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Radiation and matter: Electrodynamics postulates and Lorenz gauge

V B Bobrov[1], S A Trigger[1], G J van Heijst[2] and P P Schram[2]

[1] Joint Institute for High Temperatures of the Russian Academy of Sciences, Izhorskaya 13 Bldg 2, Moscow 125412, Russia

[2] Eindhoven University of Technology, P.O. Box 513, Eindhoven 5600 MB, Netherlands

E-mail: satron@mail.ru

Abstract. In general terms, we have considered matter as the system of charged particles and quantized electromagnetic field. For consistent description of the thermodynamic properties of matter, especially in an extreme state, the problem of quantization of the longitudinal and scalar potentials should be solved. In this connection, we pay attention that the traditional postulates of electrodynamics, which claim that only electric and magnetic fields are observable, is resolved by denial of the statement about validity of the Maxwell equations for microscopic fields. The Maxwell equations, as the generalization of experimental data, are valid only for averaged values. We show that microscopic electrodynamics may be based on postulation of the d'Alembert equations for four-vector of the electromagnetic field potential. The Lorenz gauge is valid for the averages potentials (and provides the implementation of the Maxwell equations for averages). The suggested concept overcomes difficulties under the electromagnetic field quantization procedure being in accordance with the results of quantum electrodynamics. As a result, longitudinal and scalar photons become real rather than virtual and may be observed in principle. The longitudinal and scalar photons provide not only the Coulomb interaction of charged particles, but also allow the electrical Aharonov–Bohm effect.

1. Introduction

For the thermodynamic parameters corresponding to the usual state of matter, as well as for the extremal parameters, especially, the consecutive consideration of thermodynamic properties of substances is directly related with investigation of the equilibrium system, consisting charged particles and quantized electromagnetic field. The appropriate consideration is the object of quantum electrodynamics which based on the perturbation theory on interaction between charged particles and quantized electromagnetic field (see, e.g., [1, 2]). Application of the perturbation theory is possible due to existence of the small parameter $e^2/\hbar c$. However, the essential problem of quantization of the longitudinal and scalar potentials of electromagnetic field in quantum electrodynamics still exists (for details see, e.g., [3]). For this reason, in thermodynamic and kinetic applications, for temperatures $T \ll \epsilon_0$ (where ϵ_0 is the rest energy of particles), instead the original system one consider the "model" system, consisting the quantized transversal electromagnetic field (photons) and non-relativistic charged particles interacted by the Coulomb low (see, e.g., [4]). Wherein, to use the electrostatic potential one proceed from the concepts of classical electrodynamics, where the Coulomb low is valid for the particle velocities small in comparison with the light speed [5]. However, in quantum electrodynamics interaction between charged particles is carried out via electromagnetic field and the velocity of this interaction propagation cannot be dependent on the charged particle velocities. It is clear on the example of the interaction between charged particles and the transversal electromagnetic field (photons). Thus, for the fully consistent description of thermodynamic (as well as kinetic) properties of matter it is necessary to solve the problem of the longitudinal and scalar potentials quantization. We assume that the solution of this problem is directly related with the Aharonov–Bohm (AB) effect [6, 7]. The essence of this effect is the influence of the electromagnetic field on the charged particle (or on a particle with the magnetic moment) in the space region, where the field strength equals zero, but the vector and/or scalar electromagnetic potentials possess a nonzero value.

The problem of physical reality of the electromagnetic potentials is one of the fascinating questions of electromagnetism. Although the paper by Aharonov and Bohm [6](see also [7]) was published more than 50 years ago, various aspects of the AB effect still attract great attention up to now [8–16]. Most studies, including experimental researches, are devoted to the magnetic AB effect (see, e.g., [17]).

However, in recent years the interest in the electric AB effect has significantly increased (see [13,18,19] and references therein) due to different interpretations of the experiments [20,21]. Furthermore, taking into account an analogy between electrostatic and gravitational interactions the problem of experimental detection of gravitational analog of AB effect arises [22].

