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Parametric investigation of a thermally driven QCD Deconfining Phase Transition in a finite volume at zero chemical potential

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Abstract. This work deals with a statistical description of a thermally driven deconfining phase transition (DPT) from a hadronic gas consisting of massless pions to a color-singlet Quark-Gluon Plasma (QGP), in a finite volume. The thermodynamical approach, within a coexistence model is used to investigate the Quantum Chromo-Dynamics DPT occurring between the two phases, at vanishing chemical potential. Considering the color singletness condition for the QGP phase, with massless up and down quarks, the exact total partition function of the studied system is obtained and then employed to calculate mean values of physical quantities, well characterizing the system near the transition. The finite-size effects on the DPT have been investigated through the study of the thermal behavior of the order parameter, the susceptibility and the second cumulant of the probability density. The similarity between the susceptibility and the second cumulant representing the variance is probed for the studied DPT and a parameterization of the variance is proposed for the first time.

1. Introduction

According to the state of the art in particle physics, the quark-lepton level is considered to be the fundamental level. Quarks interact with each other via the strong interaction, where the mediator bosons are the gluons [1]. The strong interaction is described through the Quantum Chromo-Dynamics (QCD) theory, which confirms that free quarks are never observed as free particles, but are confined inside hadrons such as the proton and the neutron. However, at high temperatures and/or densities a hadronic matter (HM) can turn into a new state of matter called Quark-Gluon Plasma (henceforth QGP), which is composed of the elementary particles; quarks and gluons [2]. It is worth mentioning that the quarks and gluons are still confined inside the plasma, but not inside hadrons, whence, the nomination "Deconfinement Phase Transition"(DPT) from a HM phase to a QGP phase.

The study of the finite-size effects on the DPT from the strongly interacting Hadronic Gas (HG) to the called QGP is possible through the study of several thermodynamic quantities, called response functions, among which we cite: the order parameter, its derivatives with respect to temperature and the cumulants of the probability density. Such a study has been performed by one of us in previous contributions [3, 4], allowing to note a remarkable qualitative similarity between the variations as function of temperature and volume of the order parameter derivatives and those of their homologue cumulants of the probability density, namely: between the first

derivative of the order parameter and the variance representing the second cumulant, as well as between the second derivative of the order parameter and the skewness, which is related to the third cumulant.

In this work, we present a numerical study of a thermally driven QCD-DPT in order to investigate this similarity, which does not appear to have been previously developed, focusing on the following main objectives: a numerical study of the thermal QCD-DPT in a finite volume, an investigation of the similarity between the first derivative of the order parameter with respect to temperature, i.e., the susceptibility, and its homologue cumulant, i.e., the variance, and a parametric study aiming to provide an authentic approach to estimate the effective transition temperature for the studied DPT.

2. Statistical description of the Deconfinement Phase Transition in finite volumes

In order to conceive the QCD deconfinement phase transition in a finite volume, the coexistence model reported in [5] is a powerful tool. This model considers a mixed Hadronic Gas-Quark Gluon Plasma (HG-QGP) system, which exhibits a finite volume V. The volumes of the HG and QGP phases, noted V_{HG} and V_{QGP} respectively, form together the total volume V, and the fraction of volume occupied by the HG phase is designated by the parameter h, such that:

$$\begin{cases} V = V_{HG} + V_{QGP} \\ V_{HG} = hV. \end{cases} \tag{1}$$

The system is treated as a grand canonical ensemble, i.e., the thermodynamic variables are: the temperature (T), the volume (V) and the chemical potential (μ). In the framework of the coexistence model, the probability of finding the system in the state h is defined by [5]:

$$p(h,T,V,\mu) = \frac{Z(h,T,V,\mu)}{\int_0^1 Z(h,T,V,\mu)dh}, \tag{2}$$

here, $Z(h,T,V,\mu)$ is the total partition function of the system. The mean value of a given thermodynamic quantity is defined as [5]:

$$\langle X(T,V,\mu)\rangle = \int_0^1 X(h,T,V,\mu)p(h,T,V,\mu)dh = \frac{\int_0^1 X(h,T,V,\mu)Z(h)dh}{\int_0^1 Z(h)dh}, \tag{3}$$

where, $X(h,T,V,\mu)$ is the total thermodynamic quantity of the system in the state h.

