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# Some aspects of large-$N_c$ heavy baryon chiral perturbation theory

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**Abstract.** Large-$N_c$ chiral perturbation theory is a calculational scheme for baryons that simultaneously exhibits both the $m_q$ and $1/N_c$ expansions, where $N_c$ is the number of colors of quarks. In this work an outline of the calculation of one-loop corrections to the leading baryon axial-vector and vector form factors in baryon semileptonic decays to order $\mathcal{O}(p^2)$ in the chiral expansion is provided in order to show how the expansion works.

## 1. Introduction

Chiral perturbation theory [1, 2] is a framework for studying strong-interaction processes at low energies. The global chiral symmetry $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ in QCD is spontaneously broken down to the vector subgroup $\mathrm{SU}(3)_V$ by the vacuum expectation value which yields eight massless and spinless Goldstone bosons. Those bosons correspond to the spontaneously broken internal symmetry generators, which have negative parity, baryon number zero, and transform as an octet under $\mathrm{SU}(3)_V$. The pseudoscalar octet mesons ($\pi$, $K$, $\eta$) qualify as candidates for Goldstone bosons. The finite masses of the physical mesons are interpreted as a consequence of the explicit symmetry breaking due to the finite masses of the light quarks $m_q$ in the QCD Lagrangian [3].

The effective Lagrangian is structured in terms of a simultaneous expansion in powers of covariant derivatives and $m_q$ terms so physical observables can be expanded order by order in powers of $p^2/\Lambda_\chi^2$ and $m_\Pi^2/\Lambda_\chi^2$, where $p$ and $m_\Pi$ are the meson momentum and mass, respectively, and $\Lambda_\chi \sim 1$ GeV is the scale of chiral symmetry breaking.

When chiral perturbation theory is extended to include baryons, it is convenient to introduce velocity-dependent baryon fields, so that the expansion of the baryon chiral Lagrangian in powers of $m_q$ and $1/M$ (where $M$ is the baryon mass) is manifest [4]. This approach is called *heavy baryon chiral perturbation theory*. It was used to compute the corrections to the axial-vector coupling defined in baryon semileptonic decays (BSD) due to meson loops [4]. While these corrections are large when only octet baryon intermediate states are retained, the inclusion of decuplet baryon intermediate states yields large cancellations [4].

Large-$N_c$ QCD is the generalization of QCD from three colors $N_c = 3$ to $N_c \gg 3$ [5, 6]. The baryon sector in the large-$N_c$ limit exhibits an exact $\mathrm{SU}(2N_f)$ spin-flavor symmetry, where $N_f$ is the number of light quark flavors. The difference between the SU(3) invariant masses of the decuplet and octet baryons is $\Delta \propto 1/N_c$ so decuplet and octet baryon states become degenerate in the large-$N_c$ limit. The spin-flavor symmetry allows one to classify large-$N_c$ baryon states and matrix elements and to compute static properties of baryons in an expansion in $1/N_c$, for example masses [7, 8], axial-vector couplings, magnetic moments [9] and vector couplings [10].


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Next, *large-$N_c$ heavy baryon chiral perturbation theory* is a calculational scheme that simultaneously exhibits both the $m_q$ and $1/N_c$ expansions. The $1/N_c$ chiral Lagrangian was introduced in Ref. [11] and used to compute flavor-**27** baryon mass splittings at leading order in chiral perturbation theory. Other baryon properties also computed are axial-vector couplings [12, 13, 14], magnetic moments [15, 16], vector couplings [17] and charge radii [18].

The aim is to provide an overview of the calculation of corrections to form factor of BSD.

