Theoretical Investigation of Dual Tuning of Solitonic Processes in Multiferroic Structures

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Abstract. The solitonic wave processes in a multiferroic structure based on ferroelectric and ferrite layers are studied. The influence of external electric and magnetic fields on frequency and wave-number ranges, where bright and dark solitons can exist, are analysed. The investigation was carried out with the nonlinear Schrödinger equation. Results show that an increase of the electric field shifts the boundary between bright and dark solitons to long-wave region. An increase in magnetic field results in the opposite effect.

The term "soliton" usually refers to a propagating nonlinear pulse or wave packet which preserves its shape without dispersive spreading. Study of envelope solitons is one of the fundamental problem in the modern physics. Investigation is carried out in many areas of physics, for example, the physics of wave phenomena in ferromagnetic films, nonlinear optics [1, 2], low-temperature physics [3] etc.

It is well known that the formation and propagation of the envelope solitons are usually described using the nonlinear Schrödinger equation (NLSE) [4]. This equation and its solution were intensively investigated for different waveguiding media [5-7]. It is clear that the equation coefficients describe characteristic features of waveguide medium where solitons can exist. Therefore, tuning the medium properties leads to a change in the coefficients and consequently, in the end results in a modification of the soliton parameters. From the fundamental and applied point of view, it is interesting to analyze a relation between the properties of the medium and the soliton parameters.

Media, where waves with different nature can propagate simultaneously, attracts special scientific interest. One of the most striking examples is multiferroics. Active study of artificial multiferroic media in a form of layered structures composed of ferromagnetic and ferroelectric layers has recently begun [8-10]. Their linear wave properties were well studied. At the same time, there are only few papers devoted to the investigation of nonlinear wave properties of the multiferroics [11-14]. Thus, dependences of the soliton parameters on multiferroic properties remain unexplored.

A main advantage of the multiferroic structures in comparison with other waveguide media is dual tunability of their properties by external electric and magnetic fields. This behavior is due to electrodynamic interaction between the microwave electromagnetic and spin waves in the layered ferrite-ferroelectric structures. This interaction leads to formation of hybrid spin-electromagnetic waves (SEW). Note, that electric tuning of the SEW spectrum is possible due to an explicit dependence of dielectric permittivity in the ferroelectric layer on the bias electric field whereas magnetic tuning is provided by a dependence of magnetic permeability of the ferrite layer on the bias magnetic field.

The aim of this work is to study the influence of external electric and magnetic fields on peculiarities of solitonic excitations. For that purpose, we have investigated the influence of the electric and the magnetic fields on the dispersion and nonlinear coefficients of NLSE. This investigation have enabled us to study the influence of fields changing on frequencies and wave-number ranges where bright and dark solitons can exist. It is worth noting that the bright solitons are bell-shaped and they can be described with the hyperbolic secant function. The dark solitons have the form of localized "dark" holes created on the CW background. The last one are described with hyperbolic tangent.

In contrast with the previous work [11], where our attention was focused on an influence of *AC* electric and magnetic fields on the solitonic processes, this work is devoted to dual tunability by the external *DC* electric and magnetic fields.

In the generalized coordinates the nonlinear Schrödinger equation has the following form

$$i\frac{\partial u}{\partial t}+\frac{D}{2}\frac{\partial^2 u}{\partial x^2}-N\left|u\right|^2 u=-i\Gamma u, \qquad (1)$$

where $D=\partial^2\omega/\partial k_x^2$ is the dispersion coefficient, $N=\partial\omega/\partial\left|u\right|^2$ is the nonlinear coefficient, and Γ is the relaxation frequency.

It is important to note that the envelope solitons are formed due to a balance between two opposite wave processes. The dispersion spreading and the self-phase modulation compete with each other during the propagation of waves. The first effect is described by the dispersion coefficient D; the second one is described by the nonlinear coefficient N. It is well known that if both coefficients have the same signs $DN>0$, then dark solitons can be formed. If the signs are opposite $DN<0$, then bright solitons can be excited [1]. It was recently shown that multiferroics have double wave nonlinearity [11]. This nonlinearity can lead to a change of the coefficient signs in different ranges of wave numbers. Hence, if we tune the coefficients, we obtain a modification of the ranges of the bright and dark solitons modify.

