Spectrum of Charmonia within a Contact Interaction

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Abstract. For the flavour-singlet heavy quark system of charmonia, we compute the masses of the ground state mesons in four different channels: pseudo-scalar ($\eta_c(1S)$), vector ($J/\Psi(1S)$), scalar ($\chi_{c_0}(1P)$) and axial vector ($\chi_{c_1}(1P)$), as well as the weak decay constants of the $\eta_c(1S)$ and $J/\Psi(1S)$. The framework for this analysis is provided by a symmetry-preserving Schwinger-Dyson equation (SDEs) treatment of a vector$\times$vector contact interaction (CI). The results found for the meson masses and the weak decay constants, for the spin-spin combinations studied, are in fairly good agreement with experimental data and earlier model calculations based upon Schwinger-Dyson and Bethe-Salpeter equations (BSEs) involving sophisticated interaction kernels.

1. Introduction

First explorations for heavy mesons, both charmonia and bottomonia, with a consistent use of the rainbow-ladder truncation in the kernels of the gap and Bethe-Salpeter equations (BSEs), were undertaken by Jain and Munczek in Ref. [1]. They found the mass spectrum and the decay constants of pseudoscalar mesons in good agreement with experiments. This work was repeated with the Maris-Tandy model for $\bar{c}c$ bound states in Refs. [2–4]. We use a symmetry preserving vector-vector contact interaction (CI) [5–9]. This model provides a simple scheme to exploratory studies of the spontaneous chiral symmetry breaking and its consequenses like: dynamical mass generation, a quark condensate, the rise of goldstone bosons in the chiral limit and quark confinement. The results obtained from the CI model are quantitatively comparable to those obtained using sophisticated QCD model interactions, [10–12].

We employ this interaction for the analysis of the quark model heavy mesons for spins $J = 0, 1$ and study the mass spectrum and weak decay constants for charmonia. Without parameter readjustment, we find good agreement with charmonia masses. However, we need to modify the set of parameters to simultaneously account for the weak decay constants of the $\eta_c(1S)$ and $J/\Psi(1S)$, and the charge radius of $\eta_c(1S)$.

2. The Tools

2.1. The Bethe-Salpeter and the Gap Equations

Meson bound states appear as poles in a four-point function. The condition for the appearance of these poles in a particular J^{PC} channel is given by the homogeneous BSE [13–15]

$$[\Gamma_H(k;P)]_{tu} = \int \frac{\mathrm{d}^4q}{(2\pi)^4}\, \chi(q;P)_{sr} K^{rs}_{tu}(q,k;P)\,, \tag{1}$$

where $\chi(q;P) = S_f(q_+)\Gamma_H(q;P)S_g(q_-)$; $q_+ = q + \eta P$, $q_- = q - (1-\eta)P$; k (P) is the relative (total) momentum of the quark-antiquark system; S_f is the f-flavour quark propagator; $\Gamma_H(q;P)$ is the meson BSA, where $H = f\bar{g}$ specifies the flavour content of the meson; r, s, t, u represent colour, flavour, and spinor indices; and K is the quark-antiquark scattering kernel.

The f-flavour dressed-quark propagator, S_f, that enters Eq. (1) is obtained as the solution of the quark SDE, the so called gap equation [16–19]:[1]

$$S_f^{-1}(p) = i\gamma \cdot p + m_f + \Sigma_f(p)\,, \tag{2}$$

$$\Sigma_f(p) = \int \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\Gamma^a_\nu(p,q)\,, \tag{3}$$

where g is the strong coupling constant, $D_{\mu\nu}$ is the dressed-gluon propagator, Γ^a_ν is the dressed-quark-gluon vertex, and m_f is the bare f-flavour current-quark mass.

Both $D_{\mu\nu}$ and Γ^a_ν satisfy their own SDE, which in turn are coupled to higher n-point functions and so on *ad infinitum*. Therefore, the quark SDE, Eq. (2), is only one of the infinite set of coupled nonlinear integral equations. A tractable problem is defined once a truncation scheme has been specified, i.e., once the gluon propagator and the quark-gluon vertex are defined.