## 2. The chiral Lagrangian for baryons in the $1/N_c$ expansion

The $1/N_c$ baryon chiral Lagrangian that implements nonet symmetry and contracted spin-flavor symmetry for baryons in the large-$N_c$ limit is written as [11]

$$\mathcal{L}_{\rm baryon} = i\mathcal{D}^0 - \mathcal{M}_{\rm hyperfine} + {\rm Tr}\left(\mathcal{A}^k\lambda^c\right)A^{kc} + \frac{1}{N_c}{\rm Tr}\left(\mathcal{A}^k\frac{2I}{\sqrt{6}}\right)A^k + \ldots, \tag{1}$$

where $\mathcal{D}^0 = \partial^0\mathcal{I} + {\rm Tr}\left(\mathcal{V}^0\lambda^c\right)T^c$.

The baryon operators contained in Eq. (1) have well-defined $1/N_c$ expansions; they can be expressed as polynomials in the SU(6) spin-flavor generators [8]

$$J^k = q^\dagger\frac{\sigma^k}{2}q, \qquad T^c = q^\dagger\frac{\lambda^c}{2}q, \qquad G^{kc} = q^\dagger\frac{\sigma^k}{2}\frac{\lambda^c}{2}q, \tag{2}$$

where $q^\dagger$ and $q$ are operators that create and annihilate states in the fundamental representation of SU(6), and $\sigma^k$ and $\lambda^c$ are the Pauli and Gell-Mann matrices, respectively. The flavor indices in Eqs. (1)–(2) run from one to nine to account for the full meson nonet $\pi$, $K$, $\eta$ and $\eta'$.

$\mathcal{M}_{\rm hyperfine}$ denotes the spin splittings of the tower of baryon states with spins $1/2, \ldots, N_c/2$ in the flavor representations. The vector and axial vector combinations of the meson fields,

$$\mathcal{V}^0 = \frac{1}{2}\left(\xi\partial^0\xi^\dagger + \xi^\dagger\partial^0\xi\right), \qquad \mathcal{A}^k = \frac{i}{2}\left(\xi\nabla^k\xi^\dagger - \xi^\dagger\nabla^k\xi\right), \tag{3}$$

couple to baryon vector and axial-vector currents, respectively, $\xi = \exp[i\Pi(x)/f]$, where $\Pi(x)$ is the nonet of Goldstone boson fields and $f \approx 93$ MeV is the meson decay constant. The $\ell = 1$ flavor octet axial-vector pion combination couples to the baryon axial-vector current, $A^{kc}$. It is a spin-1 object, an octet under SU(3), and odd under time reversal. Its $1/N_c$ expansion is [8]

$$A^{kc} = a_1 G^{kc} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}}\mathcal{D}_n^{kc} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}}\mathcal{O}_n^{kc}. \tag{4}$$

The first terms in expansion (4) are

$$\mathcal{D}_2^{kc} = J^kT^c, \qquad \mathcal{D}_3^{kc} = \{J^k, \{J^r, G^{rc}\}\}, \qquad \mathcal{O}_3^{kc} = \{J^2, G^{kc}\} - \frac{1}{2}\{J^k, \{J^r, G^{rc}\}\}, \tag{5}$$

and higher order terms are obtained as $\mathcal{D}_n^{kc} = \{J^2, \mathcal{D}_{n-2}^{kc}\}$ and $\mathcal{O}_n^{kc} = \{J^2, \mathcal{O}_{n-2}^{kc}\}$ for $n \geq 4$. The unknown coefficients $a_1$, $b_n$, and $c_n$ have expansions in powers of $1/N_c$ and are order unity at leading order. At the physical value $N_c = 3$ the series can be truncated as

$$A^{kc} = a_1 G^{kc} + b_2\frac{1}{N_c}\mathcal{D}_2^{kc} + b_3\frac{1}{N_c^2}\mathcal{D}_3^{kc} + c_3\frac{1}{N_c^2}\mathcal{O}_3^{kc}. \tag{6}$$

The matrix elements of the space components of $A^{kc}$ in Eq. (6) between SU(6) states give the tree-level values of the axial-vector couplings.

