

Quantum Codes From Cyclic Codes Over The Ring R_2

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Abstract. Let R_2 denotes the ring $F_2 + uF_2 + v_2 + uvF_2 + wF_2 + uwF_2 + vwF_2 + uvwF_2$. In this study, we construct quantum codes from cyclic codes over the ring R_2 , for arbitrary length n , with the restrictions $u^2 = 0, v^2 = 0, w^2 = 0, uv = vu, uw = wu, vw = wv$ and $u(vw) = (uv)w$. Also, we give a necessary and sufficient condition for cyclic codes over R_2 that contains its dual. As a final point, we obtain the parameters of quantum error-correcting codes from cyclic codes over R_2 and we give an example of quantum error-correcting codes form cyclic codes over R_2 .

1. Introduction

Quantum error-correcting codes play a prominent role both quantum communication and quantum computation. The first error-correcting code was discovered by Shor in [1] and independently by Steane in [2]. Calderbank *et al.* gave a systematic way to construct quantum error-correcting code using classical error-correcting code, which was the problem of finding quantum error-correcting codes was transformed into the problem of finding additive self-orthogonal codes over the field F_4 in [3]. Also many quantum error-correcting codes have been constructed by using classical error-correcting codes over finite field F_q in [2, 4]. In addition, quantum codes were constructed from cyclic codes of odd length over the finite ring $F_2 + uF_2$ in [5]. In the light of the paper, quaternary construction of quantum codes from cyclic codes over $F_4 + uF_4$ was studied in [6]. Besides, quantum codes from cyclic codes of odd length n over finite commutative ring $F_2 + vF_2$ was constructed in [7]. We inspired the paper in [7] and construct quantum codes from cyclic codes over the ring R_2 .

This paper is organized as follows. In Section 2, we give some basic definitions and the structure of the ring R_2 . Also, we define a Gray map and construct linear and cyclic codes over the ring R_2 . In Section 3, we give fundamental theorem for quantum error-correcting codes and we obtain the parameters of quantum error-correcting codes from cyclic codes over R_2 and give some examples.

2. Preliminaries

The ring R_2 is defined as a characteristic 2 ring, which depends on the restrictions $u^2 = v^2 = w^2 = 0, uv = vu, uw = wu, vw = wv$ and $u(vw) = (uv)w$. In addition, it is clear that R_2 is