2.2. Axial-Vector Ward-Takahashi Identtiy

The phenomenological features of chiral symmetry and its dynamical breaking in QCD can be understood by means of the axWTI. In the chiral limit, it reads

$$-\,\mathrm{i}P_\mu\Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5 + \gamma_5 S^{-1}(k_-)\,. \tag{4}$$

The axWTI relates the axial-vector vertex, $\Gamma_{5\mu}$, the pseudoscalar vertex, γ_5 and the quark propagator. This in turn implies a relationship between the kernel in the BSE and that in the quark SDE. It must be preserved by any viable truncation scheme of the SDE-BSE coupled system

$$\int \frac{d^4q}{(2\pi)^4}\, K_{tu;rs}(k,q;P)\left[\gamma_5 S(q_-) + S(q_+)\gamma_5\right]_{sr} = \left[\Sigma(k_+)\gamma_5 + \gamma_5\Sigma(k_-)\right]_{tu}\,, \tag{5}$$

thus constraining the content of the quark-antiquark scattering kernel $K(p,q;P)$ if an essential symmetry of the strong interactions, and its breaking pattern, is to be faithfully reproduced.

2.3. Rainbow-Ladder truncation and the Contact Interaction

We employ a momentum-independent vector×vector CI this interaction for the analysis of the quark model charmonia spectrum. Therefore, we use [7]

$$g^2 D_{\mu\nu}(k) = \frac{4\pi\alpha_{\mathrm{IR}}}{m_g^2}\delta_{\mu\nu} \equiv \frac{1}{m_G^2}\delta_{\mu\nu}\,, \tag{6}$$

in Eq. (3), where $m_g = 800$ MeV is a gluon mass scale which is in fact generated dynamically in QCD [20], and $\alpha_{\mathrm{IR}} = 0.93\pi$ is a parameter that determines the interaction strength. For the quark-gluon vertex, the rainbow truncation is used:

$$\Gamma^a_\mu(p,q) = \frac{\lambda^a}{2}\gamma_\mu\,. \tag{7}$$

[1] We work in a Euclidean metric where: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{mu\nu}$; $\gamma_\mu^\dagger = \gamma_\mu$; $\gamma_5 = \gamma_4\gamma_1\gamma_2\gamma_3$; $a \cdot b = \sum_{i=1}^4 a_i b_i$; and P_μ timelike $\Rightarrow P^2 < 0$.

Once the elements of the kernel in the quark SDE have been specified, we proceed to obtain and analyse its solution. Using Eq. (6) and Eq. (7), the quark equation, Eq. (2), takes the form

$$S_f^{-1}(p) = i\gamma\cdot p + m_f + \frac{4}{3}\frac{1}{m_G^2}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\gamma_\mu S_f(q)\gamma_\mu\,, \tag{8}$$

whose solution is of the form

$$S_f^{-1}(p) = \mathrm{i}\gamma\cdot p + M_f\,, \tag{9}$$

since the last term on the right-hand side of Eq. (8) is independent of the external momentum. The momentum-independent mass, M_f, is determined as the solution of

$$M_f = m_f + \frac{M_f}{3\pi^2 m_G^2}\int_0^\infty \mathrm{d}s\, s\frac{1}{s+M_f^2}\,. \tag{10}$$

