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Neutron yield and Lawson criterion for plasma with inertial electrostatic confinement

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Abstract. The physics of plasma formation is discussed in the systems with inertial electrostatic confinement (IEC) during the convergent to the axis of cylindrical geometry of the ion flow accelerated periodically in the field of virtual cathode, which is formed by the injected electrons. The ranges of plasma parameters and the resulting neutron yield are determined for different modes of ion flux formation. The requirements are formulated to the technical parameters of the system with IEC to create both a powerful neutron source with a rate of generation exceeding 10^{10}–10^{12} particles/s and to achieve a positive energy output (analogue of Lawson criterion).

1. Introduction

Interest in the inertial electrostatic confinement (IEC), the first proposals of that have been formulated in [1–8], due to the fact that such an approach, probably, will require less complicated hardware base and, by this reason, it will be less expensive way compared with laser or heavy ion inertial fusion as well as with magnetic confinement fusion. Thermonuclear neutron source with a generation rate exceeding 10^{10}–10^{11} particles/s is an important object [9] of investigation in the field of IEC, in addition to the energy aspect.

This work is devoted to theoretical analysis of the main physical processes of modern IEC-scheme based on the concept of periodically oscillating plasma spheres (POPS) [10–13]. This approach consists in periodic localization of ions near the center of spherical system or the axis of cylindrical one due to multiple ion bunch oscillations in the field of virtual cathode formed by the stream of injected fast electrons. The most important element of the POPS-concept is the electron injection into the interior of grid-like cathode to create a potential, in which the ions could perform harmonic oscillations [10]. The ion oscillations in the virtual cathode created under the braking and mutual repulsion of electrons at the system center, provide periodic convergence (collapse) of ion bunch at the bottom of potential well of the virtual cathode [13–17]. In the paper we consider a cylindrical IEC-system (with inverse polarity [6,15]) where the role of space charge effect is insignificant in comparison with a spherical electrostatic trap [18], although the latter one meets less stringent requirements to achieve a positive energy output.

As a one of the important directions of the cylindrical POPS-type system application should be mentioned the experiments [14–17] directed to investigation of a miniature nanosecond vacuum discharge (NVD) with using a deuterated Pd-anode. In these experiments it had been demonstrated the version of a virtual cylindrical cathode, where deuterium nuclei can be accelerated to energies of tens of keV, and where the pulsating DD-neutron yield had been observed [19, 20]. Such a discharge operation mode is a certain analogue of POPS [10] that was confirmed by a detailed numerical simulation of experiments with NVD [19, 20] as well as by coincidence of the measured frequency of neutron yield pulsation in NVD with the calculation of ion oscillation frequency [15] in the framework of POPS theory [10]. Already at this stage in the experiment with NVD, the virtual cathode potential wells of tens of kilovolts, the sizes of the virtual cathode of fractions of centimeter, and the ion oscillation frequencies of about 80 MHz (closed to the frequencies observed of the neutron yield pulsation) were implemented [16, 19].

Fusion reactions occurs when the maximum flux density of deuterium nuclei passing through the axis of symmetry of cylindrical POPS-type system. The total reaction time, which is the analogue of time of magnetic or inertial confinement, is determined by the number of oscillations of ion bunch $N_{\rm osc} = \tau\nu$, which can be provided by maintaining the existence of a virtual cathode in a single operation cycle of device (τ is the duration of operation cycle, ν is oscillation frequency). Usually, the problem is considered in terms of fusion energy yield in a single act of the oscillations and total duration of the ion oscillations cycle.

Below the analytical model of interaction between oscillating ion flows in cylindrical POPS-like system (section 2) and neutron yield generation (section 3) in a single act of the oscillations is developed. The criterion to achieve a positive energy output of the system in the mode of many oscillations is discussed for the case of direct ion injection (section 3) that can provide the highest compression of ion bunch. The calculated results relate to deuterium as a material of modern laboratory experiments. With taking into account the difference in the oscillation frequencies of the ions of different types the application of model can be spread to other fuels such as a mixture of deuterium and tritium nuclei, protons and boron, nuclei of deuterium and ^{3}He nuclei.

