A frequency up-converting harvester based on internal resonance in 2-DOF nonlinear systems

Yipeng Wu, Jinhao Qiu and Hongli Ji

Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China

yipeng.wu@nuaa.edu.cn

Abstract. This paper reports the design and experimental testing of a novel frequency up-converting piezoelectric energy harvester. The harvester is firstly approximated as a 2-degree-of-freedom cubic nonlinear system instead of the general Duffing systems. A 1:3 internal resonance innovatively applied in the frequency up-conversion approach is thoroughly investigated. Finally, the theoretical dynamic model confirmed by the experimental results clearly shows the effect of the frequency up-conversion.

1. Introduction

Vibration energy harvesting technology describes the process of converting ambient mechanical energy into useful electrical energy through the use of electromechanical structures. It has the ability to offer the prospect of powering autonomous electronic devices such as wireless sensor nodes without the use of conventional batteries. Piezoelectric cantilevers are the simple and typical electromechanical structures, however, real-word applications of such linear cantilevers are not practical due to the random nature of vibrations. For this reason, resonant frequency tuning harvesters, multimodal harvesters, nonlinear harvesters, etc. are proposed [1, 2]. The design of these harvesters usually follows the principle of the resonant-frequency-matching strategy.

More recently, an increasing number of researchers realize that low-frequency excitations are the most often available from the ambient environment. In this case, the utilization of the above matching strategy may unacceptably lead to increased encumbrance [3]. Consequently, the need for a technique capable of bridging between the high-frequency response and the low-frequency excitation is clearly needed. This technique is usually called the frequency up-conversion effect.

The most common frequency up-conversion approaches are implemented by mechanical plucking or impact. For example, Pozzi and Zhu proposed a knee-joint pizzicato harvester which consisted of several plectra and four piezoelectric bimorphs [3]. Gu and Livermore proposed the similar frequency up-converting harvester using vibrations of beam structures at high resonant frequencies that were excited by mechanical impact [4]. Besides the direct contact approaches, Han and Yun implemented the frequency up-conversion effect by a high impulse-like acceleration without physical contact, the harvesters apply the impact force to the high-frequency oscillators through the instantaneous state transition of a bucked bridge beam [5]. However, it is worthy of note that the above frequency up-converting harvesters usually operate at the high-level excited accelerations.

In this paper, we proposed a novel frequency up-converting harvester based on the principle of 1:3 internal resonance. The harvester can be considered as 2-Degree-Of-Freedom (DOF) cubic nonlinear system, when the internal resonance happens, part of the vibration energy can be transferred from the

low-order vibration mode to the high-order one, which realizes the frequency up-conversion without physical contact and energy losses. Finally, the simulation results validated by the experiment clearly explained the principle of this frequency up-conversion effect.

2. Theoretical model

(a) Structure schematic   (b) Dynamic model
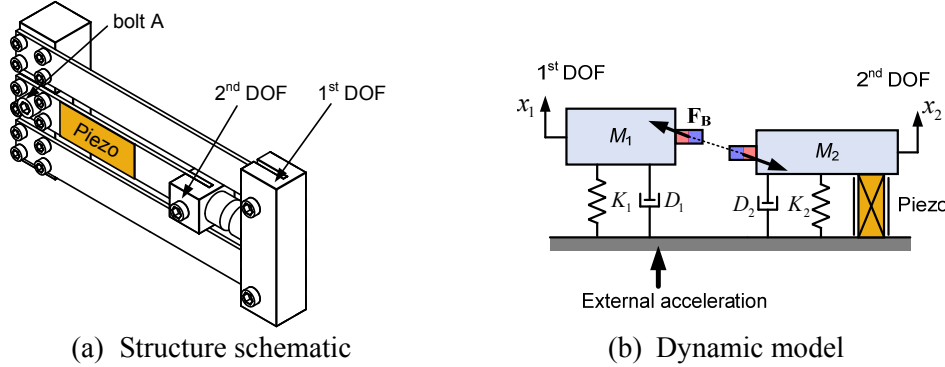
Figure 1. The novel frequency up-converting piezoelectric harvester

Figure 1 shows a compact prototype of the proposed energy harvester, it consists of two asymmetrical cantilevers and a pair of NdFeB permanent magnets. It is worthy of note that the base frame was composed of 2 sub-bases which are combined by the bolt A, so the separated distance between the two magnets can be adjusted manually by the relative position of the 2 sub-bases.