Since Eq. (10) is divergent, we have to specify a regularization procedure. We employ the proper time regularization scheme, [21]

$$\begin{aligned}\frac{1}{s+M^2} &= \int_0^\infty \mathrm{d}\tau\,\mathrm{e}^{-\tau(s+M^2)} \to \int_{\tau_{\mathrm{UV}}^2}^{\tau_{\mathrm{IR}}{}^2}\mathrm{d}\tau\,\mathrm{e}^{-\tau(s+M^2)}\,,\\ &= \frac{\mathrm{e}^{-\tau_{UV}^2(s+M^2)} - \mathrm{e}^{-\tau_{\mathrm{IR}}^2(s+M^2)}}{s+M^2}\,,\end{aligned} \tag{11}$$

where τ_{IR}^2 and τ_{UV}^2 are, respectively, infrared and ultraviolet regulators. A nonzero value for $\tau_{\mathrm{IR}} \equiv 1/\Lambda_{\mathrm{IR}}$ implements confinement by ensuring the absence of quark production thresholds [22]. Furthermore, since Eq. (6) does not define a renormalizable theory, $\tau_{\mathrm{UV}} \equiv 1/\Lambda_{\mathrm{UV}}$ cannot be removed, but instead plays a dynamical role and sets the scale for all dimensioned quantities. Thus

$$M_f = m_f + \frac{M_f}{3\pi^2 m_G^2}\mathcal{C}_{01}(M_f^2;\tau_{\mathrm{IR}},\tau_{\mathrm{UV}})\,, \tag{12}$$

where

$$\mathcal{C}_{\alpha\beta}(M^2;\tau_{\mathrm{IR}},\tau_{\mathrm{UV}}) = \frac{\left(M^2\right)^\nu}{\Gamma(\beta)}\Gamma(\beta-2,\tau_{\mathrm{UV}}^2M^2,\tau_{\mathrm{IR}}^2M^2)\,, \tag{13}$$

with $\nu = \alpha - (\beta - 2)$ and $\Gamma(a, z_1, z_2)$ is the generalized incomplete gamma function.

2.4. Classification of BSA in a Contact Interaction

We are interested in the static properties of several mesons. We begin with their classification and the general form of their BSA in the CI we are working with. Table 1 lists the spin quantum numbers of the quark model mesons under study.

L	J^{PC}	Type	L	J^{PC}	Type
0	0^{-+}	Pseudoscalars	1	0^{++}	Scalars
0	1^{--}	Vectors	1	1^{++}, 1^{+-}	Axial Vectors

Table 1. Quark model mesons

With the dependence on the relative momentum forbidden by the CI, the general form of the BSAs for the mesons listed in Table 1 are [23]:

$$\Gamma_{0^{-+}}(P) = \gamma_5\left[\mathrm{i}E_{0^{-+}} + \frac{1}{2M}\gamma\cdot PF_{0^{-+}}\right], \tag{14}$$

$$\Gamma_{0^{++}}(P) = \mathbb{1}E_{0^{++}}, \tag{15}$$

$$\Gamma_{1^{--}\mu}(P) = \gamma^T_\mu E_{1^{--}} + \frac{1}{2M}\sigma_{\mu\nu}P_\nu F_{1^{--}}, \tag{16}$$

$$\Gamma_{1^{++}\mu}(P) = \gamma_5\left[\gamma^T_\mu E_{1^{++}} + \frac{1}{2M}\sigma_{\mu\nu}P_\nu F_{1^{++}}\right], \tag{17}$$

where M is a mass scale.

2.5. Normalization of the BSA
Since the BSE is a homogeneous equation, the BSA has to be normalized by a separate condition. In the Rainbow-Ladder truncation of the BSE, that condition takes a simple form ($\eta = 1$):

$$P_\mu = N_c\frac{\partial}{\partial P_\mu}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\mathrm{Tr}\left[\overline{\Gamma}_H(-Q)S(q_+)\Gamma_H(Q)S(q)\right], \tag{18}$$

at $Q = P$, with $P^2 = -m_H^2$, which ensures that the residue at the mass pole is unity. Here, Γ_H is the normalized BSA and $\overline{\Gamma}_H$ its charge conjugated version.