2. The convergence of the ion bunch in the central area of the virtual cathode

Let us assume that the compression of ion bunch occurs from the initial radius of the inner cathode r_g (or inner anode [6, 15]) to a minimum radius r_p, when the ion density increases from the initial value n_{i0} to a maximum one n_i. Using the Poisson distribution and introducing the ratio f of ion density to electron density lead to the initial ion density in the cylindrical trap as [10]

$$n_{i0} = f\frac{\Phi}{\pi e r_g^2}, \tag{1}$$

where e is the electron charge; Φ is the depth of potential well (the potential of the electric field in the axis of the system). For generality, parameter f will be saved in the formulas discussed below. However, all assessments will be made for the value of $f \lesssim 1$. The oscillation frequency is determined by the time of the deuterium ion flight through the trap

$$\nu \equiv \frac{u_a}{2r_g} \approx 8\times10^6\frac{\Phi^{1/2}}{r_g}\ \text{Hz}, \tag{2}$$

here: $u_a = u/2 = (\Phi/4m)^{1/2}$ and u are the average and maximum speed of deuterium ion, correspondingly; m is proton mass; Φ, r_g and ν here and below are measured in kV, cm and Hz.

It is understood that the value of potential Φ should be selected from the condition that the energy of accelerated ions in the axis of system must be close to the energy corresponding to the maximum cross section of the fusion reaction. For deuterium ions, thus, this value should be in the range of 50–100 kV. When $\Phi = 100$ kV and $r_g = 8$ cm oscillation frequency is 10 MHz. For a selected potential Φ the frequency increases with radius r_g decreasing. At the same potential

$\Phi = 100$ kV, to achieve the frequency of 100 MHz the radius should be about 0.8 cm. With increasing the frequency the initial ion density increases. According to (1) and (2)

$$n_{i0} = f\frac{4\Phi\nu^2}{\pi e u_a^2} \approx 3.5 \times 10^{-5} f\nu^2 \text{ cm}^{-3}. \tag{3}$$

The ion initial density is about 10^9 cm^{-3} at the frequency of 10 MHz and about 10^{11} cm^{-3} at $\nu = 100$ MHz.

To evaluate the degree of ion bunch convergence along to the axis of trap, let us use the "ballistic" approach, in which the degree of convergence is determined by the condition of adiabatic compression of the plasma [10]. This approach does not take into account a number of important effects such as the formation of a space charge and two-stream instability, but it makes it possible to estimate the scale of a maximum degree of ion bunch compression. The degree of compression $\theta = r_g/r_p$ in this approximation is expressed through the final and initial values of temperature. So, in the frameworks of considered statement of problem with using the adiabatic exponent of ideal gas degree of compression could be written as

$$\theta = \left(\frac{\Phi}{T_0}\right)^{1/2}. \tag{4}$$

Usually, for the systems with injection of the ion beam the initial temperature T_0 is determined as the room temperature of 0.025 eV (e.g., see [10]). In this case, according to (4), the compression degree and final ion density in cylindrical trap (compare with [21–23]) are:

$$\theta = 2 \times 10^2 \Phi^{1/2}, \tag{5}$$

$$n_i = n_{i0}\theta^3 \approx 2.7 \times 10^2 f \Phi^{3/2} \nu^2 \text{ cm}^{-3}. \tag{6}$$

For $\Phi = 100$ kV the evaluation (5) gives the enough high value of compression ratio as 2×10^3. For $\Phi = 100$ kV the final density of deuterium ions is about 3×10^{19} cm^{-3} at the frequency of $\nu = 10$ MHz and 3×10^{21} cm^{-3} at the frequency $\nu = 100$ MHz.

