

Screening length in dusty plasma crystals

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Abstract. Particles interaction and value of the screening length in dusty plasma systems are of great interest in dusty plasma area. Three inter-particle potentials (Debye potential, Gurevich potential and interaction potential in the weakly collisional regime) are used to solve equilibrium equations for two dusty particles suspended in a parabolic trap. The inter-particle distance dependence on screening length, trap parameter and particle charge is obtained. The functional form of inter-particle distance dependence on ion temperature is investigated and compared with experimental data at 200–300 K in order to test used potentials applicability to dusty plasma systems at room temperatures. The preference is given to the Yukawa-type potential including effective values of particle charge and screening length. The estimated effective value of the screening length is 5–15 times larger than the Debye length.

1. Introduction

Dusty plasma is ionized gas containing nanometer to millimeter sized particles of condensed matter. Dust particles can acquire a high electric charge (up to $10^4 e$) due to the different mobility of electrons and ions [1–3]. Charge depends strongly on surrounding plasma parameters and presence of other highly charged dust particles. The process of charging has been studied in [1–5]. Big charges cause strong inter-particle interaction which leads to a natural appearance of gaseous, liquid and crystalline states in dusty plasma systems [4]. There are a lot of theoretical models describing inter-particle interactions [1, 3, 6, 7] but none of them explains all known experimental facts (rotation of dust-particle structures, crystalline grid geometry, phase transitions and so on). Difficulties appear because the interaction potential is multiparticle and shielding is non-linear in most cases. That is why inter-particle interactions are still of a great interest in dusty plasma area.

In this article, inter-particle distance connection with parameters of the surrounding plasma (mainly ion temperature) is investigated. Equilibrium equations are solved for two particles suspended in a vertical tube in a parabolic electrostatic trap. Three potentials are used to describe inter-particle interaction: linear approach gives the standard Debye–Hückel potential as the solution of the Poisson’s equation [1], non-linear approach with additional conditions gives the Gurevich potential [5] and accounting ion-neutral collisions gives the interaction potential in the weakly collisional regime [7, 8]. The results are compared with experimental data in order to test potentials applicability to real dusty plasma crystals at room temperatures. In the second section, the theoretical models are described. The third section is devoted to the solutions of equilibrium equations. The fourth section is about the comparison with experimental results.



2. Theoretical models

Spherical dust particles suspended in a vertical tube are under consideration. In the vertical direction the gravitational force is counterbalanced by the vertical electric field $\vec{E}$ which may be the result of positive column stratification in dc glow discharge. In the horizontal direction only particle interaction and the field of the electrostatic trap are taken into consideration at a first approximation. Trap-potential is considered to be parabolic $U = (1/2)\alpha\vec{r}^2$, where α is a trap parameter, $\vec{r}$ is radius vector of dust particle from the minimum of trap potential. The distribution of electric potential $\phi(\vec{r})$ around an individual spherical particle of charge Q satisfies the Poisson equation $\Delta\phi = -4\pi(n_i - n_e)$ with the boundary conditions $\phi(\infty) = 0$, $\phi(a) = \phi_0$, where a is the particle radius. The solutions of this equation under different assumptions are used.

- Debye–Hückel (Yukawa) potential. The assumption of Boltzmann distributions for ions and electrons densities gives the solution of the linearized Poisson's equation as

$$\phi(r) \approx \frac{Q}{r} \exp\left(-\frac{r}{\lambda}\right), \quad (1)$$

where λ is the screening length in plasma, in these conditions $\lambda = \lambda_D$. Linearization is often invalid in dusty plasmas due to the high value of the particle potential $\phi_0 \approx -T_e/e$, nevertheless numerical solution of non-linear Boltzmann–Poisson equations shows that this functional form persists for large distances but requires substituting of Q and λ with their effective values.

- Gurevich potential. The first approximation to the non-linearized case is given in [5]. Investigation shows that in this case potential is larger at small distances but then it becomes zero at a finite distance. Further it is matched with the Yukawa solution which takes into account presence of other particles because in most experiments inter-particle distance is not so large as compared to non-linear screening distances. Simple explicit expression for Gurevich Polarization potential is given as

$$\phi = \frac{Q}{r} \left(1 - \frac{r}{\lambda}\right)^4, \quad (2)$$

where λ is the equivalent of the screening length for the Yukawa-type potentials.