The investigation was carried out in several steps. The linear dispersion relation has been derived and solved numerically. The double wave nonlinearity in the dispersion relation was considered. The dispersion and nonlinear coefficients were calculated by taking the derivatives of the relation. Then we have investigated the influence of the external electric and magnetic fields on the coefficients. Finally, we have analyzed the dual tunability of frequency and wave-number ranges, where bright and dark solitons can exist.

The investigated multiferroic structure is shown in figure 1. The bilayer structure was considered. A substrate for the ferrite layer is negligible in view of its relatively small dielectric permittivity and absence of magnetic properties.

We assume that the SEW propagate along the x-axis in the tangentially magnetized structure. The z-axis is parallel to the direction of the bias magnetic field. The structure is infinite in the x-z-plane and is surrounded by free space. Therefore, in the considered case we can use the linear dispersion equation of the SEW from the [15].

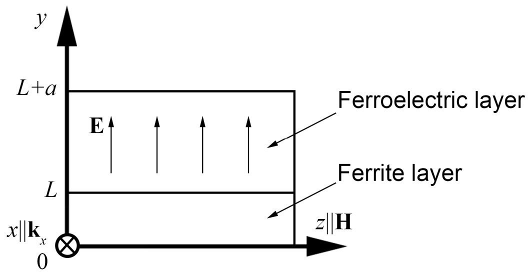

Figure 1. Geometry of the investigated structure

Solving the linear dispersion equation, we have used parameters which are typical for an yttrium iron garnet (YIG), namely, the saturation magnetization was $M_0 = 1750$ G, the permittivity was $\varepsilon_f = 14$ and the thickness was $L = 20\ \mu$m. The value of the external magnetic field was $H = 1500$ Oe. The ferroelectric layer thickness was $a = 500\ \mu$m, dielectric permittivity was equal to 660 at zero electric field. Note that these parameters correspond to the barium strontium titanate (BSTO) ceramic in paraelectric state at room temperature [16].

In addition, we have varied the external electric and magnetic fields. It is necessary to point out that the parameter, which describes the influence of the magnetic field, has been included in the dispersion equation. However, this equation does not involve the electric filed itself. To take into account the electric field we used the following dependence of the BSTO dielectric permittivity:

$$\varepsilon_a = \varepsilon_{a0} - rE^2, \quad (2)$$

where the coefficient r was calculated from the experimental work [16] and its value was equal to 7.7×10^{-8} m^2 / V^2.

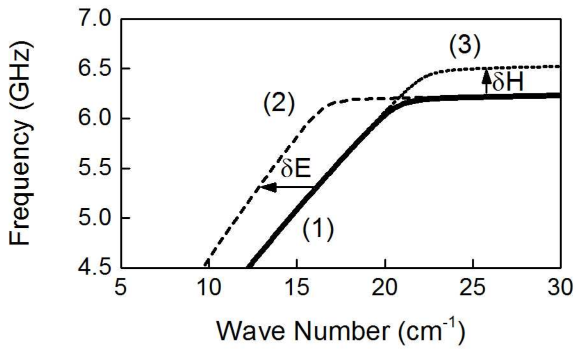

Figure 2. Dispersion characteristic of the SEW, where $\delta E = 20$ V/cm, $\delta H = 100$ Oe.

The numerical solutions of the dispersion equation for different values of the fields are shown in figure 2. The curves on the figure are numbered. These numbers are corresponded to the different parameter sets. The parameters from the first set are mentioned above. The other sets differ only by values of external electric and magnetic fields. The difference is given below.

It is well known that the SEW spectrum consists of the fast and the slow dispersion branches, but only slow branches are plotted for better visibility. Note that the slow SEW are usually excited in experiments. For that reason, our main attention was focused on these waves.

The spectrum of the SEW is formed because of hybridization of spin waves in the ferrite layer and electromagnetic waves in the ferroelectric layer. There is the point of the phase synchronism between these two types of waves. This point has the following coordinates $f = 6.214$ GHz, $k_x = 20$ cm^{-1} at $\delta E = 0$ and $\delta H = 0$. Due to the hybridization, the waves before and after the phase synchronism point have different nature. This statement has been corroborated by the investigation of the electric and magnetic fields variation.

In our investigation we have found that the increase of the electric field leads to a shift of that part of spectrum which corresponds to the electromagnetic-like waves. The dashed curve in the figure 2 demonstrates this. The curve was calculated using $\delta E = 20$ V/cm and it corresponds to the second parameter set. The increase of the magnetic field shifts the other part of the dispersion curve. This part demonstrates spin-wave-like behavior. In this case we used $\delta H = 100$ Oe. This case corresponds to the third parameter set. The point of the phase synchronism moves to a short-wave region with the magnetic field increase and vice versa to a long-wave region with the electric field increase.