Electric AB effect concerns the basic concepts of the electromagnetic field gauge theory [23]. Since the magnetic field is completely defined by the transverse part of the vector potential, the magnetic AB effect does not depend on the gauge invariance of the electrodynamic equations [24]. At the same time, the electric AB effect is in obvious contradiction with electrodynamics postulates.

One of the main arguments against the physical reality of the electromagnetic potential is the fact that they depend on the gauge choice. The physical reality of some variable means that its average value (at least for some ranges of the parameters and arguments, this variable depends on) can be determined uniquely by means of some experiment. In most cases it is necessary to use a theory (always approximate) for the transition (or conversion) from a directly measured variable to the sought physically real variable. From a purely theoretical point of view we can consider some variable as physically real if it possesses a single-valued operator representation and carries physically valued information. It is necessary to stress that there is a well-marked difference between the microscopical variables (the variables represented in an operator form) and their averaged (macroscopic) values which only can be found from the experiments, if these variables are physically real. In brief, the average values of the physically real variables are referred to as observable.

The gauge invariance is known to be associated with the property of the Maxwell stress tensor of the electromagnetic field being unchanged when adding a four-dimensional gradient $\partial\chi/\partial x^{\mu}$ of an arbitrary scalar function χ of space–time coordinates. The requirement of relativistic invariance slightly narrows the arbitrariness in the choice of this function (the so-called Lorenz gauge). Using the Lorenz gauge creates significant problems in electromagnetic field quantization, since the electromagnetic potential components become dependent on each other (see, e.g., [3]). However, if only the electromagnetic field (i.e., electric and magnetic field strengths) is accepted as a physical reality, then the requirement for relativistic invariance of the potential seems to become excessive. Therefore, the Coulomb and other obviously non-covariant gauges are used in many studies.

A question arises why the fields (rather than the potentials) are considered to be real? It is easy to see that the reason is the "tacit" assumption that Maxwell equations for microscopic fields are valid (see, e.g., [5], chapter 4). However, the Maxwell equations (which are generalizations of available experimental data on the macroscopic behavior of systems of charged particles and associated with finite propagation velocity of the interaction between them) are valid only for averaged (or observed) values of dynamic variables (operators in quantum electrodynamics) entering in these equations.

2. Quantum description and the Lagrangian choice

Due to linearity of the Maxwell equations, the "tacit" assumption of the validity of these equations for dynamic variables (in operator form) naturally leads to their validity for average values as well. However, the opposite statement is not necessarily valid. The requirement of the fulfillment of this assumption follows neither from experimental data nor from the logical construction of the theory itself.

The only requirement for microscopic fields is that the corresponding equations of motion satisfy the Maxwell equations after the averaging procedure. Indeed, constructing microscopic electrodynamics, we should not be restricted to the introduction of electromagnetic field strengths only, but we are "forced" to take into consideration the field potentials to describe the field interaction with charged particles [5]. One of the key features of Maxwell equations is absence of longitudinal waves in vacuum (i.e., the absence of waves whose electric field vector is collinear to the wave vector). Nevertheless, longitudinal and scalar photons are used in quantum electrodynamics to describe the Coulomb interaction of charged particles (see, e.g., [25]). These photons are considered as "nonphysical" (virtual), since, otherwise, the microscopic Maxwell equations become invalid. However, as follows from the above consideration, there is no need for the requirement of the validity of the microscopic Maxwell equations.

The following question then arises: what can be proposed instead? Quantum electrodynamics gives a factual answer to this question. Within the concept of the free electromagnetic field, it is quite natural to postulate that the microscopic four-vector potential of the electromagnetic field satisfies the d'Alembert equation [25] (chapters II, VII). In order that this equation would correspond to macroscopic Maxwell equations, it is necessary to require the validity of the Lorenz gauge condition, but only for average (observed) values of the four-potential of the electromagnetic field. However, this condition is completely consistent with the modern results of quantum electrodynamics from the viewpoint of the above mentioned difficulties in electromagnetic field quantization [3,25]. In this case, the Lorenz condition for averages is only indicative of the choice of feasible quantum states, providing the independence of longitudinal and scalar photons in their microscopic description.