According to the previous literature [6], the proposed harvester can be considered as 2 spring-mass-damper models which are coupled by the magnetic force $\mathbf{F_B}$. The parameters K_i and D_i ($i = 1, 2$) are the global equivalent stiffness and the damping coefficient, respectively. An external acceleration γ is applied to the model, induces a relative displacements x_1 and x_2 between the masses and the base frame. The governing equations of the system is then obtained as shown in equation (1), where M_i are the proof masses of the corresponding cantilevers, F_B is the magnitude of the magnetic force $\mathbf{F_B}$, α is the piezoelectric force-voltage coefficient of the cantilever, V is the output voltage of the piezoelectric element.

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \gamma = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -F_B \\ \alpha V + F_B \end{bmatrix} \tag{1}$$

Because the shape and size of the magnets are not so important compared with the whole harvester device, the permanent repulsive magnets can be modeled as a dipole-dipole magnetic interaction in this article [7]. Finally, the simplified magnetic force F_B is expressed in equation (2), where a_i, b_i, c_i are the constant values for a given PEG.

$$F_B = a_1 x_1 - a_2 x_2 - b_1 x_1^3 + b_2 x_2^3 + c_1 x_1^2 x_2 - c_2 x_1 x_2^2 \tag{2}$$

From the above 2 equations, it is clearly found that the dynamic system has two DOFs (x_1 and x_2), the equivalent nonlinear stiffness only has linear and cubic terms. Hence, the proposed harvester can be considered as a 2-DOF cubic nonlinear system. A typical nonlinear vibration phenomenon called 1:3 internal resonance will be excited, provided that the nature frequency of the 2nd cantilever is the triple-frequency of the 1st one. This phenomenon has been thoroughly explained in multi-DOF nonlinear dynamics theory.

3. Simulation performances and experimental validation

Since it is very complicated to solve the above dynamic equation analytically, a numerical integration techniques using MATLAB software will be discussed in this section. All simulation parameters are set according to the experimental prototype of the harvester.

Figure 2 shows the photo of the experimental setup and the proposed frequency up-converting harvester. The experimental platform is mainly composed of a vibration controller, a power amplifier, a vibration table and a feedback accelerometer. The experimenter can easily establishes a PID feedback loop to make sure the table vibrate at a constant acceleration amplitude. The harvester is fixed on the vibration table by the screws, one Macro-fiber-composite (MFC) is attached on the inner cantilever for energy generation, the other is attached on the outer cantilever for comparison. It is worthy of note that in this article, we mainly focus on the frequency up-conversion effect of the piezoelectric harvester, the optimization of the electromechanical coupling coefficient will not be discussed here.

Based on the measured data, the key parameters of the model utilized in the simulation are given in table 1. Some of the parameters are identified using the equation given in reference [6].
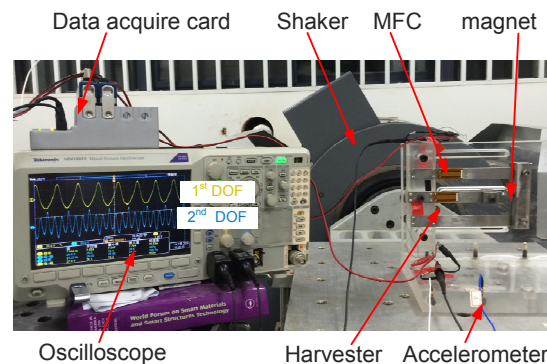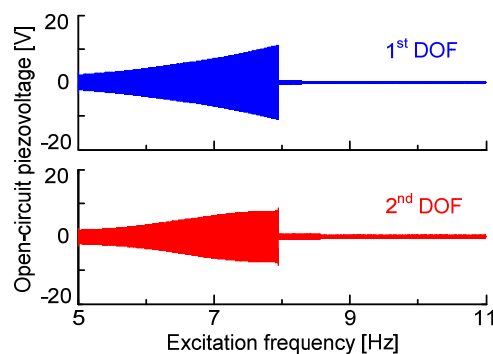

Figure 2. Experimental setup and the photo of the proposed harvester

Table 1. Parameters used for numerical computation

|	Parameters	1st DOF	2nd DOF
M_i	Effective mass (g)	782.9	104.6
l_i	Equivalent length (mm)	125	100
f_i	Resonant frequency (Hz)	8.03	23.27
ξ_i	Damping ratio	0.011	0.007
α	Force-voltage coefficient (N/V)		8.4×10^{-5}
C_0	Clamped capacitance (nF)		47.58

3.1. Simulation performances of the frequency up-conversion

Figure 3. Experimental setup and the photo of the proposed harvester

To study the performances of the frequency up-converting harvester, a forward sweep signal with a rate of 0.04 Hz/min is used as an excited acceleration, two magnets are placed with the distance of 23 mm.