Taking into account the color-charge confinement property, requiring that the QGP phase must be a colorless object in the color space [6], and under the assumption of non-interacting phases, the total partition function of the system can be expressed as follows:

$$Z(h) = \frac{4}{9\pi^2}\exp\left[\frac{\pi^2}{30}VT^3\right]\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} d\varphi d\psi M(\varphi,\psi)\exp\left[(1-h)VT^3\Big(g(\varphi,\psi) - \frac{\pi^2}{30} - \frac{B}{T^4}\Big)\right], \tag{4}$$

where, $M(\varphi,\psi)$ represents the weight function (Haar measure) having the following expression:

$$M(\varphi,\psi) = \left[\sin\left(\frac{1}{2}\Big(\psi+\frac{\varphi}{2}\Big)\right)\sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{1}{2}\Big(\psi-\frac{\varphi}{2}\Big)\right)\right]^2, \tag{5}$$

and:

$$g(\varphi,\psi) = \frac{\pi^2}{12}\Big(\frac{21}{30}d_q + \frac{16}{15}d_g\Big) + \frac{\pi^2}{12}\frac{d_q}{2}\sum_{q=r,b,g}\left[\left(\left(\frac{\theta_q}{\pi^2}\right)^2 - 1\right)^2 - 1\right] - \frac{\pi^2}{12}\frac{d_g}{2}\sum_{g=1}^{4}\left[\left(\frac{\theta_g-\pi}{\pi}\right)^2 - 1\right]^2, \tag{6}$$

here, $d_q = 2N_f$ (N_f: is the number of flavors) and $d_g = 2$ being the degeneracy factors of quarks and gluons respectively. The angles $\theta_q (q = r, b, g)$ are given by:

$$\theta_r = \frac{\varphi}{2} + \frac{\psi}{3},\ \theta_b = -\frac{2\psi}{3},\ \theta_g = -\frac{\varphi}{2} + \frac{\psi}{3}, \tag{7}$$

and $\theta_g (g = 1, ..., 4)$ are expressed as follows:

$$\theta_1 = \theta_r - \theta_g,\ \theta_2 = \theta_g - \theta_b,\ \theta_3 = \theta_b - \theta_r,\ \theta_4 = 0. \tag{8}$$

We note that our study is performed at a vanishing chemical potential ($\mu = 0$), taking into account massless u and d quarks, i.e., $N_f = 2$ in the QGP phase, and the pionic degrees of freedom in the HG phase. The value of the bag constant $B^{1/4} = 200\ MeV$ is adopted in all our calculations, with the units chosen as: $k_B = \hbar = c = 1$.

Based on the definition (3), the mean value of any physical quantity within the phase coexistence model may be calculated using the exact total partition function defined in (4). The first quantity of interest for the thermal DPT is the order parameter, which is the mean value of the volume fraction occupied by the HG phase, $\langle h(T, V) \rangle$, given by:

$$\langle h(T, V) \rangle = \frac{\int_0^1 h Z(h) dh}{\int_0^1 Z(h) dh}. \tag{9}$$

The second quantity is the first derivative of the order parameter with respect to temperature T, representing the susceptibility, χ, defined as:

$$\chi(T, V) = \left. \frac{\partial \langle h(T, V) \rangle}{\partial T} \right|_V. \tag{10}$$