- Interaction potential in the weakly collisional regime [7, 8]. Ion absorption by particles and ion-neutral collisions impact screening. Taking them into account leads to the expression

$$\phi(r) = \phi_1(r) + \phi_2(r) = \frac{Q}{r} \exp(-k_D r) - \frac{e}{r} \int_0^\infty \frac{k \sin(k_D r) f(\theta)}{k^2 + k_D^2} dk, \quad (3)$$

where

$$f(\theta) = \frac{8n_0}{\pi^{3/2}k_D} \frac{\int_0^\infty \sigma(\xi) \xi^2 \arctan(\frac{\xi}{\theta}) \exp(-\xi^2) d\xi}{1 - \sqrt{\pi} \theta \exp(\theta^2) [1 - \text{erf}(\theta)]},$$

k_D is the inverse linearized Debye radius,

$$\theta = \frac{\nu}{\sqrt{2}k v_{T_i}}, \quad \xi^2 = \frac{\nu^2}{2v_{T_i}^2},$$

ν is the effective ion-neutral collision frequency, n_0 is the ion density. In the weakly collisional regime when $l_i \geq \lambda_D$ where l_i is ion mean free path, λ_D is the Debye length, the second term in (3) is approximately

$$\phi_2(r) \approx -\frac{e}{r} \frac{\sqrt{\pi}}{4\sqrt{2}} \frac{J_i}{k_D v_{T_i}} \left(F(k_D r) + \frac{0.60\pi^{3/2}}{l_i k_D} [1 - \exp(-k_D r)] \right), \quad (4)$$

where J_i is ion flux to the particle, $F(x) = \exp(-x)E_i(x) - \exp(x)E_i(-x)$. This expression is used to solve the equilibrium equation.

3. Theoretical results

The equilibrium equation for a system containing 2 particles is

$$F_{\text{trap}} = F_{\text{electrostatic}}, \quad (5)$$

where $F_{\text{trap}} = -\partial U_{\text{trap}}/\partial r$, $F_{\text{electrostatic}} = -\partial \phi/\partial r$. These equations are solved with respect to r for all three given potentials assuming that the distance between particles approximately equals the screening length (so that $\delta r/\lambda \ll 1$). This assumption is due to the weakness of particle electric field outside the sphere with radius $R = \lambda_{\text{scr}}$ (field potential falls exponentially inside of this sphere in Debye–Hückel model, for example).

The solutions of the equilibrium equations are:

- for the Debye–Hückel potential:

$$r_D = \lambda \frac{1}{\frac{\exp(1)\alpha\lambda^3}{7Q} + 5/7}; \quad (6)$$

- for the Gurevich potential:

$$r_G = \lambda(1 + \lambda^3 \sqrt{\frac{\alpha}{4Q^2}}); \quad (7)$$

- for the interaction potential in the weakly collisional regime:

$$r_Y = \lambda \left(1 + \frac{\frac{2(Q-bc)}{\lambda^2 \exp(1)} - \frac{bc}{\lambda^2} - 1.4 \frac{b}{\lambda^2} - \alpha\lambda}{\frac{5(Q-bc)}{\lambda^2 \exp(1)} - 2 \frac{bc}{\lambda^2} - 5 \frac{b}{\lambda^2} + \alpha\lambda} \right), \quad (8)$$

where

$$b = -e \frac{\sqrt{\pi}}{4\sqrt{2}} \frac{J_i}{k_D v_{T_i}}, \quad c = \frac{0.60\pi^{3/2}}{l_i k_D}.$$

In assumptions that:

- ion temperature equals neutral component temperature $T_i = T_n = T$ because ions and neutrals masses are almost equal (differ by one electron mass) and electric fields taken into consideration are less than 10 V/cm [9];
- Q and α do not depend on T which is a reasonable first approximation as they are mainly defined by electron temperature [1]

$$Q \approx \frac{kT_e a}{e}, \quad \alpha \approx \frac{1}{r} \frac{T_e}{e} \frac{1}{n_e} \frac{\partial n_e}{\partial r} = \left(\frac{2.4}{R} \right)^2 \frac{(3/2)kT_e}{2e},$$

where R is the tube radius [10]) and electron temperature $T_e \propto \frac{E}{N} \approx \text{const}$ for dc glow discharge positive column [9, 11–13];

- the Debye length is

$$\lambda = \sqrt{\frac{kT_i}{8\pi n_i e^2}} \propto \sqrt{T}$$

as ions concentration is defined by ionization frequency which depends on electron temperature and ionization potential [9];

- $l_i = \frac{1}{n_i \sigma} \propto T$, $v_{T_i} = \sqrt{\frac{2kT_i}{m}} \propto \sqrt{T}$,

inter-particle distance depends on temperature as:

- for the Debye–Hückel potential:

$$r_D = AT^{1/2} \frac{1}{BT^{3/2} + 5/7}; \quad (9)$$

- for the Gurevich potential:

$$r_G = CT^{1/2}(1 + DT^{1/2}); \quad (10)$$

- for the interaction potential in the weakly collisional regime:

$$r_Y = ET^{1/2} \left(1 + \frac{F/T - GT^{1/2}}{GT^{1/2} + H/T} \right). \quad (11)$$

Here, A , B , C , D , E , F , G , H are numeric constants that are counted by experimental graphs approximation.

