

Influence of nonlinear constitutive relations in unimorphs piezoelectric harvesters

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Abstract. This paper presents the influence of nonlinear terms of a previously proposed constitutive piezoelectric equation on the dynamics of a cantilever aluminium beam with a piezoelectric unimorph PZT (MIDE QP16N) attached to it. The system is subjected to different levels of base acceleration with the intention to evidence the limits of the linear model. To carry out the analysis, a one-dimensional model is applied and solved employing a single-term solution of the harmonic balance method to compare with the experiments. A model identification of linear and nonlinear parameters such as dissipation, stiffness, and electromechanical coupling were then performed. From the results, it is possible to observe the departure of the linear model even for very low acceleration levels (0.1G). It can be concluded that the nonlinearity plays an unavoidable roll in predicting electric generation for the considered systems.

1. Introduction

Nonlinearity in piezoelectric beams is a challenging and complex phenomenon. Although it is commonly supposed that a linear description captures the principal features of the mechanical and electrical response of a piezoelectric material under usual practical situations, its behavior is far from being linear even for low values of mechanical and electrical excitation. There are several factors that make the dynamics to be nonlinear. Between them, it is possible to mention strong electric fields [1, 2], damping, influence of adhesive layer, nonlinear beams theories, nonlinear constitutive equations under nonlinear elastic and electromechanical coupling, among others [3]. The observed discrepancy between linear model and experiments is evident even at weak electric fields.

Non-linear theory of dielectrics has been developed by the pioneering works of Toupin [4] and Tiersten [5]. Joshi [6] employed the thermodynamic Gibbs potential to derive constitutive equations including higher order effects up to second order terms in the linear formulation. Aurelle et. al. [7] studied the effect of a nonlinear electromechanical coupling parameter and nonlinear elasticity on the response of a piezoelectric transducer. Wagner and Hagedorn [2] studied the nonlinear elastic and coupling behavior of piezoelectric beam actuators, proposing an enthalpy density with quadratic and cubic nonlinearities of strain and coupling. The first approach to study nonlinear piezoelectricity in a harvester was published by Hu [8]. His results show a hardening response of a piezoelectric plate due to a shear mode vibration. Triplett and Quinn [9] proposed a strain dependence of the piezoelectric



constant as responsible for nonlinearity in the electromechanical equations. In a first work Stanton et. al. [10] used the nonlinear constitutive equations of ref. [8] in a bimorph PZT 5H and proposed a nonlinear quadratic damping to account for the theoretical overshoot of experimental data. In a second work, the same author and collaborators [11] extended the nonlinear constitutive relations to include non-conservative stress and electric displacement to introduce dissipation within electroelastic media. Abdelkefi et. al. [12] performed a nonlinear analysis of a multi-layered piezoelectric beam with a tip mass taking into account geometric and piezoelectric nonlinearities into the electromechanical model. The constitutive relations of the piezo are derived using the approach of Joshi [6]. Goldshmidtboeing et. al. [13] attributed to ferroelastic hysteresis the source of nonlinearities in PZT cantilever beams. They modelled the constitutive nonlinear equations and nonlinear dissipation effects with the simplest hysteresis model, the Rayleigh model, to obtain their theoretical predictions. More recently, Leadenham and Erturk [14] based on the work of refs [2,13] proposed their own enthalpy density expression which predicts a linear backbone curve in the response amplitude of a bimorph piezoelectric sheet. Finally, Yang and Upadrashta [15] considered a nonlinear electromechanical model and damping nonlinearities to model a cantilever macro-fiber composite for energy harvesting. They proposed to identify nonlinear elastic and damping coefficients by matching analytical responses and experimental results. As a consequence, a high-order stress/strain relationship is obtained which contains seven polynomial terms to fit the experimental data up to very high strains.

In this paper we used the model of Leadenham and Erturk [14] to investigate the validity of their model applied to an energy harvesting system (EH) containing a unimorph piezoelectric sheet. In most of the works mentioned previously, high resonant frequencies were considered to study the behaviour of piezoelectric materials [10 - 14]. Only Yang and Upadrashta [15] investigated a harvester working at low frequencies with large deformation but with another model of dissipation, not considering the ferroelastic hysteresis.