Table 1. Ideals and The number of the elements of the ideals

The ideals	The number of the elements	The ideals	The number of the elements
$I_0 = \{0\}$	$ I_0 = 1,$		
I_{uvw}	$ I_{uvw} = 2,$	$I_{u,vw}$	$ I_{u,vw} = 32,$
I_{uv}	$ I_{uv} = 4,$	$I_{v,uw}$	$ I_{v,uw} = 32,$
I_{uw}	$ I_{uw} = 4,$	$I_{w,uv}$	$ I_{w,uv} = 32,$
I_{vw}	$ I_{vw} = 4,$	$I_{uv,u+w}$	$ I_{uv,u+w} = 32,$
I_{uv+uw}	$ I_{uv+uw} = 4,$	$I_{uv,v+w}$	$ I_{uv,v+w} = 32,$
I_{uv+vw}	$ I_{uv+vw} = 4,$	$I_{uv,u+v+w}$	$ I_{uv,u+v+w} = 32,$
I_{uw+vw}	$ I_{uw+vw} = 4,$	$I_{uw,u+v}$	$ I_{uw,u+v} = 32,$
$I_{uv+uw+vw}$	$ I_{uv+uw+vw} = 4,$	$I_{uv,uw,v+w}$	$ I_{uv,uw,v+w} = 32,$
$I_{uv,uw} = I_{uv} \oplus I_{uw}$	$ I_{uv,uw} = 8,$	$I_{uv,vw,u+w}$	$ I_{uv,vw,u+w} = 32,$
$I_{uv,vw}$	$ I_{uv,vw} = 8,$	$I_{uv,vw,u+v+w}$	$ I_{uv,vw,u+v+w} = 32,$
$I_{uv,uw+vw}$	$ I_{uv,uw+vw} = 8,$	$I_{u+v,uv+uw}$	$ I_{u+v,uv+uw} = 32,$
$I_{uw,vw}$	$ I_{uw,vw} = 8,$	$I_{u,v}$	$ I_{u,v} = 64,$
$I_{uw,uv+vw}$	$ I_{uw,uv+vw} = 8,$	$I_{u,w}$	$ I_{u,w} = 64,$
$I_{vw,uv+uw}$	$ I_{vw,uv+uw} = 8,$	$I_{v,w}$	$ I_{v,w} = 64,$
$I_{uv+uw,uv+vw}$	$ I_{uv+uw,uv+vw} = 8,$	$I_{u,v+w}$	$ I_{u,v+w} = 64,$
I_u	$ I_u = 16,$	$I_{v,u+w}$	$ I_{v,u+w} = 64,$
I_v	$ I_v = 16,$	$I_{w,u+v}$	$ I_{w,u+v} = 64,$
I_w	$ I_w = 16,$	$I_{u+v,u+w}$	$ I_{u+v,u+w} = 64,$
I_{u+v}	$ I_{u+v} = 16,$	$I_{u+v,u+vw}$	$ I_{u+v,u+vw} = 64,$
I_{u+w}	$ I_{u+w} = 16,$	$I_{u+v,w+uv}$	$ I_{u+v,w+uv} = 64,$
I_{u+vw}	$ I_{u+vw} = 16,$	$I_{u+w,v+w}$	$ I_{u+w,v+w} = 64,$
I_{v+w}	$ I_{v+w} = 16,$	$I_{u+w,u+vw}$	$ I_{u+w,u+vw} = 64,$
I_{v+uw}	$ I_{v+uw} = 16,$	$I_{v+w,v+uw}$	$ I_{v+w,v+uw} = 64,$
I_{w+uv}	$ I_{w+uv} = 16,$	$I_{u+vw,v+uw}$	$ I_{u+vw,v+uw} = 64,$
I_{u+v+w}	$ I_{u+v+w} = 16,$	$I_{u+vw,w+uv}$	$ I_{u+vw,w+uv} = 64,$
I_{u+v+uw}	$ I_{u+v+uw} = 16,$	$I_{v+uw,w+uv}$	$ I_{v+uw,w+uv} = 64,$
I_{u+v+uv}	$ I_{u+v+uv} = 16,$	$I_{uv,u+w,v+w}$	$ I_{uv,u+w,v+w} = 64,$
I_{v+w+uv}	$ I_{v+w+uv} = 16,$	$I_{vw,u+v,u+w}$	$ I_{vw,u+v,u+w} = 64,$
$I_{u+v+w+uv}$	$ I_{u+v+w+uv} = 16,$	$I_{u,v,w}$	$ I_{u,v,w} = 128,$
$I_{uv,vw,uw}$	$ I_{uv,vw,uw} = 16,$	$I_1 = R_2$	$ I_1 = 256.$

isomorphic to

$$F_2[X, Y, Z] / \langle X^2, Y^2, Z^2, XY - YX, XZ - ZX, YZ - ZY \rangle.$$

We list all the ideals of R_2 as can be seen at Table 1. Due to the structure of the ideals, we can see the ideals are not all principal just as $I_{u,v,w}$ is not a principal ideal. The ideal $I_{u,v,w} = M$ is a maximal ideal of R_2 and R_2 is a local ring since it has a only one maximal ideal. And it is known that if a ring R is a local ring with maximal ideal M , then an element in $R - M$ is a unit in R . So, we can determine the units of R_2 as $R_2^* = R_2 - I_{u,v,w}$.