Therefore, the variations of electric and magnetic fields result in different spectrum transformation. This difference is defined by hybrid nature of the SEW. It is the striking example that SEW are highly tuned and as a consequence of that devices based on the SEW will have high tunability, respectively.

The nonlinear dispersion relation has been derived in order to obtain the NLSE coefficients. This relation includes the double wave nonlinearity. Previously it was shown that the double nonlinearity should be taken into account for the multiferroic structures [11]. In accordance with this paper, we have transformed the *linear* dispersion equation into *nonlinear*. Subsequently, the formulae describing the dispersion and nonlinear coefficients were obtained.

In spite of the nonlinear coefficient of spin-waves in a free ferrite film, which can be either positive or negative in the whole wave-number range, the SEW nonlinear coefficient changes the sign in the different ranges of the wave numbers. The curves in figure 3(a) demonstrate this statement. Changing in signs is conditioned by the double wave nonlinearity of the SEW.

The dispersion coefficient of the SEW does not change the sign and it remains negative in the whole wave-number range. The values of the expression $|DN| / \max(|DN|)$ are shown in figure 3(b). As it was mentioned above the sign of DN defines the type of the envelope solitons that can propagate in media described by the NLSE.

It is clearly seen from the figure 3 that the electric and magnetic variation have different influence on the nonlinear and dispersion coefficients. The increase of the electric filed leads to the shift of the nonlinear coefficient curve to a long-wave region, while the magnetic field variation acts oppositely. The ranges of wave numbers, where either bright or dark solitons can exist, are shifted synchronously with the nonlinear coefficient. Hence, these ranges can be tuned by the electric and magnetic field. These fields variation impacts oppositely.

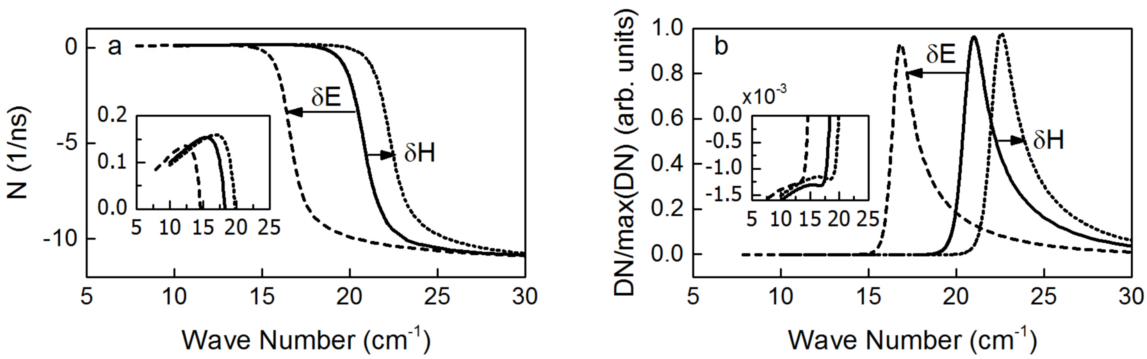

Figure 3. The nonlinear coefficient (a). The product of the dispersion and the nonlinear coefficients normalized on maximum value (b).

Table 1. Summary table of the parameter sets.

No	H, Oe	E, V/cm	ε_a	k_b, cm^{-1}	f_b, GHz
1	1500	0	660	18.5	5.770
2	1500	20	456	14.6	5.720
3	1600	0	660	20	6.070

The table 1 provides a summary of the results. In this table k_b and f_b designate the boundary wave number and the boundary frequency respectively. This boundary values separate the ranges, where bright and dark solitons can exist. The bright solitons exist if a carrier wave number and frequency are less than k_b and f_b respectively and contrariwise if a carrier wave number and frequency are greater than k_b and f_b respectively, then the dark solitons can exist.

In summary, as a result of the investigation it has been obtained that, the nonlinear solitonic excitations can be tuned by variation of the external electric and magnetic fields. As it was shown the tuning is possible due to hybrid nature of the SEW. The magnetic field influences spin-wave nature of the SEW whereas electromagnetic nature of the SEW can be controlled by the electric field. The effect of these fields is the opposite. The increase of the electric field shifts the phase synchronism point and the boundary between bright and dark solitons to long-wave region. On the other hand, the increase of the magnetic field can lead to the reverse effect.

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