2. Design and Modelling

The model of the proposed EH system comprises an aluminum cantilever beam with a piezoelectric unimorph PZT sheet (MIDE QP16N) attached at its root. The system is excited by its base with constant acceleration level, observed schematically in figure 1. The mathematical model is based on the Bernoulli-Euler theory for flexural beams. The electric field is considered uniform and linear over the thickness of the piezo sheet and the constitutive relations (stress/strain+electric field/electric displacement) contain nonlinear elastic terms, linear and nonlinear electromechanical coupling and dissipation effects due to ferroelastic hysteresis as stated in [13].

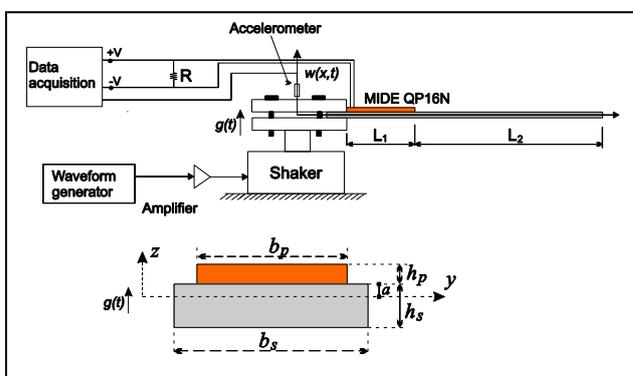


Figure 1: Schematic picture of the system and its geometric parameters.

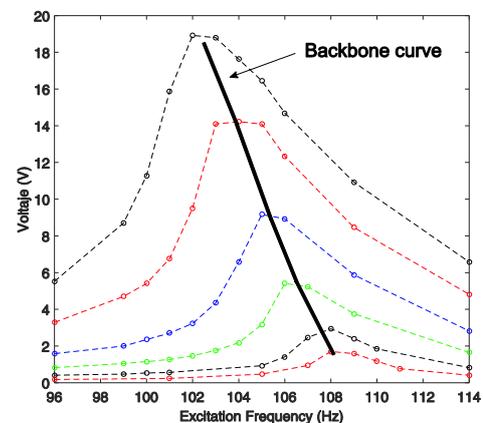


Figure 2: Experimental response curves at various excitation amplitudes.

2.1. Mathematical formulation

Following the conclusions of Leadenham and Erturk [14], we also observed in the experimental results that the backbone curve changes linearly with the response amplitude (see figure 2). Therefore, to address the nonlinear problem of the constitutive model, we consider the enthalpy density for the substrate H_s and for the piezo beam H_p as [14]:

$$H_s = \frac{1}{2} c_{11}^s S_1^2 \quad (1)$$

$$H_p = \frac{1}{2} c_{11}^p S_1^2 + \frac{1}{3} c_{111}^p S_1^3 \operatorname{sgn}(S_1) - e_{31} S_1 E_3 - \frac{1}{2} e_{311} S_1^2 \operatorname{sgn}(S_1) E_3 - \frac{1}{2} \epsilon_{33} E_3^2 \quad (2)$$

where c_{11}^s y c_{11}^p are the Young modulus of the aluminum and the piezo sheet respectively, c_{111}^p is the third order nonlinear elastic modulus, e_{31} is the linear piezoelectric constant, e_{311} is the third order nonlinear piezoelectric constant, ϵ_{33} is the piezo permittivity and E_3 is the electric field in the transverse direction given by $E_3 = v(t)/h_p$ ($v(t)$ is the voltage).

As mentioned previously, to account for the dissipation, the model of ferroelastic hysteresis of [13] provides a dissipation energy as $D_p = \frac{1}{3} b_{111} S_1^3$, whereas the damping for the substructure is modelled as simple proportional damping. Kinetic and potential energy of the piezo and the substructure can be obtained in a straightforward manner following [14] with the only difference that here, we consider a unimorph piezoelectric beam [16].