To define the dual code of a code in R_2^n , we give an inner product which is, in fact, the usual

inner product as:

$$\langle (x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \rangle = x_1 y_1 + x_2 y_2 + \dots + x_n y_n, \quad (1)$$

where the operations belong to the ring R_2 . Now we define the inner product for the ring R_2 at the same way concerning the dot product in F_2 as follows :

$$\begin{aligned} \langle \bar{a}_0 + u\bar{a}_1 + v\bar{a}_2 + uv\bar{a}_3 + w\bar{a}_4 + uw\bar{a}_5 + v\bar{a}_6 + uvw\bar{a}_7, \bar{b}_0 + u\bar{b}_1 + v\bar{b}_2 + uv\bar{b}_3 + \\ w\bar{b}_4 + uw\bar{b}_5 + v\bar{b}_6 + uvw\bar{b}_7 \rangle = \bar{a}_0 \cdot \bar{b}_0 + u(\bar{a}_0 \cdot \bar{b}_1 + \bar{a}_1 \cdot \bar{b}_0) + v(\bar{a}_0 \cdot \bar{b}_2 + \bar{a}_2 \cdot \bar{b}_0) + \\ uv(\bar{a}_0 \cdot \bar{b}_3 + \bar{a}_1 \cdot \bar{b}_2 + \bar{a}_2 \cdot \bar{b}_1 + \bar{a}_3 \cdot \bar{b}_0) + w(\bar{a}_0 \cdot \bar{b}_4 + \bar{a}_4 \cdot \bar{b}_0) + uw(\bar{a}_0 \cdot \bar{b}_5 + \bar{a}_1 \cdot \bar{b}_4 + \bar{a}_4 \cdot \bar{b}_1 + \\ \bar{a}_5 \cdot \bar{b}_0) + v\bar{a}_6 \cdot \bar{b}_6 + \bar{a}_2 \cdot \bar{b}_4 + \bar{a}_4 \cdot \bar{b}_2 + \bar{a}_6 \cdot \bar{b}_0 + uvw(\bar{a}_0 \cdot \bar{b}_7 + \bar{a}_1 \cdot \bar{b}_6 + \bar{a}_3 \cdot \bar{b}_4 + \bar{a}_4 \cdot \bar{b}_3 + \\ \bar{a}_5 \cdot \bar{b}_2 + \bar{a}_6 \cdot \bar{b}_1 + \bar{a}_0 \cdot \bar{b}_7 + \bar{a}_7 \cdot \bar{b}_0), \end{aligned}$$

where the symbol $[\cdot]$ denotes the dot product in binary vectors.

Definition 1 A code C of length n over the ring R_2 is a subset of R_2^n and C is linear if it is a submodule.

Definition 2 Let C be a linear code over R_2 of length n , then the dual of C is defined as

$$C^\perp = \{\bar{y} \in R_2^n : \langle \bar{y}, \bar{x} \rangle = 0, \forall \bar{x} \in C\}.$$

C^\perp is also a linear code over R_2 of length n resulting from the definition of the inner product in Eq. 1. A code C is self orthogonal if $C \subseteq C^\perp$ and self dual if $C = C^\perp$.

We denote that $A \oplus B = \{a + b : a \in A, b \in B\}$ and $A \otimes B = \{(a, b) : a \in A, b \in B\}$.

Definition 3 For any ring R , a cyclic shift on R^n is a permutation σ such that $\sigma(q_0, q_1, \dots, q_{n-1}) = (q_{n-1}, q_0, \dots, q_{n-2})$. And a linear code C over a ring R that is invariant under the cyclic shift is called a cyclic code.

Definition 4 A Gray map ψ from R_2^n to F_2^{8n} can be defined as

$$\psi(\bar{a}_0 + u\bar{a}_1 + v\bar{a}_2 + uv\bar{a}_3 + w\bar{a}_4 + uw\bar{a}_5 + v\bar{a}_6 + uvw\bar{a}_7) = \left(\bar{a}_7, \sum_{i=6,7} \bar{a}_i, \sum_{i=5,7} \bar{a}_i, \sum_{i=4,5,6,7} \bar{a}_i, \sum_{i=3,7} \bar{a}_i, \sum_{i=2,3,6,7} \bar{a}_i, \sum_{i=1,3,5,7} \bar{a}_i, \sum_{i=0}^7 \bar{a}_i \right).$$

The Gray map ψ is a weight preserving map from R_2^n (Lee weight) to F_2^{8n} (Hamming weight). And if C is self orthogonal, so is $\psi(C)$.