On the other hand, we are interested in the cumulants of the probability density, $p(h)$, where the first cumulant is the order parameter $\langle h \rangle$ itself, and the second cumulant is the variance, σ^2, given by [7, 8]:

$$\sigma^2(T, V) = \langle h^2 \rangle - \langle h \rangle^2, \tag{11}$$

where $\langle h^2 \rangle$ is, using the definition (3), expressed as follows:

$$\langle h^2 \rangle = \frac{\int_0^1 h^2 Z(h) dh}{\int_0^1 Z(h) dh}. \tag{12}$$

The involved integrals in the considered response functions may be calculated numerically at each temperature, T, and volume, V, and this allows us to illustrate the variations of the order parameter, the susceptibility and the variance, on the whole adopted range of temperature, for various volumes.

3. Thermal behavior of the studied response functions

In the following, the thermal behavior of the considered response functions, namely, the order parameter, its first derivative with respect to temperature, as well as the second cumulant of the probability density is investigated on a range of temperature around the transition region at different finite-volumes. The considered volumes, i.e., 550, 700, 850 and 1000 fm^3 are included in the RHIC (Relativistic Heavy Ion Collider) realized sizes; (268-2144 fm^3) [9], where the QGP is eventually formed during ultra-relativistic heavy ion collisions.

The order parameter behavior in the range of temperature $143 - 146\ MeV$ at the considered finite volumes is illustrated in figure 1. The curves show a sharp transition in the infinite volume

limit, characterized by a step-like jump at a transition temperature noted $T_c(\infty)$. This latter has been found to depend on the number of quark flavors, N_f, as well as on the bag constant, B, and it is obtained for $N_f = 2$ according to the following relationship (see for example [10]):

$$T_c(\infty) = \left(\frac{45B}{17\pi^2}\right)^{1/4}, \tag{13}$$

for $B^{1/4} = 200\ MeV$, we find $T_c(\infty) = 143.93\ MeV$, and this value of the transition temperature agrees very well with that obtained from recent lattice simulations [10, 11]. In small volumes, the transition is smeared out over a transition region that is larger, smaller is the volume. Another characteristic of the obtained curves is also the shift of the transition temperature to higher values for small volumes ($T_c(V) > T_c(\infty)$). This last finding is due to the color-singletness requirement in the QGP partition function [5, 6].

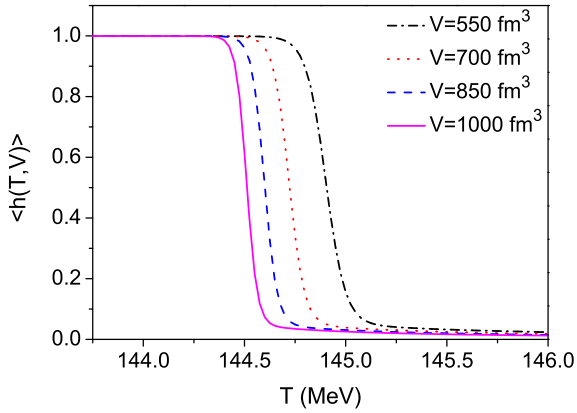

Figure 1. The order parameter variations with temperature for different volumes.

The variations of the susceptibility, $\chi(T, V)$, and its homologue cumulant, i.e., the variance, $\sigma^2(T, V)$, are displayed in figures 2 and 3, respectively. Both quantities behave like bell function, with the peaks broadened, smaller is the volume. The recorded peaks are located at the effective transition temperature $T_c(V)$, seemingly tending to $T_c(\infty)$ with increasing volume.

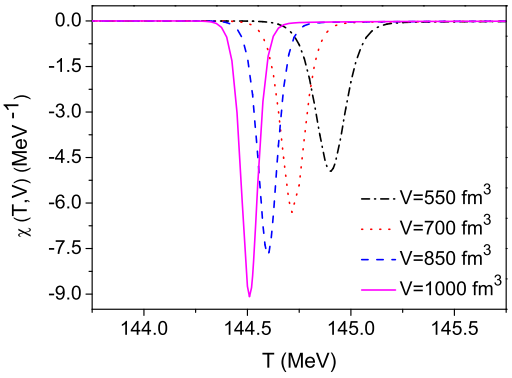

Figure 2. Thermal behavior of the susceptibility χ, for various system volumes.

