two-neutron separation energies is important from the point of view of neutron drip line. One neutron separation energies govern the asymptotic density of neutrons [21], [22], whereas the two-neutron separation energies reveal the shell structure in an isotopic chain [23]. It is seen in Table VI [2] that these quantities, which are, $\sigma(S1)$, $\sigma(S2)$ and $\sigma(Q_\beta)$, are quite reasonable. These are somewhat larger for the no-clustering case (line 2) implying that clustering does affect the fits though quite modestly; it was more so in case of 367 spherical nuclei the difference is somewhat significant. Line 5 in Table VI [2] gives the results with Wigner term excluded. The value of $\sigma(E)$ increases to 0.12 MeV. This demonstrates the importance of Wigner term. Our preferred results are for $AV8' + V_{2\pi}^{PW} + V_{\mu=300}^{R}$ (line 3) and $AV8'$ (line 4), since the *rms* deviations for these are small compared to other results. Last line of the Table VI [2] give results for calculation with $K = 260$ MeV, which demonstrates that dependence on K is weak. It is largely compensated by a change in the parameter M.

Table I Results for neutron skin thickness for a number of nuclei. All items are in fermis. Outcomes for no-clustering correlate with the second row of Table VI [2]. Outcomes for clustering correlate with the averages of the first, third and the fourth rows of Table VI [2]. Observe the significant deduction in δR when clustering is involved.

Nucleus	δR (fm), neutron skin thickness					
	Clustering	No-clustering	HFB-17 [24]	Sky-HF [16], [17], [25], [26]	RMF [18]	Experiment
^{208}Pb	0.10	0.16	0.15	0.22±0.04	0.21	0.16±0.06 [27] 0.18±0.035 [28]
^{132}Sn	0.15	0.23	–	0.29±0.04	0.27	0.24±0.04 [28]
^{124}Sn	0.10	0.16	–	0.22±0.04	0.19	0.185±0.017 [20]
^{90}Zr	0.04	0.07	–	0.09±0.04	–	0.07±0.04[19], [29]
^{48}Ca	0.10	0.16	–	–	–	–

In Table I, we give results of our calculations for clustering and no-clustering. The results for no-clustering are in reasonable agreement with Skyrme-HF [16], [17], [25], [26] and RMF [18] calculations, and experimental deductions [19], [20], [29]. However, there is clear discrepancy between the clustering and the other results including the experiment. We find much lower values for δR. Does this imply that experimental deductions are implemented assuming no-clustering? In fact, they are model dependent. A recently completed parity-violating electron scattering experiment [30] at the Jefferson Laboratory will greatly help to clarify this inconsistency. Our L value for both the cases of clustering and no-clustering is similar; $L \approx 68$ MeV well within the range of values taken from isospin diffusion data.

Clustering at low densities significantly reduces the neutron skin thickness in nuclei. The conventional Skyrme density functionals do not incorporate clustering at low densities; thus their estimates of neutron skin thickness can be considerably off. Also, since they do not contain a quartic isospin term, it would be difficult to explain quantitatively the nuclear binding energies in a fully microscopic density-functional theory and at the same time conform to the realistic EOS of neutron matter, particularly in situations where they are steep, for example with UIX.

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