Energy transfer between degrees of freedom of a dusty plasma system

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Abstract. Dust particles under certain conditions can acquire kinetic energy of the order of 10 eV and higher, far above the temperature of gas and temperatures of ions and electrons in the discharge. Such heating can be explained by the energy transfer between degrees of freedom of a dusty plasma system. One of the mechanisms of such energy transfer is based on parametric resonance. A model of dust particles system in gas discharge plasma including fluctuations of dust particles charge and features of near-electrode layer is presented. Molecular dynamics simulation of the dust particles system is performed. Conditions of the resonance occurrence are obtained for a wide range of parameters.

1. Introduction

Dust particles in gas discharge plasma may have an abnormally large kinetic energy [1–14]. Furthermore, horizontal and vertical kinetic energy of the dust particles may be different [3–6, 15–18]. So phenomenon and mechanisms of an energy transfer between degrees of freedom of a dusty plasma system are of great interest in the field of dusty plasma. One of such mechanisms [6, 18–22] is based on parametric resonance [3–5, 16, 23]. The charged dust particles oscillate in the sheath of the gas discharge. The dust particle charge is determined by electron and ion fluxes on the surface of dust grain. The concentration of electrons and ions depend on the distance from the electrode, so periodical vertical motion of dust particles in the sheath leads to synchronous oscillations of the dust particles charge [24]. The mechanism of energy transfer based on such phenomena and parametric resonance is studied in [3–5, 16].

Articles [3,4] are focused on studying of the equilibrium kinetic energy. In [5,16] a Tailor series expansion is used and only the most significant terms are taken into account. This approach works only when amplitudes of particles oscillations are much smaller than interparticle distance. So only initial stages of energy transfer between degrees of freedom can be described using this method, but not the saturation stage.

In the second section a model of dust particles system in gas discharge plasma including fluctuations of dust particles charge and features of near-electrode layer is presented. In the third section conditions of the resonance occurrence and equilibrium horizontal and vertical kinetic energy are presented. Obtained results are discussed in the fourth section. Conclusions are performed in the last section.

2. Equations of dust particles motion

The motion of N dust particles, forming a single a horizontal layer in the gas discharge near-electrode layer is considered. Dust particles in gas-discharge plasma have a significant negative charge $Q = Ze$, where Z is a charge number and e is electron charge. Dust particles are oscillating in the near-electrode layer near values of z for which electric field $\mathbf{E}(z)$ is strong enough to compensate gravity force $\mathbf{F}_{\text{grav}} = m\mathbf{g}$. The starting point of z is the point where gravity is balanced by the electric force:

$$Q\mathbf{E}(0) = m\mathbf{g}. \tag{1}$$

The electric field $\mathbf{E}$ depends on the vertical coordinate z in the near-electrode layer of gas discharge. Since dust particles are forming a single horizontal layer their deviations from the equilibrium position is small so this dependence is considered to be linear:

$$\mathbf{E}(z) = \mathbf{E}(0)\left(1 + e'z\right), \tag{2}$$

where e' is a normalized vertical gradient electric field component in the near-electrode layer.

The dust particle charge [23, 25, 26] is determined by electron and ion fluxes on the surface of dust grain, which depends on electric field. So the dust particle charge depends on z:

$$Q(z) = Q_0\left(1 + q'_z z\right), \tag{3}$$

where Q_0 is an average charge of a dust particle and q'_z is a normalized vertical gradient of the dust particle charge, caused by changes in the concentrations of electrons and ions due to the electric field change in the near-electrode layer.

Ions and electrons are absorbed (emitted) by the surface of dust particles discretely in random times and order. Thus, the charge of dust particles in plasma stochastically fluctuates in time:

$$Q(z) = Q_0\left(1 + \delta q(t)\right), \tag{4}$$

where the normalized charge fluctuations $\delta q(t)$ is described by the correlation

$$\left\langle \delta q(t)\,\delta q(t')\right\rangle = \delta q^2 \exp\left[-\Omega\left(t - t'\right)\right], \tag{5}$$

where Ω^{-1} is the characteristic relaxation time of the small deviations of the charge and δq is normalized amplitude of charge fluctuations.