Definition 5 For any $q \in R_2$, we define the Lee weight as

$$w_L(q) = w_H(\psi(q)),$$

where w_H denotes well-known Hamming weight for binary codes.

Let C be a linear code of length n over R_2 . Define

$$\begin{aligned} C_1 &= \{a_7 \in F_2^n : \exists a_i \in F_2^n, 0 \leq i \leq 6, q \in C\} \\ C_2 &= \{a_6 + a_7 \in F_2^n : \exists a_i \in F_2^n, 0 \leq i \leq 5, q \in C\} \\ C_3 &= \{a_5 + a_7 \in F_2^n : \exists a_0, a_1, a_2, a_3, a_4, a_6 \in F_2^n, q \in C\} \\ C_4 &= \{a_4 + a_5 + a_6 + a_7 \in F_2^n : \exists a_i \in F_2^n, 0 \leq i \leq 3, q \in C\} \\ C_5 &= \{a_3 + a_7 \in F_2^n : \exists a_0, a_1, a_2, a_4, a_5, a_6 \in F_2^n, q \in C\} \\ C_6 &= \{a_2 + a_3 + a_6 + a_7 \in F_2^n : \exists a_0, a_1, a_4, a_5 \in F_2^n, q \in C\} \\ C_7 &= \{a_1 + a_3 + a_5 + a_7 \in F_2^n : \exists a_0, a_2, a_4, a_6 \in F_2^n, q \in C\} \\ C_8 &= \left\{ \sum_{i=0}^7 a_i \in F_2^n : q \in C \right\}, \end{aligned}$$

where $q = a_0 + ua_1 + va_2 + uva_3 + wa_4 + uwa_5 + vwa_6 + uvwa_7$. Then C'_i 's are binary codes at length n for every $i = 1, 2, \dots, 8$. Furthermore, the linear code C is uniquely defined as

$$\begin{aligned} C = & (1 + u + v + uv + w + uw + vw + uvw) C_1 \oplus (u + uv + uw + uvw) C_2 \oplus \\ & (v + uv + vw + uvw) C_3 \oplus (w + uw + vw + uvw) C_4 \oplus (uv + uvw) C_5 \oplus \\ & (uw + uvw) C_6 \oplus (vw + uvw) C_7 \oplus (uvw) C_8. \end{aligned}$$

Theorem 1 Let C be a linear code of length n over R_2 , then $\psi(C) = \bigotimes_{i=1}^8 C_i$ and $|C| = \prod_{i=1}^8 |C_i|$.

Here,

$$|C| = \prod_{i=1}^8 |C_i| = 2^{n-\deg f_1} \cdot 2^{n-\deg f_2} \cdot \dots \cdot 2^{n-\deg f_8} = 2^{8n - \sum_{i=1}^8 \deg f_i},$$

where f'_i 's are generator polynomials of C' 's respectively for $i = 1, 2, \dots, 8$.

$d_L = \min \{d_H(C_i) : i = 1, 2, \dots, 8\}$ where d_L denotes the minimum Lee weight of linear code C over R_2 and $d_H(C_i)$ denotes the minimum Hamming weights of binary codes C'_i 's respectively.

Proposition 1 Let

$$\begin{aligned} C = & (1 + u + v + uv + w + uw + vw + uvw) C_1 \oplus (u + uv + uw + uvw) C_2 \oplus \\ & (v + uv + vw + uvw) C_3 \oplus (w + uw + vw + uvw) C_4 \oplus (uv + uvw) C_5 \oplus \\ & (uw + uvw) C_6 \oplus (vw + uvw) C_7 \oplus (uvw) C_8 \end{aligned}$$

be a linear code over R_2 . Then C is a cyclic code over R_2 if and only if C'_i 's are binary cyclic codes.

3. Quantum Codes From Cyclic Codes Over R_2

We start to this section by giving well-known theorem on quantum codes:

Theorem 2 (CSS Construction) Let C and C' be two binary codes with parameters $[n, k_1, d_1]$ and $[n, k_2, d_2]$, respectively. If $C^\perp \subseteq C'$, then an $[[n, k_1 + k_2 - n, \min \{d_1, d_2\}]]$ quantum code can be constructed. Especially, $C^\perp \subseteq C$, then there exists an $[[n, 2k_1 - n, d_1]]$ quantum code.