As a result, the fundamental difference in our interpretation of quantum electrodynamics is reduced to the statement that longitudinal and scalar photons are quite physical, rather than virtual. Their "non-physical" character only means that they "are absent" in the macroscopic Maxwell equations for the free electromagnetic field. However, the Maxwell equations themselves are not unique from the viewpoint of the description and manifestation of electromagnetic field. In this sense, the analogy between the Klein–Gordon and Dirac equations is relevant: the former is used to determine Dirac matrices. However, the Dirac equation does not follow from the Klein–Gordon equation (see, e.g., [25]).

Instead of the d'Alembert equation for the four-vector potential of the electromagnetic field, we can postulate, following to Fock and Podolsky [26], the corresponding Lagrangian for the system of charged particles and electromagnetic field

$$L = -\frac{1}{16\pi}\sum_{\mu,\nu}\left(\frac{\partial A^\mu}{\partial x^\nu} - \frac{\partial A^\nu}{\partial x^\mu}\right)^2 - \frac{1}{8\pi}\sum_{\mu}\left(\frac{\partial A^\mu}{\partial x^\mu}\right)^2 + \sum_{\mu}\left(\frac{j^\mu A^\mu}{c}\right), \tag{1}$$

where $A^\mu \equiv (\varphi, \mathbf{A})$ is the four-vector operator of the electromagnetic potential. The Lagrangian (1) is valid also for the quantum description of particles and electromagnetic field. The expression

(1) implies that the Lorenz gauge, as well as any other relation between the A^μ operators and their derivatives, is absent.

It should be kept in mind that it is not a single Lagrangian providing the validity of d'Alembert equation. However, the Fock–Podolsky Lagrangian provides the "required" quantization procedure of the electromagnetic field and its interaction with charged particles (described by the Dirac equation). Furthermore, the Lagrangian of the free electromagnetic field is not completely defined by the Maxwell stress tensor.

Then, how can we explain the fact that many results of applying gauge quantum electrodynamics lead to equivalent results using various gauges, first of all, the Lorenz and Coulomb ones (see, e.g., [27])? This is due to the so-called "heuristic" quantization proposed by Feynman [28] (see [29] for more details). Calculating the radiative corrections to scattering, Feynman drew attention to the fact that scattering amplitudes of elementary particles are independent of the reference frame and gauge choice. In fact, the gauge choice became a formal procedure of choosing gauge-invariant field variables. However, the range of application of heuristic quantization is limited only by the problem of elementary particle scattering, where this quantization appeared. The elementary particle scattering does not exhaust the problems of quantum electrodynamics (see, e.g., [30, 31]).

3. D'Alembert equations as the conceptual basis of quantum electrodynamics

Our concept may be formulated as follows: microscopic values of the vector potential $\mathbf{A}$ and scalar potential φ, forming in quantum case the four-vector operator A^μ, satisfy d'Alembert equations

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{4\pi \mathbf{j}}{c}, \quad \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi = 4\pi\rho, \tag{2}$$

or in covariant form

$$\mathrm{D}A^\mu = 4\pi j^\mu. \tag{3}$$

where $j^\mu \equiv (\rho, \mathbf{j}/c)$ is the four-vector of the charge current and D is the d'Alembert operator

$$\mathrm{D} \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \equiv \partial^\mu \partial_\mu, \tag{4}$$

$$\partial^\mu = \left(\frac{\partial}{\partial ct}, \nabla\right), \quad \partial_\mu = \left(\frac{\partial}{\partial ct} - \nabla\right). \tag{5}$$

In the brackets the time and space components of the four-vector divergence operator are placed. Due to linearity of the d'Alembert equations (2), (3) they are also valid for the average values $\langle A^\mu \rangle$ and $\langle j^\mu \rangle$. In the framework of quantum electrodynamics [25] the quantized free electromagnetic field should satisfy the homogeneous d'Alembert equations (2),(3) (in absence of charges)

$$\mathrm{D}A^\nu = 0. \tag{6}$$

However, for realization of the non-contradicted quantization of the free (vacuum) electromagnetic field it is necessary to fulfill the Lorenz gauge for the averages $\langle A^\mu \rangle$ (see in detail [3, 5, 13])

