Perturbation of circumsolar dust ring on stability of Sun-Earth triangular libration points

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Abstract. Asteroid collisions and cometary outgassing produce grains and dust that cause to fill the interplanetary space. The particles are temporarily trapped into orbit of the Earth to develop a circumsolar dust ring. The ring encloses the Sun-Earth equilateral triangular libration points in terms of Restricted Three-Body Problem (RTBP). On the other hand, regions close to the triangular points are preferable as locations for placing astronomical satellite. Here we study the planar-circular RTBP of Sun-Earth system with considers the oblateness of Earth and the presence of a circumsolar dust ring. Perturbation of the dust ring on linear stability of the points is discussed and we find that the points are still stable. However, in general the presence of the circumsolar (resonant) dust ring decreases value of the critical mass parameter.

1. Introduction

It has been known that Earth has a circumsolar dust ring of asteroidal and cometary origin [1]. The dust and grains are temporarily trapped into orbit of the Earth (figure 1). The ring embeds Earth along orbiting the Sun. From observations taken by on board infrared instruments of some satellites there is an asymmetry zodiacal cloud that envelops the inner solar system. The asymmetry of the zodiacal cloud can be accounted for this dust ring. The cloud has micrometer-sized particles and can be removed by drag forces (Poynting-Robertson and solar wind) that eventually spiral in the particles towards the Sun.

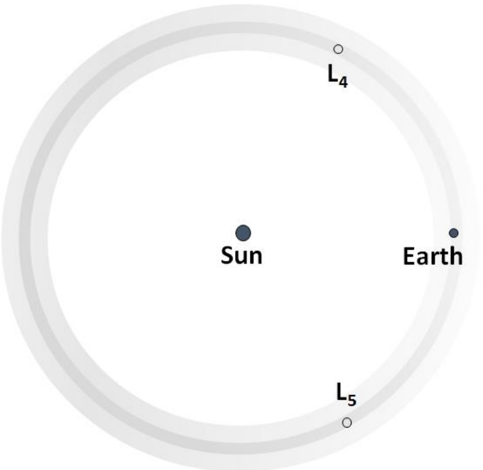

Figure 1. Illustration of circumsolar dust ring. Darker color represents higher number particle density.

The dust ring is located in Earth's orbit and consequently, it covers the Sun-Earth equilateral triangular libration (Lagrangian) points L_4 and L_5 (see figure 1) with respect to the Restricted Three-Body Problem (RTBP). The triangular points are considered as future potential and promising locations for hovering satellites such as that for Earth-Moon system [2]. There are at least two scientific interests for placing satellites around the Sun-Earth triangular points. First, the points are ideal locations for monitoring the space-weather away from the Sun-Earth line; and second, the L_4 and L_5 regions may harbor asteroids [3] that of significant interest to small body experts. Recent study [4] describes an attractive behavior and transfer mechanism between the triangular points.

Because the triangular points may efficiently harbor satellite for various astronomical purposes, understanding accurate locations of the triangular points becomes important. Singh and Taura [5,6] considered effects of radiation, oblateness, and the presence of circular cluster of material points on the equilibrium points in RTBP. They found that locations of the triangular points are influenced by the additional effects. Here we study the Sun-Earth triangular libration points in Planar-Circular RTBP under the influences of oblateness of Earth and the presence of circumsolar dust ring.

2. Equations of motion

Classical RTBP studies motion of a third mass-less body under the gravitational forces of the point-mass primaries (massive bodies). Orbits of the primaries around their mutual center of mass are circular and the third body has a planar orbit. In this work the classical RTBP is modified by including additional effects. Potential due to the dust ring [7] is:

$$\frac{m_r}{\sqrt{r^2+d^2}},$$

where m_r is the total mass of the dust, r the radial distance of the third infinitesimal body, and d the parameter that represents the flatness and the core size of the dust ring.

In rotational coordinate system, locations of the Sun, Earth, and the third infinitesimal body are, respectively, (ξ_1,η_1), (ξ_2,η_2), and (ξ,η). Under the effects of oblateness and circular dust ring, equations of motion of the third body in barycentric and dimensionless coordinate system $\left(\overline{\xi}=\xi/1\,\mathrm{au}, \overline{\eta}=\eta/1\,\mathrm{au}\right)$ are:

$$\frac{d^2\overline{\xi}}{df^2}-2\frac{d\overline{\eta}}{df}=\frac{d\overline{V}}{d\overline{\xi}},\quad \frac{d^2\overline{\eta}}{df^2}+2\frac{d^2\overline{\xi}}{df}=\frac{d\overline{V}}{d\overline{\eta}},$$

where the potential

$$\overline{V}=\frac{n^2}{2}\left(\overline{\xi}^2+\overline{\eta}^2\right)+\frac{(1-\mu)}{r_1}+\frac{\mu}{r_2}\left(1+\frac{A_E}{2r_2^2}\right)+\frac{m_r}{\sqrt{r^2+d^2}}, \tag{1}$$

with

$$n^2=1+\frac{3}{2}A_E+\frac{2m_r r_c}{\left(r_c^2+d^2\right)^{3/2}},\quad \mu=\frac{m_E}{m_S+m_E},$$

$$r_i^2=\left(\overline{\xi}-\overline{\xi}_i\right)^2+\left(\overline{\eta}-\overline{\eta}_i\right)^2,\ i=1,2;$$

A_E is Earth's oblateness coefficient derived from its equatorial and polar radii [5], n is known to be the mean motion, r_c is radial distance of the third body in the classical RTBP, μ is the mass parameter with m_E is Earth mass and m_S is mass of the Sun.