Proposition 2 Suppose

$$\begin{aligned} C = & (1 + u + v + uv + w + uw + vw + uvw) C_1 \oplus (u + uv + uw + uvw) C_2 \oplus \\ & (v + uv + vw + uvw) C_3 \oplus (w + uw + vw + uvw) C_4 \oplus (uv + uvw) C_5 \oplus \\ & (uw + uvw) C_6 \oplus (vw + uvw) C_7 \oplus (uvw) C_8 \end{aligned}$$

is cyclic code of length n over R_2 . Then

$$\begin{aligned} C = & \langle (1 + u + v + uv + w + uw + vw + uvw) f_1, (u + uv + uw + uvw) f_2, \\ & (v + uv + vw + uvw) f_3, (w + uw + vw + uvw) f_4, (uv + uvw) f_5, \\ & (uw + uvw) f_6, (vw + uvw) f_7, (uvw) f_8 \rangle \end{aligned}$$

and

$$|C| = 2^{8n - \sum_{i=1}^8 \deg f_i},$$

where f_i' 's are generators polynomials of C_i' 's respectively. Also,

$$C^\perp = \langle (1 + u + v + uv + w + uw + vw + uvw) h_1^* + (u + uv + uw + uvw) h_2^* + \\ (v + uv + vw + uvw) h_3^* + (w + uw + vw + uvw) h_4^* + (uv + uvw) h_5^* + \\ (uw + uvw) h_6^* + (vw + uvw) h_7^* + (uvw) h_8^* \rangle$$

and

$$|C^\perp| = 2^{\sum_{i=1}^8 \deg f_i},$$

where for $i = 1, 2, \dots, 8$, h_i^* are the reciprocal polynomials of h_i i.e., $h_i(x) = (x^n - 1)/f_i(x)$, $h_i^*(x) = x^{\deg h_i} h_i(\frac{1}{x})$ for $i = 1, 2, \dots, 8$.

Proposition 3 Suppose C is a cyclic code of length n over R_2 , then there is a unique polynomial $f(x)$ such that $C = \langle f(x) \rangle$ and $f(x)/(x^n - 1)$, where

$$f(x) = (1 + u + v + uv + w + uw + vw + uvw) f_1(x) + (u + uv + uw + uvw) f_2(x) + \\ (v + uv + vw + uvw) f_3(x) + (w + uw + vw + uvw) f_4(x) + (uv + uvw) f_5(x) + \\ (uw + uvw) f_6(x) + (vw + uvw) f_7(x) + (uvw) f_8(x).$$

Lemma 1 [3] A binary cyclic code C with generator polynomial $f(x)$ contains its dual code if and only if $x^n - 1 \equiv 0 \pmod{f^*}$ where f^* is the reciprocal polynomial of f .

Theorem 3 Let $C = \langle f(x) \rangle$ be a cyclic code of length n over R_2 and $f(x)/(x^n - 1)$, where

$$f(x) = (1 + u + v + uv + w + uw + vw + uvw) f_1(x) + (u + uv + uw + uvw) f_2(x) + \\ (v + uv + vw + uvw) f_3(x) + (w + uw + vw + uvw) f_4(x) + (uv + uvw) f_5(x) + \\ (uw + uvw) f_6(x) + (vw + uvw) f_7(x) + (uvw) f_8(x).$$

Then $C^\perp \subseteq C$ if and only if $x^n - 1 \equiv 0 \pmod{f_i f_i^*}$ for $i = 1, 2, \dots, 8$.

Corollary 1 Let

$$C = (1 + u + v + uv + w + uw + vw + uvw) C_1 \oplus (u + uv + uw + uvw) C_2 \oplus \\ (v + uv + vw + uvw) C_3 \oplus (w + uw + vw + uvw) C_4 \oplus (uv + uvw) C_5 \oplus \\ (uw + uvw) C_6 \oplus (vw + uvw) C_7 \oplus (uvw) C_8$$

is a cyclic code of length n over R_2 . Then $C^\perp \subseteq C$ if and only if $C_i^\perp \subseteq C_i$ for $i = 1, 2, \dots, 8$.