Let us consider the state of plasma systems with electrostatic confinement in terms of ion distribution on energy. Using the well known expression for ion-ion collision time [24]

$$\tau_{ii} = \frac{3}{4\sqrt{\pi}} \frac{m_i^{1/2} \epsilon_i^{3/2}}{e^4 Z^4 n_i \Lambda} \approx 4.2 \times 10^{10} \frac{A^{1/2} \epsilon_i^{3/2}}{Z^4 n_i} \text{ s},$$

where Λ is Coulomb logarithm (its value is approximately 15 in frames of the problem), Z is ion charge, ϵ_i is ion energy (here and below measured in keV) and using the expressions (5) and (6), lead to the characteristic time for establishing the energy equilibrium of deuterium ions in the electrostatic trap as

$$\tau_{ii} \approx 1.5 \times 10^8 f^{-1} \nu^{-2} \text{ s}.$$

Comparison of the above written value τ_{ii} with the time of ion flight $\sim 1\nu^{-1}\theta^{-1}$ through the region of compressed bunch $\tau_{ii} < \nu^{-1}\theta^{-1}$ gives the following condition for the establishing of equilibrium state

$$\nu > 4.5 \times 10^{10} \frac{\Phi^{1/2}}{f} \text{ s}^{-1},$$

which indicates that the real IEC-systems with oscillation frequencies of 10–100 MHz correspond to a nonequilibrium state of the colliding ions. Thus, ion-ion relaxation does not take place during the time t_p of ion collapse. In the case of POPS-system the conclusion on non-equilibrium ion state in particular ion collapse can be extended to a full operation cycle of the trap, because the oscillations are not correlated, and the additive accumulation of angular momentum does not occur properly by the reason of a strong non-ideal state of the ions in the final moment of their expansion [10]. In the next section, the model of DD-neutron generation and the condition of positive energy output (analogue of Lawson criterion [25]) in a cylindrical IEC-system under non-equilibrium state of the interacting nuclei of deuterium is presented. Note, the necessary to reconsider the Lawson criterion for inertial electrostatic confinement namely for non-equilibrium ion energy spectra have been remarked earlier (see., e.g, [9, chap 13]).

3. The neutron yield and Lawson criterion for IEC in cylindrical POPS-like system

In a single act of oscillation the neutron generation occurs during the time $t_p = 1/\nu\theta$ for which the deuterium ions pass the area of a compressed bunch. In approximation of the utmost non-equilibrium state of the ions, when it is assumed that all ions have an energy $\epsilon_i \approx \Phi$, the rate of DD-neutron generation is then given by

$$\dot{N} = \frac{1}{2} n_i^2 \sigma(\epsilon_i) u_i \Omega, \tag{7}$$

where σ is cross-section of the fusion reaction, Ω is the volume corresponding to the maximum ion density. Then, for a cylindrical system with a longitudinal size L, using the expression (1), we obtain the expression recorded in a general way—without specifying the compression degree θ

$$\dot{N} = \frac{f^2 \theta^4 \Phi^2 \sigma(\epsilon_i) u_i L}{2\pi e^2 r_g^2}. \tag{8}$$

It is important to underline the inverse-quadratic dependence of the generation rate on the radius r_g that is stronger than the well known hyperbolic dependence for a spherical trap [10]. So, for cylindrical system (see real geometry, for example, in [26]) it is even more manifested the benefits of reduction r_g to increase the fusion power output in IEC-systems.

With taking into account the periodical operation of IEC-systems, the effective neutron generation rate is introduced for the period of a single oscillation as $\dot{N}_{\text{eff}} = \dot{N}/\theta$. Using this value gives a possibility to calculate the total number of neutrons produced to the current time t as $N = \dot{N}_{\text{eff}} t$. According to (1), (2), (5), (6) and (8), the effective generation rate of DD-neutrons in cylindrical trap is

$$\dot{N}_{\text{eff}} \approx 2.4 \times 10^{26} f^2 \Phi^{7/2} \nu \sigma \frac{L}{r_g}. \tag{9}$$

For a system with $L \approx r_g$ DD-neutron generation rate reaches the practically demanded values of $\dot{N}_{\text{eff}} \approx 10^{10}$–$10^{12}$ particles/s, for example, specifically, the value of $\dot{N}_{\text{eff}} \approx 10^{11}$ particles/s, when

$$\nu > 4 \times 10^{-16} \sigma^{-1} \Phi^{-7/2}.$$

If the value of potential is 25 kV, this condition is satisfied for frequency larger than 1 MHz, if $\Phi = 50$ kV it is satisfied for $\nu > 10$ kHz.