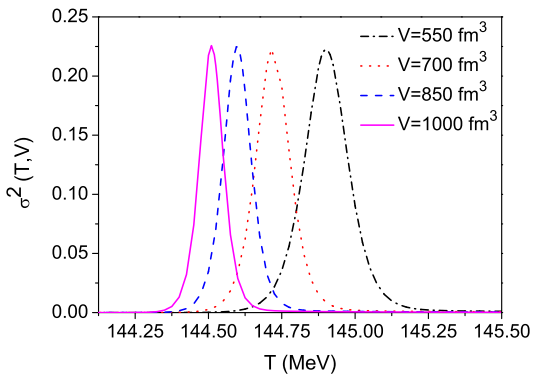

Figure 3. The variance σ^2 vs. temperature for different system volumes.

4. Correlation study and parametric investigation

4.1. Correlation between susceptibility and variance

In the following, let us investigate the similarity between the thermal susceptibility, χ, and the variance, σ^2, for the studied DPT. In a first step, we plot both χ and σ^2 with varying temperature, at a fixed volume ($V = 550\ fm^3$), on figure 4, considering a logarithmic scale, since their order of magnitude are different and do not allow to plot them clearly on the same graph. As expected, a remarkable similarity is observed with the peaks occurring at the same effective transition temperature $T_c(V)$.

In a second step, let us plot σ^2 vs. χ; the variations of the variance with the susceptibility for the considered values of the system volume are illustrated graphically in figure 5. The obtained plots reflect a linear proportionality, where the slope is inversely proportional to the system size. Thus, the variance is proportional to the susceptibility with a proportionality factor depending on the volume V.

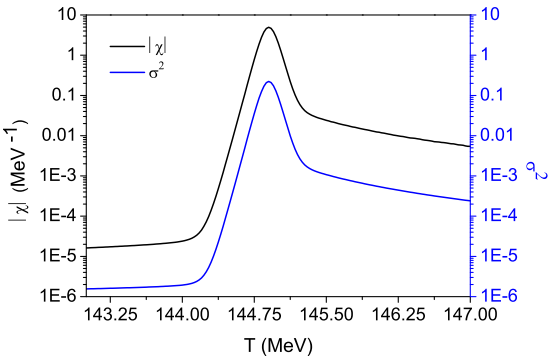

Figure 4. Evolution of the susceptibility and the variance at $V = 550\ fm^3$.

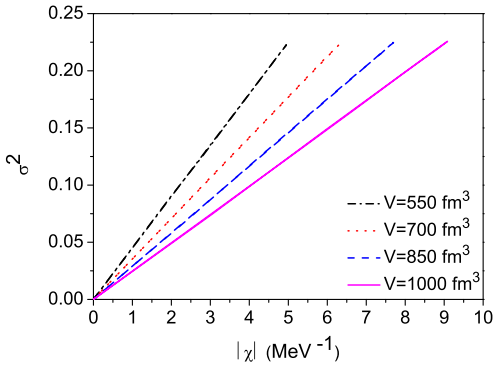

Figure 5. Variation of the variance, σ^2, with the system susceptibility, χ.