Example 1 For $n = 63$,

$$x^{63} - 1 = (x + 1)(x^2 + x + 1)(x^3 + x + 1)(x^3 + x^2 + 1)(x^6 + x + 1) \\ (x^6 + x^3 + 1)(x^6 + x^4 + x^2 + x + 1)(x^6 + x^4 + x^3 + x + 1) \\ (x^6 + x^5 + 1)(x^6 + x^5 + x^2 + x + 1)(x^6 + x^5 + x^3 + x^2 + 1) \\ (x^6 + x^5 + x^4 + x + 1)(x^6 + x^5 + x^4 + x^2 + 1) \\ = f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 f_9 f_{10} f_{11} f_{12} f_{13}$$

in $F_2[x]$. Hence, $f_1^* = f_1$, $f_2^* = f_3$, $f_3^* = f_2$. So, $x^{63} - 1$ is divisibly by $f_i f_i^*$, $i = 3, 4, 5, 7, 8, 9, 10, 11, 12, 13$. Thus we have a binary cyclic code over R_2

$$C = \langle (1 + u + v + uv + w + uw + vw + uvw) f_3, (u + uv + uw + uvw) f_4, \\ (v + uv + vw + uvw) f_5, (w + uw + vw + uvw) f_7, (uv + uvw) f_8, \\ (uw + uvw) f_9, (vw + uvw) f_{10}, (uvw) f_{11} \rangle$$

and its dual code is generated as following over R_2 by Proposition 2:

$$C^\perp = \{(1 + u + v + uv + w + uw + vw + uvw) f_3^* + (u + uv + uw + uvw) f_4^* + (v + uv + vw + uvw) f_5^* + (w + uw + vw + uvw) f_7^* + (uv + uvw) f_8^* + (uw + uvw) f_9^* + (vw + uvw) f_{10}^* + (uvw) f_{11}^*\}$$

Thus, the code C contains its dual code C^\perp .

By using Theorem 2 and Theorem 3 we can construct the parameters of quantum codes.

Theorem 4 Let

$$C = (1 + u + v + uv + w + uw + vw + uvw) C_1 \oplus (u + uv + uw + uvw) C_2 \oplus (v + uv + vw + uvw) C_3 \oplus (w + uw + vw + uvw) C_4 \oplus (uv + uvw) C_5 \oplus (uw + uvw) C_6 \oplus (vw + uvw) C_7 \oplus (uvw) C_8$$

be a cyclic code of arbitrary length n over R_2 with size

$$256^{k_1} 128^{k_2} 64^{k_3} 32^{k_4} 16^{k_5} 8^{k_6} 4^{k_7} 2^{k_8}.$$

If $C_i^\perp \subseteq C_i$ where $i = 1, 2, \dots, 8$ then $C^\perp \subseteq C$ and there exists a quantum error-correcting code with parameters

$$[[8n, 8k_1 + 7k_2 + 6k_3 + 5k_4 + 4k_5 + 3k_6 + 2k_7 + k_8 - 8n, d_H]].$$

Example 2 For $n = 4$, in $F_2[x]$

$$(x^4 - 1) = (x + 1)^2 (x^2 + 1).$$

Let $C = \langle (vw + uvw) f_1(x), (uvw) f_2(x) \rangle$, where $f_1(x) = (x + 1) = f_1^*(x)$ and $f_2(x) = (x^2 + 1) = f_2^*(x)$. In the light of Theorem 1, we know that C is linear cyclic codes with the parameters $[32, 3, 16]$. Also, with the help of Proposition 2, $C^\perp = \langle uvw(x^3) + vw(x^2) + uvw(x) + vw \rangle$ is a linear codes with the parameters $[32, 29, 2]$. We can clearly see $C^\perp \subseteq C$. Thus, we obtain a quantum code with parameters $[[16, 13, 2]]$.

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