Combining equations (3) and (4) in the final expression for dust particle charge:

$$Q(z,t) = Q_0\left(1 + q'_z z + \delta q(t)\right). \tag{6}$$

Considering equations (2) and (6) the electric force is

$$\mathbf{F}_{\text{el}} = Q_0\mathbf{E}(0)\left(1 + q'_z z + \delta q(t)\right)\left(1 + e'z\right). \tag{7}$$

Since plasma is weakly ionized, the main contribution to the friction force is given by the neutral particles. So it can be simulated by the Langevin thermostat:

$$\mathbf{F}_{\text{fr}} = -2m\lambda\dot{\mathbf{r}} + \sqrt{4m\lambda k_B T_n/\Delta t}\,\mathbf{h}(t), \tag{8}$$

where λ is a friction coefficient, k_B is the Boltzmann constant, $\mathbf{r} = \{x, y, z\}$ is a three-dimensional position vector of the dust particle, T_n is the temperature of the neutral gas, Δt is integration step, and $\mathbf{h}(t)$ is a normally distributed stochastic variable.

The dust particles interaction potential is chosen in the form of the Yukawa potential:

$$U_{\text{ij}}(\mathbf{r}_i - \mathbf{r}_j) = Q_i Q_j \exp(-k|\mathbf{r}_i - \mathbf{r}_j|) / (\mathbf{r}_i - \mathbf{r}_j), \tag{9}$$

where k is a screening parameter. The trap potential that holds charged dust particles from spreading horizontally is a parabolic $U_{\text{trap}} = \varepsilon\left(x^2 + y^2\right)$, where ε is a trap-potential parameter.

So the motion of dust particles in gas-discharge plasma is described by the system of equations

$$m\ddot{\mathbf{r}}^i = \mathbf{F}^i_{\text{inter}} + \mathbf{F}^i_{\text{trap}} + \mathbf{F}^i_{\text{fr}} + \mathbf{F}^i_{\text{el}} + \mathbf{F}^i_{\text{grav}}. \tag{10}$$

3. Extended Mathieu equation and energy transfer in dusty plasma

As it was shown in [3–5, 16] initial stages mechanism of energy transfer between degrees of freedom of dusty-plasma system can be described by extended Mathieu equation:

$$\ddot{x} + 2\lambda\dot{x} + \omega_0^2(1 + h\cos\omega_p t) = \eta(t), \tag{11}$$

where λ is a friction coefficient, ω_0 is a frequency of horizontal oscillations, $\omega_p = 2\omega_z \approx \sqrt{-g\left(e' + q_z'\right)}$ is doubled frequency of vertical oscillations [3], $\eta(t)$ is stochastic force, which obeys a normal distribution law with variance determined by the characteristics of neutral gas and the amplitude of fluctuations of the dust particles charge, and h is a coefficient, determined by system parameters. It can be approximated [3] as

$$h \approx -aA_z^2/\omega_0^2, \tag{12}$$

where A_z is an amplitude of vertical oscillations and

$$a \approx \frac{Q_0^2 \exp\left[-k\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle\right]}{m\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle^5}\left[12 + 12k\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle + 5\left(k\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle\right)^2 + \left(k\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle\right)^3\right]. \tag{13}$$

Amplitude of stochastic force $\eta(t)$ doesn't affect conditions of the resonance occurrence or the grows rate of energy [16]. So charge fluctuations may substantially affects only amplitude of vertical oscillations A_z, but not the mechanism of energy transfer between degrees of freedom itself.

So, using these approximations, results obtained by modeling can be compared with ones obtained in [5, 16].

4. Results of modeling

Using molecular modeling approach [27] dependence of horizontal and vertical energy on time was obtained for a various sets of parameters. Graphics presented in this paper are corresponding to the next one: $\lambda = 0.01$ s^{-1}, $\delta q = 0.1$, $\Omega = 4 \times 10^4$ s^{-1}, $Z = 10^4$, $e' = -40$ cm^{-1}, $\varepsilon = 0.08$, $k = 30$ cm^{-1}, $q_z' = 0.03$ cm^{-1}. Since power spectra of the vertical kinetic energy of such dust particles system has one strongly marked peak (figure 1), it can be considered, that only one harmonic of vertical oscillations is heating horizontal oscillations. Since kinetic energy is proportional to velocity squared and oscillates twice more frequent than particles, frequency of this harmonic is $\nu_z \approx 31.5$ Hz which is equal to the theoretical approximation.