The range of the excited frequency is from 5 Hz to 11 Hz, the acceleration amplitude is a constant value of 1 m/s^2. Figure 3 shows the open-circuit piezoelectric output voltages of the two DOFs, because the piezoelectric outputs are proportional to the corresponding vibration displacements of the cantilevers, it can be observed when the excited frequency is near the resonance of the 1st beam, this outer beam can amplify the vibration displacement at its DOF, leads to the drastic alteration of the magnetic coupling force in the nonlinear structure. Consequently, although the excited frequency is much lower than the resonance of the 2nd cantilever, the 2nd cantilever also vibrates strongly at its DOF.
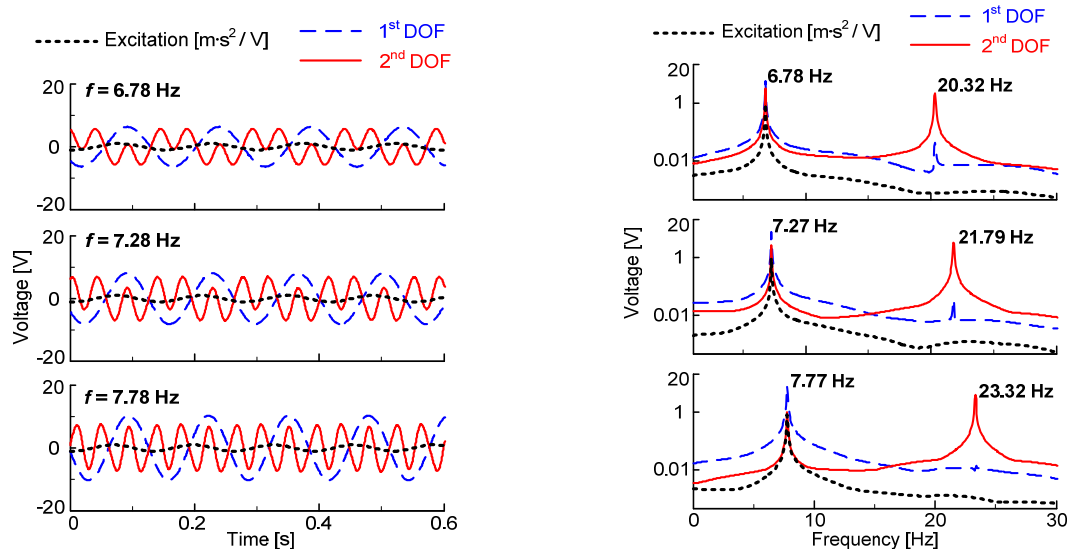
(a) Waveforms of the piezoelectric voltages and the excitation signal

(b) Frequency response of the piezoelectric voltages and the excitation signal

Figure 4. Simulation results of the frequency up-conversion effect under the different excited frequencies