2.2. Governing equations

Two governing electromechanical equations have been derived using Lagrange equations, with a single-mode solution for the first bending mode:

$$m\ddot{q}[t] + (bq[t]\operatorname{sgn}(q[t]) + b_n q[t]^2)\operatorname{sgn}(\dot{q}[t]) + kq[t] + k_n q[t]^2 \operatorname{sgn}(q[t]) - (\theta + \theta_n q[t]\operatorname{sgn}(q[t]))v[t] = -m_a \ddot{g}[t] \quad (3)$$

$$C_p \dot{v}[t] + \frac{v[t]}{R_l} + (\theta + \theta_n q[t])\dot{q}[t] = 0 \quad (4)$$

Where $m, b, b_n, k, k_n, \theta, \theta_n, m_a, C_p, R_l$ are the modal mass, linear and nonlinear damping, linear and nonlinear electromechanical coupling, modal forcing mass, internal capacitance of the piezoelectric unimorph sheet and load resistance. These lumped parameters are functions of the device dimensions and material properties and are given in [14]. The nonlinear differential equations are further solved analytically using a single-term solution in the harmonic balance method, so that the dynamic response of the system is obtained. From this, it is possible to find the voltage $v[t]$ to compare it with the experiments.

3. Experiments and Results

The experiments were carried out using the setup of figure 3. We considered a large range of accelerations ranging from 0.1-3.0 G (G=gravity =9.8m/sec²). The sweeps for each acceleration amplitude were performed at constant amplitude. Material and geometrical constants of the EH system are given in table 1.

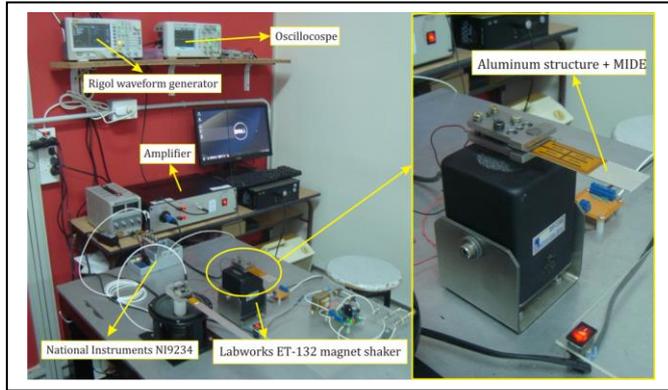


Figure 3. Experimental setup (left), zoom in of the EH system (right)

Table 1. Geometrical and material constants.

Parameter	Alum.	QP16N
Length (mm)	80	45.9
Width (mm)	21	20.57
Thickness (mm)	0.5	0.25
Density (kg/m ³)	2700	7800
Young's modulus (GPa)	67	67
Coupling d_{31} (pm/V)		-190
Capacitance (nF)		90.78
Permittivity		1500 ϵ_0