Similarly, the $1/N_c$ expansion for the baryon mass operator $\mathcal{M}$ can be written as [7, 8]

$$\mathcal{M} = m_0 N_c \mathcal{I} + \sum_{n=2,4}^{N_c-1} m_n \frac{1}{N_c^{n-1}} J^n, \tag{7}$$

where $m_n$ are unknown coefficients. The first term on the right-hand side of Eq. (7) is the overall spin-independent mass of the baryon multiplet and is removed from the chiral Lagrangian by the heavy baryon field redefinition [4]. The additional spin-dependent terms define $\mathcal{M}_{\rm hyperfine}$.

## 3. One-loop corrections to the baryon axial-vector current

$A^{kc}$ is renormalized by the one-loop diagrams displayed in Fig. 1, which are given by the product of a group theoretic structure times a loop integral [12, 13].

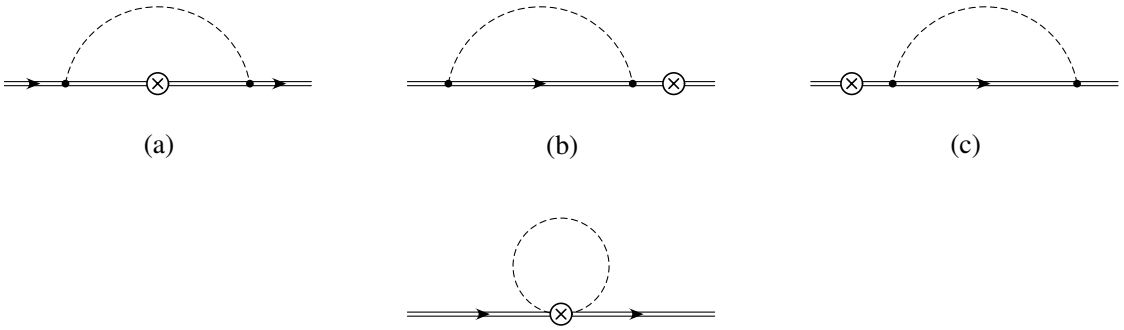



**Figure 1.** One-loop corrections to the baryon axial-vector current.

Specifically, the correction arising from the sum of the diagrams of Figs. 1(a,b,c) reads [12]

$$\begin{aligned}\delta A^{kc} &= \frac{1}{2}[A^{ja},[A^{jb},A^{kc}]]\Pi^{ab}_{(1)} - \frac{1}{2}\{A^{ja},[A^{kc},[\mathcal{M},A^{jb}]]\}\Pi^{ab}_{(2)} \\ &\quad + \frac{1}{6}\left([A^{ja},[[\mathcal{M},[\mathcal{M},A^{jb}]],A^{kc}]] - \frac{1}{2}[[\mathcal{M},A^{ja}],[[\mathcal{M},A^{jb}],A^{kc}]]\right)\Pi^{ab}_{(3)} + \ldots \end{aligned} \tag{8}$$

Here, $A^{ia}$ and $A^{jb}$ are used at the meson-baryon vertices and $\Pi^{ab}_{(n)}$ is a symmetric tensor which contains meson loop integrals $F^{(n)}(m_\Pi, \Delta, \mu)$ with the exchange of a single meson. It decomposes into flavor singlet, flavor **8**, and flavor **27** representations as [11]

$$\Pi^{ab}_{(n)} = F^{(n)}_{\mathbf{1}}\delta^{ab} + F^{(n)}_{\mathbf{8}} d^{ab8} + F^{(n)}_{\mathbf{27}}\left[\delta^{a8}\delta^{b8} - \frac{1}{8}\delta^{ab} - \frac{3}{5}d^{ab8}d^{888}\right], \tag{9}$$

where $F^{(n)}_{\mathbf{k}}$ are linear combinations of $F^{(n)}(m_\Pi, \Delta, \mu)$ [12].