The zoomed waveforms of the piezoelectric output voltages are plotted in figure 4(a), where the excited acceleration signal is also plotted in the same coordinate axes. From the figure, it is seen that the vibration of the 2nd DOF has two major vibration modes: low-order vibration mode whose frequency corresponds to the excited frequency, and high-order vibration mode whose frequency is three times increased. Obviously, the higher vibration value of the high-order mode corresponds to the better performance of the frequency up-conversion.
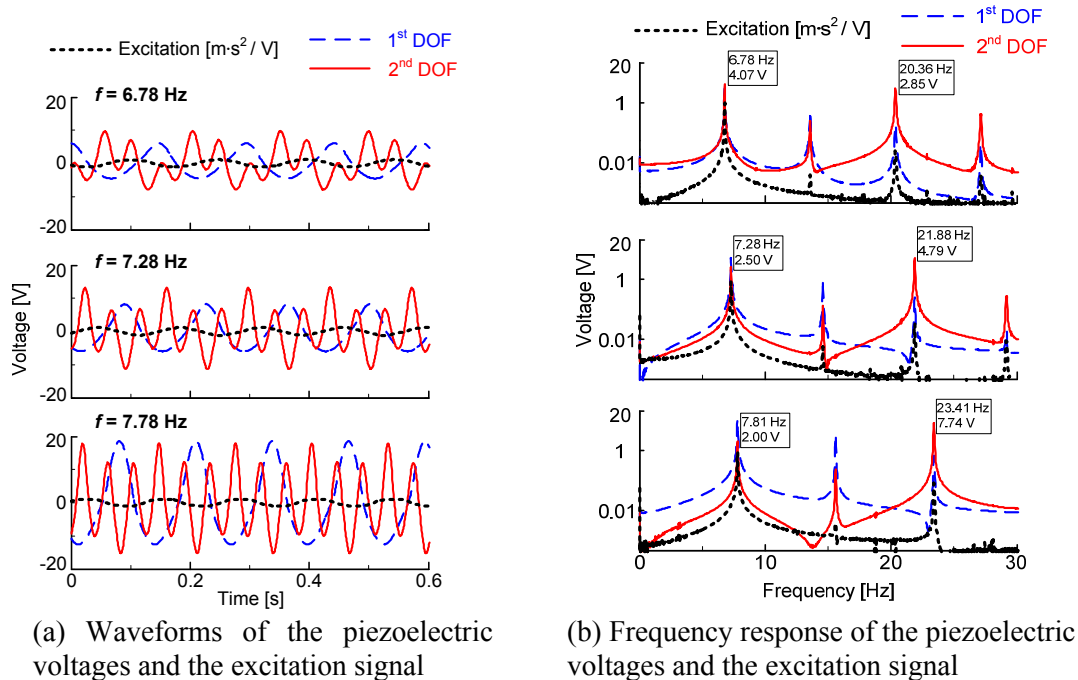
The performances of frequency up-converting harvester in the frequency domain are obtained by fast Fourier transformation, for several sweep excitations (around 6.78 Hz, 7.28 Hz and 7.78 Hz), as shown in Figure 4(b). It can be observed that the high-order vibration mode of the 2nd cantilever is excited, and the vibration frequency is exactly three times higher than the excited frequency. In addition, the excited frequency is more close to 7.78 Hz, the vibration amplitude of the 1st beam is larger, the energy percentage of the high-order vibration mode is also enlarged.

3.2. Experimental results and validation
The results in this sub-section were obtained from the experiment setup, shown in figure 2. As presented in the simulation sub-section, the experimental results of a typical frequency up-conversion effect is shown in figure 5. A forward sweep signal with a rate of 0.05 Hz/min is used as an excitation, two magnets are placed with the distance of 22.65 mm. The range of the excited frequency is 5~11 Hz and the acceleration amplitude is a constant value of 1 m/s^2.

However, due to the large vibration amplitude of the outer beam and the nonlinear magnetic force, it seems that there exists the harmonic distortion in the 1st beam, and the third harmonic component may cause the vibration of the 2nd cantilever. Based on the simulation analysis, the authors still think that the 1:3 internal resonance existed in the 2-DOF cubic nonlinear system is the main reason. In addition, when

the excited frequency is converging with the natural frequency of the 1st beam, the fundamental component of the 1st DOF is drastically enhanced, while the corresponding component in the 2nd DOF is decreased, which also validates the above conclusion on the other hand.

(a) Waveforms of the piezoelectric voltages and the excitation signal

(b) Frequency response of the piezoelectric voltages and the excitation signal

Figure 5. Experimental results of the frequency up-conversion effect under the different excited frequencies

4. Conclusion

In summary, the proposed compacted harvester design, which is comprised of the magnets and the simple cantilevers, offers the capability of the frequency up-conversion effect which is benefit to low-frequency vibration energy harvesting. Ongoing studies can aim at researching the frequency up-conversion effect as functions of the different design parameters and optimizing the coupling coefficient of the piezoelectric harvester.

References
[1]   Tang L, Yang Y and Soh C K 2013 *Advances in Energy Harvesting Methods* ed N Elvin and A Erturk (New York: Springer) chapter 2 pp 17-61
[2]   Daqaq M F, Masana R, Erturk A and Quinn D D 2014 *Applied Mechanics Reviews* **66** 040801
[3]   Pozzi M and Zhu M 2013 *Advances in Energy Harvesting Methods* ed N Elvin and A Erturk (New York: Springer) chapter 5 pp 119-140
[4]   Gu L and Livermore C 2011 *Smart Materials and Structures* **20** 045004
[5]   Han D and Yun K S 2015 *Microsystem Technologies* **21** 1669-1676
[6]   Guyomar D, Badel A, Lefeuvre E and Richard C 2005 *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* **52** 584-595
[7]   Wu H, Tang L, Yang Y and Soh C K 2014 *Journal of Intelligent Material Systems and Structures* **25** 1875-1889