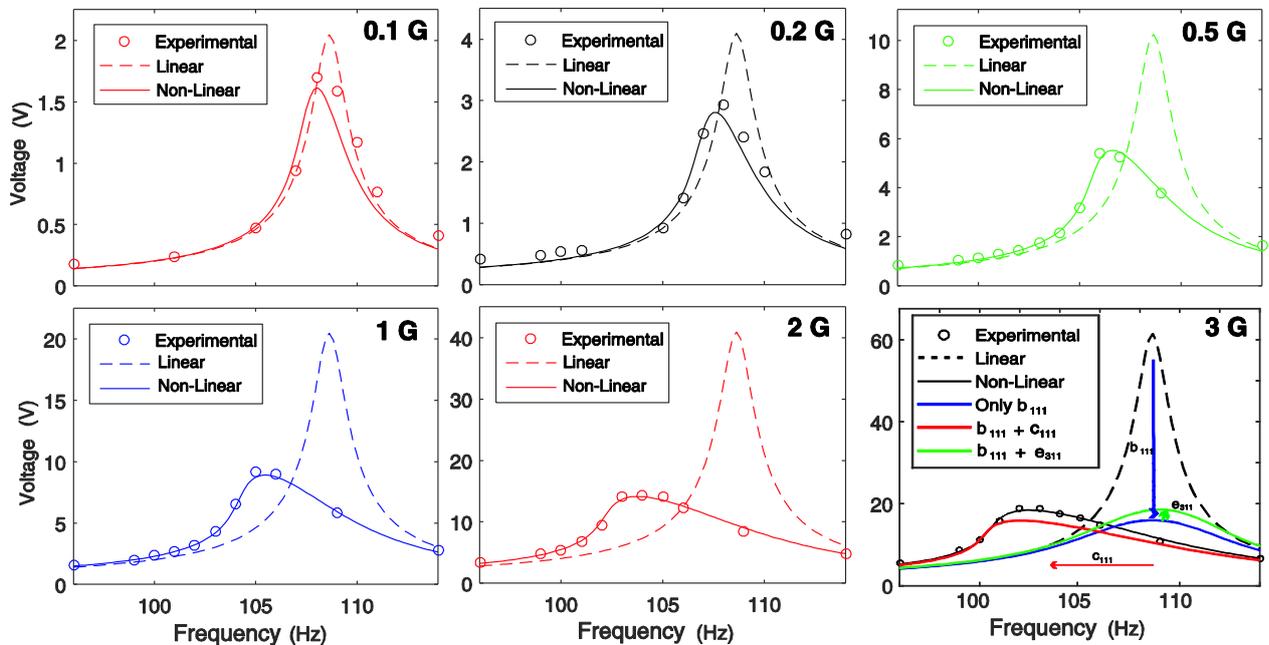


Figure 4: Comparison between experimental and linear/nonlinear responses for accelerations ranging from 0.1 to 3 G

In order to apply and solve the model of the proposed device, it was necessary to make an identification of linear and nonlinear parameters (b , b_{111} , c_{111} , e_{311}). To this end, the experimental tests for an acceleration level of 3 G were used. After that, we employ a least squares technique (*lsqnonlin* command in Matlab) along with the proposed analytical model to extract the more approximate values in a least squared sense. The obtained values are: $b_{11} = 3.59 \cdot 10^9 \text{ N/m}^2$, $b_{111} = 6 \cdot 10^{13} \text{ N/m}^2$, $c_{111} = -303.44 \text{ TPa}$ and $e_{311} = -68.15 \frac{\text{kC}}{\text{m}^2}$. These values are fed into the analytical model and plot together with the experimental results. The comparison can be observed in figure 4 where both linear and nonlinear responses are shown along with the experiments for accelerations ranging from 0.1-3G. As a first singular feature, it is possible to observe that the curves bend to the left (softening response) as the amplitude of the base acceleration increases. This clearly represents a significant departure of the linear regime. Even for low accelerations levels (0.1G) the linear prediction is poor. For larger accelerations, instead, the nonlinear model represents an accurate description of the voltage generation. A very good

agreement can be appreciated for all the performed tests including the maximum value of 3G. In this case, the voltage peak amplitude diminishes four times compared with its linear counterpart and the system's natural frequency (frequency peak) decreases up to 102 Hz, almost 6 Hz less than the linear frequency of 108.6 Hz. An interesting analysis can be observed for the 3G case. There, we plot the contribution of the nonlinear parameters (b_{111} , c_{111} and e_{311}) on the FRF curves. The linear response amplitude is reduced considerably by means of nonlinear damping b_{111} while the nonlinear elastic modulus c_{111} is responsible for the softening behavior of the EH system. The third order nonlinear piezoelectric constant e_{311} plays a secondary role in this case.

4. Conclusions

In this paper, the dynamic behavior of a cantilever beam with a piezoelectric unimorph sheet (MIDE QP16N) attached to an aluminum beam subjected to different levels of acceleration, was studied for low frequency energy harvesting purposes. A non-linear constitutive model for the piezoelectric sheet was used, which was previously proposed by Leadenham and Erturk [14] for a bimorph EH system. The model takes into account the effect of ferroelastic hysteresis resulting in terms of nonlinear stiffness and nonlinear dissipation. The results evidence a very good agreement between the nonlinear analytical model and the experiments, even though for low levels of base acceleration. Therefore, it is concluded that the model correctly predicts the dynamic behavior of the studied system. This contrasts definitively with the application of the linear model for these cases. Future efforts will be conducted to explore the effect of ferroelastic hysteresis in macro-fiber composite EH, considering the piezoelectric 33 and 31-effects.

Acknowledgments

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