On (figure 2) energy of horizontal oscillation obtained by modeling and the growth rate of energy obtained by numerical solving of extended Mathieu equation is shown. It shows that

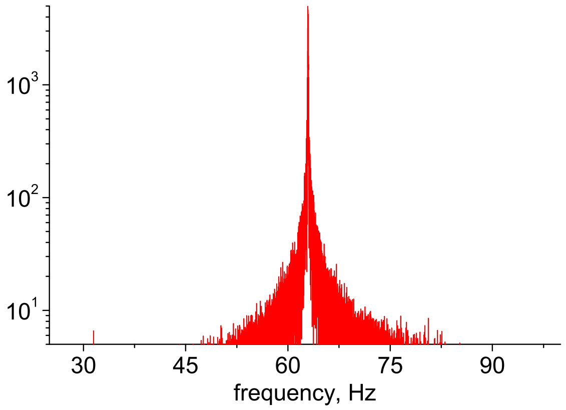

Figure 1. Power spectra of the vertical kinetic energy of dust particles system.

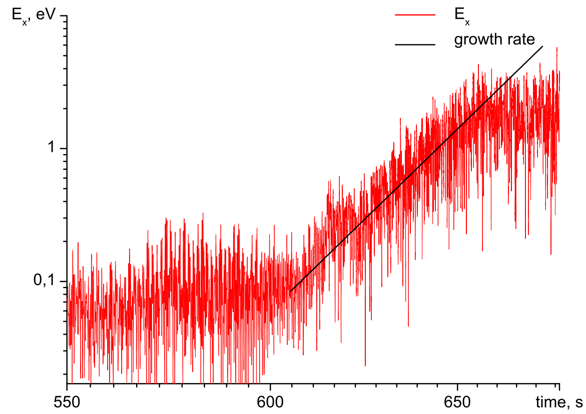

Figure 2. Energy of horizontal oscillations obtained by modeling and numerical solving of the extended Mathieu equation.

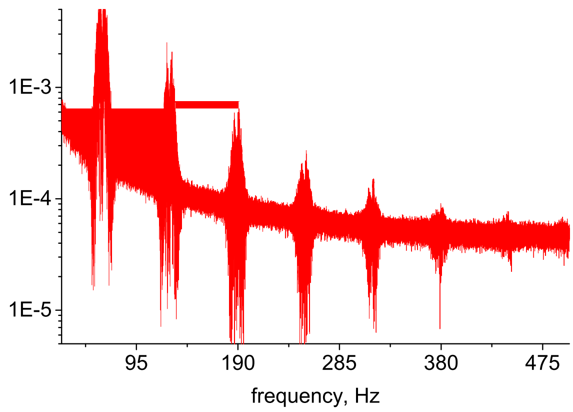

Figure 3. Power spectra of the horizontal kinetic energy of dust particles system.

grows rate of energy is constant for the growth stage and can be obtained by solving extended Mathieu equation for corresponding parameters.

Power spectra of the horizontal kinetic energy of dust particles system obtained by modeling (figure 3) is also in agreement with results obtained by solving extended Mathieu equations. Location and width of these seven peaks are corresponding to resonance regions of the extended Mathieu equation. So, extended Mathieu equation is describing growth rate of energy and specter of heated harmonics of horizontal oscillations, but not the saturation stage.

5. Conclusions

Mechanism of energy transfer in dusty plasma based on parametric resonance is described. Model describing dust particle motion is proposed. Using molecular modeling heating of horizontal motion of dust particles caused by energy transfer between degrees of freedom is shown. Conditions of resonance occurrence and growth rates of energy are in agreement with results obtained by solving the extended Mathieu equation in a wide range of parameters. Width of spectral peaks obtained by modeling corresponds to width of the resonance region of the extended Mathieu equation obtained for the same parameters. Saturation stage is shown.

Obtained results allow describing energy transfer between degrees of freedom of dusty plasma system more accurate.

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