$\delta A^{kc}$ contains $n$-body operators,[1] with $n > N_c$, which are complicated commutators and/or anticommutators of the one-body operators $J^k$, $T^c$, and $G^{kc}$. All these structures should be reduced and rewritten as linear combinations of the operator basis, with $n \leq N_c$. Although the operator basis is complete and independent, the reduction could be rather difficult.

[1] An $n$-body operator is one with $n$ $q$'s and $n$ $q^\dagger$'s and can be written as a polynomial of order $n$ in $J^i$, $T^a$, and $G^{ia}$ [8].

The $N_c$ dependence of the matrix elements of $J^i$, $T^a$, and $G^{ia}$ is nontrivial [8]. A simplified counting rule for baryons with spins of order unity is [12]

$$T^a \sim N_c, \qquad G^{ia} \sim N_c, \qquad J^i \sim 1. \tag{10}$$

Similarly, $f \propto \sqrt{N_c}$, so the functions $F^{(n)}(m_\Pi, \Delta, \mu)$ introduce a $1/N_c$ suppression.

Naively, the double commutator would be $\mathcal{O}(N_c^3)$: one factor of $N_c$ from each $A^{kc}$. However, there are large-$N_c$ cancellations between the diagrams of Figs. 1(a,b,c), *provided all baryon states in a complete multiplet of the large-$N_c$ spin-flavor symmetry are included in the sum over intermediate states* [12]. The terms with one and two baryon mass insertions also contain large-$N_c$ cancellations [12]. Thus, the correction $\delta A^{kc}$ is $\mathcal{O}(1)$, or $1/N_c$ times the tree level value.

In a similar way, the one-loop correction to $A^{kc}$ from the diagram of Fig. 1(d) is given by

$$\delta A^{kc} = -\frac{1}{2}[T^a, [T^b, A^{kc}]]\Pi^{ab}, \tag{11}$$

where $\Pi^{ab}$ is a tensor similar to the one in Eq. (9) with a different loop integral [12].

The double commutator in Eq. (11) is proportional to $A^{kc}$ [12] so the correction is at most $\mathcal{O}(1)$ and is of the same order as the one arising from the sum of Figs. 1(a,b,c).

Finally, in the limit $\Delta/m_\Pi = 0$ the one-loop correction to $A^{kc}$ becomes

$$\delta A^{kc} = \frac{1}{2}[A^{ja}, [A^{jb}, A^{kc}]]\Pi^{ab}_{(1)} - \frac{1}{2}[T^a, [T^b, A^{kc}]]\Pi^{ab}. \tag{12}$$

The matrix elements between spin-$\frac{1}{2}$ baryon states of the space components of the renormalized baryon axial-vector current, $A^{kc} + \delta A^{kc}$, give the actual value of $g_1$, as defined in BSD, with a normalization such that $g_1 \approx 1.27$ for neutron decay.

A comparison can be done between the $g_1$ obtained in the combined formalism and the one obtained in heavy baryon chiral perturbation theory [4] through the relations between the SU(3) invariant couplings $D$, $F$, $\mathcal{C}$ and $\mathcal{H}$ and the operator coefficients of the expansion (6),

$$D = \frac{1}{2}a_1 + \frac{1}{6}b_3, \quad F = \frac{1}{3}a_1 + \frac{1}{6}b_2 + \frac{1}{9}b_3, \quad \mathcal{C} = -a_1 - \frac{1}{2}c_3, \quad \mathcal{H} = -\frac{3}{2}a_1 - \frac{3}{2}b_2 - \frac{5}{2}b_3. \tag{13}$$

*Both approaches yield the same results.*

## 4. One-loop corrections to the baryon vector current

The baryon vector current $V^{0c}$ is a spin-0 object and an octet under SU(3). At $q^2 = 0$, $V^{0c}$ is the generator of SU(3) symmetry transformations, so to all orders in the $1/N_c$ expansion [10]

$$V^{0c} = T^c. \tag{14}$$

The matrix elements of $V^{0c}$ of Eq. (14) between SU(6) baryon states yield the form factors $f_1$ in the limit of exact SU(3) symmetry, $f_1^{\mathrm{SU(3)}}$. One-loop corrections to these form factors for $|\Delta S| = 1$ processes arise from the diagrams displayed in Fig. 2. Due to the Ademollo-Gatto theorem, these corrections appear to order $\epsilon^2 \sim (m_s - \hat{m})^2$. Details can be found in Ref. [17].