2.6. Pseudoscalar and Vector Decay Constants
Once the BSA has been normalized canonically, we can calculate observables from it. The pseudoscalar leptonic decay constant, $f_{0^{-+}}$, is defined by

$$P_\mu f_{0^{-+}} = N_c\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_5\gamma_\mu S(q_+)\Gamma_{0^{-+}}(P)S(q_-)\right]. \tag{19}$$

Similarly, the vector decay constant, $f_{1^{--}}$, is defined by

$$m_{1^{--}}f_{1^{--}} = \frac{N_c}{3}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_\mu S(q_+)\Gamma_{\mu 1^{--}}S(q_-)\right], \tag{20}$$

where $m_{1^{--}}$ is the mass of the 1^{--} bound state, and the factor of 3 in the denominator comes from summing over the three polarizations of the spin-1 meson.

3. Results
The mass and BSA of a meson depend on its quantum numbers and can be found by solving Eq. (1). In order to do this, a fictitious eigenvalue, λ_H, is introduced to the bound state equation. Thus, the mass of the bound state in a particular channel, m_H, will be such that $\lambda_H(P^2 = -m_H^2) = 1$, where P is the meson's momentum. In any channel, the form of the homogeneous BSE for the CI is

$$K_H(m_H)\cdot\Gamma_H(m_H) = \lambda_H(m_H)\Gamma_H(m_H), \tag{21}$$

where K_H is a 2×2 matrix, and the subscript H indicates the dependence of the explicit expressions on the quantum numbers of the meson under consideration, see Eqs. (14-17). Equation (21) is an eigenvalue equation for the vector $\Gamma_H(m_H) = (E_H(m_H), F_H(m_H))^T$ with solutions for discrete values of $P^2 = -m_H^2$. Explicit expressions for every channel given in Table 1 in the reference [24].

	masses			
	$m_{\eta_c(1S)}$	$m_{J/\Psi(1S)}$	$m_{\chi_{c_0}(1P)}$	$m_{\chi_{c_1}(1P)}$
Experiment [29]	2.983	3.096	3.414	3.510
Contact Interaction	2.983*	2.979	3.412	3.442
	-	-	3.293	3.344
JM [1]	2.821	3.1	3.605	-
BK [25]	2.928	3.111	3.321	3.437
S1rp [4]	3.035	3.192	-	-
RB1 [26]	3.065	-	-	-
RB2 [26]	3.210	-	-	-
	decay constants			
Experiment [29]	0.361	0.416	-	-
Contact Interaction	0.084	0.080	-	-

Table 2. Ground state charmonia masses, BS amplitudes and decay constants obtained with the light sector parameter set: $m_g = 0.8\,\text{GeV}$, $\alpha_{IR} = 0.93\pi$, $\Lambda_{\text{IR}} = 0.24\,\text{GeV}$, $\Lambda_{\text{UV}} = 0.905\,\text{GeV}$. The current-quark mass is $m_c = 1.578^*\,\text{GeV}$, and the dynamically generated constituent-like mass is $M_c = 1.601\,\text{GeV}$. The value immediately below the CI results is obtained without a spin-orbit coupling $g_{so} = 0.24$. For a direct comparison, we quote values from other SDE approaches to calculate the masses of low lying charmonia. Dimensioned quantities are in GeV. (* = The current quark mass was fitted to obtain the mass of the pseudoscalar meson).

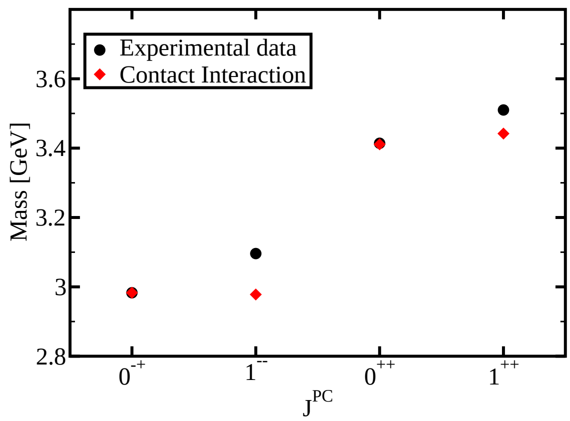

Figure 1. Contact interaction results for the $\bar{c}c$ mass spectrum using model parameters fitted in the light sector, see Table 2 PDG-labeled data is from [29].