$$\partial^\mu \langle A^\mu \rangle = 0. \tag{7}$$

The essential point is the requirement that the Lorenz condition (7) should be satisfied not for the microscopic values of the four-vector A^μ, as it is traditionally accepted in quantum electrodynamics in the so-called Lorenz gauge [5], but only for the average values $\langle A^\mu \rangle$. The components of the microscopic four-vector A^μ are to be considered independently in the quantization procedure of the vacuum electromagnetic field. The Lorenz condition (7) leads only to limitations in choice of the state vectors of the electrodynamic system under consideration. Since the condition (7) is evidently independent on whether or not charges are present it can be postulated also in the presence of charged particles. The Maxwell equations for the average fields then follow directly from d'Alembert equations (2), (3) and equation (7) for averages.

Taking into account that the average values of the electric and magnetic fields $\langle \mathbf{E}(\mathbf{r},t)\rangle$ and $\langle \mathbf{H}(\mathbf{r},t)\rangle$ are determined by

$$\langle \mathbf{E}\rangle = -\frac{1}{c}\frac{\partial\langle \mathbf{A}\rangle}{\partial t} - \nabla\langle\varphi\rangle, \quad \langle \mathbf{H}\rangle = \nabla\times\langle \mathbf{A}\rangle, \tag{8}$$

Maxwell equations for the averaged $\langle A^\mu\rangle$ take the form

$$\mathrm{D}\langle A^\mu\rangle - \partial^\mu\left(\partial_\alpha\langle A^\alpha\rangle\right) = 4\pi\langle j^\mu\rangle. \tag{9}$$

Using condition (7) we arrive at the d'Alembert equations for the average values $\langle A^\mu\rangle$

$$\mathrm{D}\langle A^\mu\rangle = 4\pi\langle j^\mu\rangle. \tag{10}$$

These equations correspond to the averaged initial equations (2), (3) for the microscopical values of the potentials.

4. Conclusions

The use of Maxwell equations for *microscopic* description of electromagnetic field can not be considered as axiom. Instead, we discussed the hypothesis that quantum electrodynamics (and, consequently, thermodynamic and kinetic properties of the system of charged particles and electromagnetic field [32]) may be constructed basing on microscopic d'Alembert equation (3) for 4-vector electromagnetic potential.

As is shown, the averaged microscopic equation (3) used together with the Lorenz gauge (7), formulated only for averages, yields the traditionally accepted Maxwell equations for *averaged* electromagnetic fields and, hence, all results of the conventional electrodynamics. However, there are advantages of the suggested hypothesis and description. First, we are able to overcome difficulties associated with the electromagnetic field quantization, which is the quantization procedure facilitated. Therefore, the gauge problem and respective uncertainty of the microscopic description of the electromagnetic potentials disappears. Second, longitudinal waves (quanta of the electromagnetic potential A^μ) in vacuum are not forbidden anymore. Namely, the d'Alembert equations for potentials $(\mathbf{A},\varphi)$ in vacuum are satisfied by solutions in the form of both transversal and longitudinal waves. As a result, longitudinal and scalar quanta of electromagnetic potentials (in contrast with scalar and longitudinal photons) may become real (rather than virtual as within conventional electrodynamics), and as such can being in principle observable.

To summarize:

(i) The formalism considered above eliminates the necessity to postulate the relation between the electromagnetic potential operators A^μ and, therefore, avoids the known difficulty of QE. According to the presented concept, the Lorenz gauge is valid only for the averages potentials;

(ii) This formalism extends the class of observable physical variables. In particular, the existence and observation of the longitudinal and scalar quanta of electromagnetic potentials are possible in parallel with the transversal quanta of the electromagnetic field (photons). Hence, the theoretical basis for experimental manifestation of the electromagnetic potentials is formulated. Evidently, experimental investigations of the additional observable variables (see, e.g., [20–22]) can lead to interesting practical findings;

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(iii) The proposed microscopic basis of QE reproduces all known effects in QE, as well as the Maxwell equations for averages. At the same time, the Maxwell equations for microscopic variables (operators) of the electromagnetic fields do not exist, contrary to the usual theoretical formalism.

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