For instance, the contribution to $V^{0c}$ arising from Fig. 2(a) can be written as

$$\delta V^c_{(\mathrm{a})} = \sum_{\mathsf{j}} A^{ia}\mathcal{P}_{\mathsf{j}} A^{ib} P^{abc}(\Delta_{\mathsf{j}}). \tag{15}$$

Here $\mathcal{P}_{\mathsf{j}}$ is the baryon propagator for spin $J = \mathsf{j}$ [11]

$$\frac{i\mathcal{P}_{\mathsf{j}}}{k^0 - \Delta_{\mathsf{j}}}, \tag{16}$$

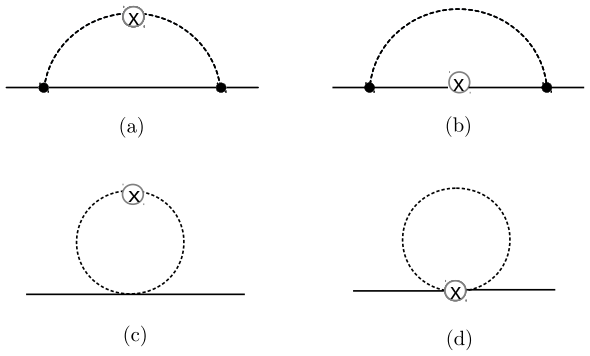



**Figure 2.** One-loop corrections to the vector current..

which satisfies by definition $\mathcal{P}_{\rm j}^2 = \mathcal{P}_{\rm j}$ and $\mathcal{P}_{\rm j}\mathcal{P}_{\rm j'} = 0$ for ${\rm j} \neq {\rm j'}$. Similarly, $\Delta_{\rm j}$ stands for the difference of the hyperfine mass splitting between the intermediate baryon with spin $J = {\rm j}$ and the external baryon. The remaining corrections can be obtained without difficulty in the combined expansion in $m_q$ and $1/N_c$. Details can be found in Ref. [17].

The one-loop corrections to order $\mathcal{O}(p^2)$ in the chiral expansion to $f_1$ for $\Lambda p$ process is [17]

$$\begin{aligned}\left[\frac{f_1}{f_1^{\rm SU(3)}}\right]_{\Lambda p} &= 1 + \left[\frac{3}{8} + \frac{17}{32}a_1^2 + \frac{3}{16}a_1b_2 + \frac{17}{48}a_1b_3 + \frac{1}{32}b_2^2 + \frac{1}{16}b_2b_3 + \frac{17}{288}b_3^2\right] H(m_\pi, m_K) \\ &\quad + \left[\frac{3}{8} + \frac{9}{32}a_1^2 + \frac{3}{16}a_1b_2 + \frac{3}{16}a_1b_3 + \frac{1}{32}b_2^2 + \frac{1}{16}b_2b_3 + \frac{1}{32}b_3^2\right] H(m_K, m_\eta) \\ &\quad + \left[-\frac{1}{4}a_1^2 - \frac{1}{4}a_1c_3 - \frac{1}{16}c_3^2\right] K(m_\pi, m_K, \Delta), \end{aligned} \tag{17}$$

where $H(m_1, m_2)$ and $K(m_1, m_2, \Delta)$ arise from loop integrals. When relations (13) are used in Eq. (17), the expression obtained in heavy baryon chiral perturbation theory [19] is recovered.