Thermonuclear gain that is the ratio of fusion reaction energy released for a total time of ion flights through the region of compressed bunch $t_s = \tau/\theta$ during the operation cycle time τ to the total energy of electrons and ions acquired in a trap is

$$Q = \frac{f}{1+f} \frac{\epsilon_r}{\epsilon_i} \frac{n_i \sigma(\epsilon_i) u_i}{\theta} \tau, \tag{10}$$

here $\epsilon_r = 3.2$ MeV is the energy of DD-reaction. Substituting the expression (2), (5) and (6) in (10), we obtain

$$Q = 7 \times 10^{10} \frac{f^2}{1+f} \Phi^{1/2} \sigma(\epsilon_i) \nu^2 \tau. \tag{11}$$

Thermonuclear gain can be expressed in terms of the radius r_g as

$$Q = 4.5 \times 10^{24} \frac{f^2}{1+f} \frac{\Phi^{3/2}\sigma(\epsilon_i)\tau}{r_g^2}.$$

The gain decreases inversely proportional to the square of the trap radius that is, as already mentioned in the introduction, an important property of the IEC systems. Equation (11) with the additional requirement $Q > 2$ makes it possible to establish an analogue of Lawson criterion [25] (criterion of positive energy yield of thermonuclear reaction) for the systems with inertial electrostatic confinement. Implementation of such a criterion represents that the energy released in fusion reactions must be larger than the energy expended to create the plasma with taking into account the 30% efficiency of energy conversion into electrical energy. When $\Phi = 100$ kV such a criterion for the deuterium plasma in a cylindrical electrostatic trap, according to (11), has the form

$$\tau\nu^2 > 3 \times 10^{13}\ \mathrm{s}^{-1}. \tag{12}$$

Using (2), this criterion can be written in the terms of the radius r_g (at $\Phi = 100$ kV)

$$\frac{\tau}{r_g^2} > 7.5 \times 10^{-3}\ \mathrm{s\,cm}^{-2}.$$

Thus, according to the criterion for a positive energy yield of DD-reaction, for example, when the oscillation frequency is $\nu = 100$ MHz ($r_g = 0.8$ cm) the duration of operation cycle of IEC-installation with direct injection of ions must exceed about 3 ms.

4. Concluding remarks

The analytical requirements are formulated for achieving a positive energy yield of nuclear reaction (analogue of Lawson criterion) and intense neutron generation with rate of 10^{10}–10^{12} particles/s at the inertial confinement of deuterium plasma in a cylindrical electrostatic trap. Both requirements correspond to a high degree of ion bunch convergence at the direct injection of ions. High final density of the convergent ion bunch is a natural requirement for inertial confinement of plasma. Also, we supposed that the shape and depth of potential well, injected ion energy, distribution function for ion energy as well as resonant heat (with high quality factor) of ions were optimal ones for our purposes (like chosen in [27] to get $Q > 1$).

Earlier, the process of nuclear burning at IEC has been described by J Lawson qualitatively in terms of "wet wood burning" (see [9, p 391]). In fact, it should be stressed that the positive fusion output at the inertial electrostatic confinement is produced in a result of repeated initiation of fusion reactions in the periodically generated plasma at a head-on collision of ion bunches in the center of the IEC-system. In this case, the gain in each individual act of synthesis is much less than unity ("non-igniting" plasma, which in contrast to the tokamak plasma or plasma of laser fusion target, when a fusion energy release is a continuous burning process). Meanwhile, the summation of relatively small gains over the all periodic short "burning" time $t_p N_{\mathrm{osc}}$ during the whole time of operation cycle τ would provide $Q > 1$. Accordingly, the obtained analogue of Lawson criterion (12) for the lower limit of value of $\tau\nu^2$ has the same dependence on τ as the standard one [25] for the value $n\tau$ but still depends on the oscillation frequency that is specific result for IEC in POPS-concept.

Thus, the overcoming of the physical and technological problems to achieve the required time of operation cycle of low-loss IEC-device [10] with POPS-like low-damping regime [11] with corresponding up to $\geq 10^5$ oscillations per cycle is rather complex task which is partially similar to providing the necessary confinement times for tokamak or laser target plasmas. Remark also, much detailed study remains to verify the potential for the IEC with POPS-like oscillations to achieve proton–boron burning [9, 17, 26] with positive energy output.

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