4.2. Susceptibility and variance parametrization

In the case of coexisting HM and QGP phases, the order parameter in a finite volume, $\langle h(T,V)\rangle$ may be expressed using one of the known mathematical representations of the Heaviside step-function as follows [3, 12]:

$$\langle h(T,V)\rangle = \frac{1}{2}\left(1 - \tanh\left[\frac{T - T_c(V)}{\Gamma_T(V)}\right]\right), \tag{14}$$

where, $T_c(V)$ is the effective transition temperature and $\Gamma_T(V)$ is the half-width of the rounded transition region. This leads directly to the susceptibility expression:

$$\chi(T,V) = \left.\frac{\partial\langle h(T,V)\rangle}{\partial T}\right|_V = -\frac{1}{2\Gamma_T(V)\cosh^2\left[\frac{T-T_c(V)}{\Gamma_T(V)}\right]}. \tag{15}$$

Based on the parametrization of the susceptibility with equation (15), and since a linear proportionality has been found between variance σ^2 and susceptibility χ, we propose a parametrization for the variance, similarly to the susceptibility, on the form:

$$\sigma^2(T,V) = \frac{1}{\Gamma_{\sigma^2}(V)\cosh^2\left[\frac{T-T_c(V)}{\Gamma_T(V)}\right]}, \tag{16}$$

where, $\Gamma_{\sigma^2}(V)$ is a dimensionless coefficient. An illustration of such parametrizations of the susceptibility and the variance at the volume $V = 1000\ fm^3$, are presented in figure 6 and figure 7, respectively. The regression coefficients are close to one for both performed fits. The obtained fit parameters T_c and Γ_T for $V = 1000\ fm^3$ from the proposed parametrization of the variance are in very good agreement with those obtained with the parametrization of the susceptibility, as it can be noted from the results displayed on figures 6 and 7.

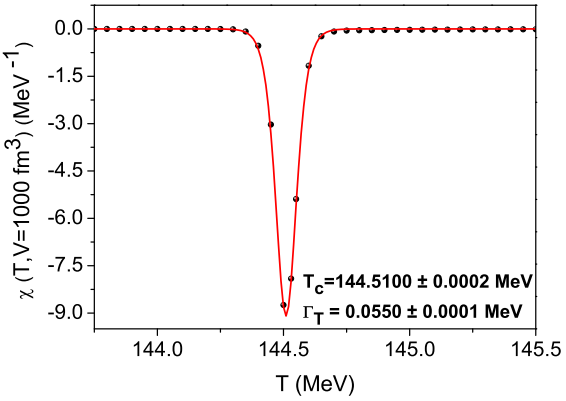

Figure 6. Parameterization of the susceptibility to the equation (15).

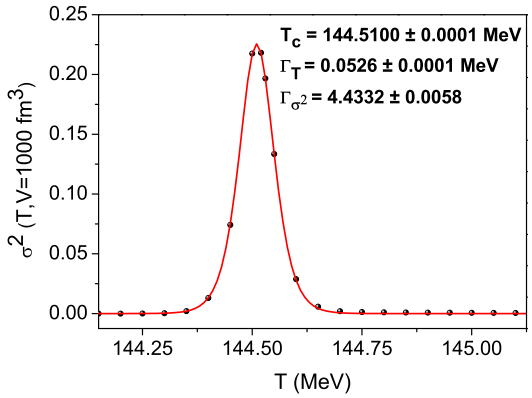

Figure 7. Parameterization of the variance to the equation (16).

5. Conclusion

In this communication, a thermally driven deconfinement phase transition from a Hadronic Gas consisting of massless pions, to a color-singlet QGP, containing gluons and massless up and down quarks, has been studied numerically. Based on a model considering the coexistence of both phases in a finite volume, a statistical approach was used in the framework of the QCD formalism to describe the thermal phase transition occurring between the two phases at vanishing chemical potential ($\mu = 0$). The finite size effects on the DPT have been investigated through the study of the thermal behavior of the order parameter, the susceptibility and the second cumulant of the probability density representing the variance, at different volumes, for $B^{1/4} = 200\ MeV$. The correlation between the susceptibility, χ, and the variance, σ^2, has been probed for the studied DPT, and the linearity between the two quantities is obtained. Furthermore, we proposed a parametrization of the variance, and this is done for the first time, to the best of our knowledge. The obtained results show that the proposed parametrization may be considered as an authentic method to estimate the effective transition temperature.

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