The chiral corrections to the vector form factor can be parametrized as

$$f_1 = f_1^{\rm SU(3)}(1 + \delta^{(2)} + \ldots), \tag{18}$$

where $\delta^{(2)}$ is the leading SU(3)-breaking loop correction $\mathcal{O}(p^2)$ and the dots stand for higher chiral corrections not computed yet within the present formalism.

## 5. Fit to data

A comparison of the theoretical expressions with the available experimental data [20] through a least-squares fit can be performed. For octet baryons the data are the decay rates $R$, the ratios $g_1/f_1$, the angular correlation coefficients $\alpha_{e\nu}$ and the spin-asymmetry coefficients $\alpha_e$, $\alpha_\nu$, $\alpha_B$, $A$ and $B$. For decuplet baryons, the axial couplings $g$ for the processes $\Delta \to N\pi$, $\Sigma^* \to \Lambda\pi$, $\Sigma^* \to \Sigma\pi$, $\Xi^* \to \Xi\pi$ are determined. The free parameters are the operator coefficients of $A^{kc}$ plus a set of parameters $d_i$ which are introduced by explicit SU(3) symmetry breaking Ref. [17].

The fit yields [17]

$$\begin{aligned} &a_1 = 0.95 \pm 0.14, \quad b_2 = -1.10 \pm 0.19, \quad b_3 = 1.10 \pm 0.09, \quad c_3 = 1.07 \pm 0.15, \\ &d_1 = 0.62 \pm 0.13, \quad d_2 = -0.57 \pm 0.24, \quad d_3 = 0.39 \pm 0.05, \quad d_4 = -0.06 \pm 0.08. \end{aligned} \tag{19}$$

with $\chi^2 = 5.5/4\,{\rm dof}$. With the best-fit parameters, the predicted values of observables in BSD as well as the pattern of SU(3) symmetry breaking in $f_1$ are listed in Table 1 for completeness.

**Table 1.** Predicted observables in BSD. $R$ is given in $10^{-3}\,\mathrm{s}^{-1}$ for $np$ process and $10^{6}\,\mathrm{s}^{-1}$ for the rest.

| | $np$ | $\Sigma^+\Lambda$ | $\Sigma^-\Lambda$ | $\Lambda p$ | $\Sigma^- n$ | $\Xi^-\Lambda$ | $\Xi^-\Sigma^0$ | $\Xi^0\Sigma^+$ |
|---|---|---|---|---|---|---|---|---|
| $R$ | 1.128 | 0.232 | 0.387 | 2.949 | 6.284 | 2.844 | 0.454 | 0.820 |
| $\alpha_{e\nu}$ | −0.079 | −0.406 | −0.414 | −0.026 | 0.340 | 0.559 | | |
| $\alpha_e$ | −0.087 | | | 0.014 | −0.627 | | | |
| $\alpha_\nu$ | 0.987 | | | 0.977 | −0.357 | | | |
| $\alpha_B$ | | | | −0.586 | 0.668 | | | |
| $A$ | | | 0.049 | | | 0.583 | | |
| $B$ | | | 0.888 | | | | | |
| $g_1/f_1$ | 1.270 | | | 0.718 | −0.340 | 0.254 | 1.217 | 1.217 |
| $f_1/f_1^{\mathrm{SU(3)}}$ | | | | 0.952 | 0.966 | 0.953 | 0.962 | 0.962 |

To order $\mathcal{O}(p^2)$ in the chiral expansion, corrections to the form factors $f_1$ produce a systematic decrease of around 5% with respect to their SU(3) symmetric values. Also, the values of the predicted observables are consistent with the measured ones.

## 6. Conclusions

The combined formalism in $m_q$ and $1/N_c$ expansions is an outstanding approach to analyze the static properties of baryons in a systematic way. Here the analysis of leading axial-vector and vector form factors that appear in BSD has been presented to exemplify how the method works. A number of baryon properties can be studied with it, which constitutes an important advancement in our understanding of the low-energy consequences of QCD.

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