4. Comparison with experimental results

Experimental data was taken from [14] and from the experiments conducted in JIHT RAS in 2010–2012 by I S Samoylov, V P Baev *et al* [15]. Video clips of observed dust particle structures and experimental parameters records were generously provided by I S Samoylov and V P Baev who also gave important comments on details of the experiment. The inter-particle distance has been measured by the video sequence. Results have been combined with points from S N Antipov [14]. Functional connection between inter-particle distance and temperature has been plotted for temperatures from 200 to 275 K and pressure 8.5 mBar. Then it has been approximated by the obtained expressions (9)–(11). The value of the screening length in dusty plasma has been estimated by the ratio of the value of the numeric coefficient μ standing before $T^{1/2}$ in (9)–(11) to the value of this coefficient for the Debye length. For the Debye potential $\mu = A$, for the Gurevich potential $\mu = C$, for the widened Yukawa potential $\mu = E$, for the Debye length in helium plasma

$$\mu = \sqrt{\frac{k}{8\pi n e^2}} \approx 1.5 \frac{\mu\text{m}}{\text{K}^{1/2}},$$

if ion concentration value is taken as standard for the glow discharge positive column $n = 10^9 \text{ cm}^{-3}$.

In figure 1 the example of approximation for the Debye–Hückel potential is shown. The value of the screening length in this case is $\lambda_{\text{scr}} \approx 8\lambda_D$. The approximation line is in good agreement with the experimental error which is 20–30%. For lower temperatures fitting by expressions (9)–(11) is not satisfactory. It may be connected with appearance of metastables and subsequent change of ions concentration.

The value of the screening length in dusty plasma for all three potentials in the temperature range 200–275 K is $\lambda_{\text{scr}} \approx (5 - 15)\lambda_D$. This numeric coefficient well coincides with the average value of the ratio of inter-particle distances in the experiment to the Debye length.

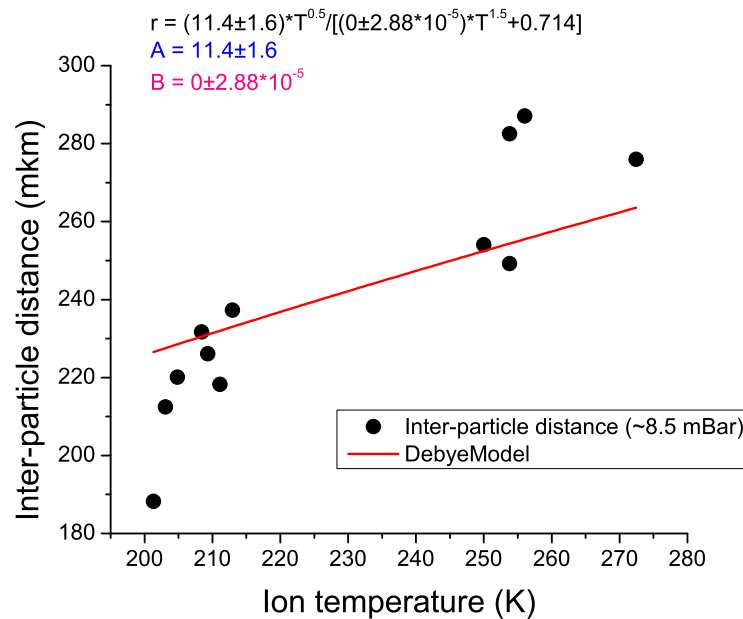


Figure 1. Approximation of the experimental data for the temperature range 200–275 K and pressure 8.5 mBar for the Debye–Hückel potential.

5. Conclusions

The estimated value of the screening length in dusty plasma is in one order of magnitude larger than the ion Debye length.

Good experimental data fitting is provided by the solution of the power equilibrium equations for the Debye–Hückel potential with the effective screening length $\lambda_{scr} \approx 8\lambda_D$ in the temperature range 200–275 K for pressure 8.5 mBar, at lower temperatures expressions (9)–(11) only reflect the tendency of inter-particle distance decrease for fixed pressures. The power equilibrium equation solution for the potential described by a superposition of two exponentials with different screening constants [11] will also be studied. Horizontal and vertical directions will be analyzed separately in the experimental video clips. This investigation is important for the correct description of structures in dusty plasma.

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