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3. Location of triangular equilibrium points

Conditions of equilibrium are satisfied when the partial derivatives of components of the potential (1) are zero: $\overline{V}_{\bar{\xi}} = \overline{V}_{\bar{\eta}} = 0$. After some algebra, the conditions produce coordinate of the triangular equilibrium points $(\bar{\xi}_0, \bar{\eta}_0)$, i.e. $\bar{\xi}_0 \neq 0$ and $\bar{\eta}_0 \neq 0$:

$$\begin{aligned}\bar{\xi}_0 &= \frac{1}{2} - \mu - \frac{A_E}{2}, \\ \bar{\eta}_0 &= \pm\sqrt{3}\left[\frac{1}{2} - \frac{A_E}{6} - \frac{2m_r(2r_c - 1)}{9(r_c^2 + d^2)^{3/2}}\right].\end{aligned} \tag{2}$$

(+ for L_4 and – for L_5)

In classical RTBP coordinates of the triangular points are $\left(\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$. From (2) it is clear that effects of Earth oblateness and the dust rings shift locations of the triangular points of classical RTBP. Earth oblateness affects both abscissa and ordinates of the triangular points. While the mass parameter changes only the abscissa of the points, the dust ring (mass and physical properties) influences only on the ordinates.

4. Linear stability and critical mass parameter

To examine the linear stability of the triangular equilibrium points, small displacements to the points are added: $u = \bar{\xi} - \bar{\xi}_0$, $v = \bar{\eta} - \bar{\eta}_0$. By taking linear part of Taylor expansion, the equations of motion become

$$\ddot{u} - 2\dot{v} = \overline{V}_{\bar{\xi}\bar{\xi}} u + \overline{V}_{\bar{\xi}\bar{\eta}} v, \quad \ddot{v} + 2\dot{u} = \overline{V}_{\bar{\xi}\bar{\eta}} u + \overline{V}_{\bar{\eta}\bar{\eta}} v, \tag{3}$$

where dots represent derivatives with respect to time (t). Arranging $u = C_1 e^{\lambda t}$ and $v = C_2 e^{\lambda t}$, where C_1 and C_2 are constant coefficients, and second partial derivatives of the potential (1) with respect to its components, yields the corresponding characteristic equation

$$\lambda^4 + \left(4 - \overline{V}^0_{\bar{\eta}\bar{\eta}} - \overline{V}^0_{\bar{\xi}\bar{\xi}}\right)\lambda^2 + \overline{V}^0_{\bar{\xi}\bar{\xi}} \overline{V}^0_{\bar{\eta}\bar{\eta}} - \left(\overline{V}^0_{\bar{\xi}\bar{\eta}}\right)^2 = 0,$$

that the second partial derivatives are evaluated at the triangular points $(\bar{\xi}_0, \bar{\eta}_0)$ to obtain the characteristic's roots (λ). Distinct pure imaginary roots reveal stable motion of the third body around the triangular equilibrium points.

By taking the discriminant of the characteristic equation equals zero, the critical mass parameter (μ_c) can be derived,

$$\mu_c = \frac{1}{2}\left(1 - \sqrt{\frac{23}{27}}\right) - \frac{1}{9}\left(1 - \frac{13}{\sqrt{69}}\right)A_E + \left[\frac{3}{2} - \frac{83 + 12r_c^2}{6\sqrt{69}} + \frac{(76 - 8r_c)(r_c^2 + d^2)}{27\sqrt{69}}\right]\frac{m_r}{(r_c^2 + d^2)^{5/2}}. \tag{4}$$

Motion of the third infinitesimal body is stable when $0 \leq \mu \leq \mu_c$.

From (4), by inserting values of A_E, r_c, m_r, and d, we obtain value of μ_c. Comparing value of μ for Sun-Earth system to the μ_c, we find that μ is much less than μ_c. Therefore, motion of the third body, such as a satellite, close to the Sun-Earth triangular equilibrium points remains stable under the influence of the Earth oblateness and a circumsolar dust ring. In general, the denser the dust ring, the lower value the critical mass parameter.

5. Conclusions

Linear stability of the Sun-Earth triangular equilibrium points has been studied by including effects of Earth oblateness and circumsolar dust ring. Both effects shift coordinate of the triangular points compared to that of the classical one. Particularly, effect of the circumsolar dust ring changes slightly the ordinates of the points. We also find that the denser the dust ring, the lower value the critical mass parameter. Although under the influence of a circumsolar dust ring, motion of the third infinitesimal body, such as a satellite, remains stable around the triangular equilibrium points.

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