3.1. Charmonia mass spectrum

The parameter set used in this calculation is the same as that obtained used in the light sector [5]. Only the current-quark mass for the charm quark is an input parameter, and it is fixed such that the experimental mass of the pseudoscalar is reproduced. The rest of the meson masses are predictions of the model. As can be seen from Table 2, the predictions for the masses of the remaining mesons are in good agreement with the results obtained from more sophisticated SDE-BSE model calculations [25,26], lattice QCD for the charm sector [27,28] as well as experimental values [29].

On the other side, Table 2 also shows that the pseudoscalar and vector decay constants, for the model parameters used, are strongly underestimated, in disagreement both with model calculations and experimental data. As noticed in Refs. [30–32], the decay constant is influenced by the high momentum tails of the dressed-quark propagator and the BSAs. This high momentum region probes the wave-function of quarkonia at origin. The CI, on the other hand, yields constant mass with no perturbative tail for large momenta. Therefore, this artefact of quarkonia has to be built into the model in an alternative manner. We know that with increasing mass of the heavy quarks, they become increasingly point-like in the configuration space. The closer the quarks get, the further the coupling strength between them decreases. Therefore, we cannot expect the decay constants to be correctly reproduced with the parameters of the light quark sector. The next step is to consider the possibility of extending the simple CI model to the heavy sector by reducing the effective coupling. However, the reduction in the strength of the kernel has to be compensated by increasing the ultraviolet cut-off. This makes sense by observing that the Λ_{UV} (highest energy scale associated with the system) used in the light quark sector is, in fact, less than the current charm quark mass. Therefore, it needs to be modified.

3.2. Contact Interaction Model for Charmonia
We look a balance between the effective coupling and the ultraviolet cut-off to describe the static properties of charmonia. For this purpose, we set out to redefine the parameters of the CI to study the masses and weak decay constants.

	masses			
	$m_{\eta_c(1S)}$	$m_{J/\Psi(1S)}$	$m_{\chi_{c_0}}(1P)$	$m_{\chi_{c_1}}(1P)$
Experiment [29]	2.983	3.096	3.414	3.510
Contact Interaction	2.950*	3.129	3.407	3.433
			3.194	3.254
JM [1]	2.821	3.1	3.605	-
BK [25]	2.928	3.111	3.321	3.437
RB1 [26]	3.065	-	-	-
RB2 [26]	3.210	-	-	-
	decay constants			
	f_{η_c}	$f_{J/\Psi}$		
Experiment [29]	0.361	0.416		
S1rp [4]	0.239	0.198		
S3ccp [4]	0.326	0.330		
BK [25]	0.399	0.448		
Contact Interaction	0.305	0.220		

Table 3. Ground state charmonia masses and decay constants obtained with the best-fit parameter set: $m_g = 0.8\,\text{GeV}$, $\alpha_{IR} = 0.93\pi/20$, $\Lambda_{\text{IR}} = 0.24\,\text{GeV}$, $\Lambda_{\text{UV}} = 2.788\,\text{GeV}$. The current-quark mass is $m_c = 0.956^*\,\text{GeV}$, and the dynamically generated constituent-like mass is $M_c = 1.497\,\text{GeV}$. The value immediately below the CI results is obtained without a spin-orbit coupling $g_{so} = 0.24/3$. Dimensioned quantities are in GeV. ($^* =$ This parameter set was obtained from a best-fit to the mass and decay constant of the pseudoscalar and vector channels).

We retain the parameters m_g and Λ_{IR} of the light sector. Modern studies of the gluon propagator indicate that in the infrared, the dynamically generated gluon mass scale virtually remains unaffected by the introduction of heavy dynamical quark masses [33, 34]. The rest of

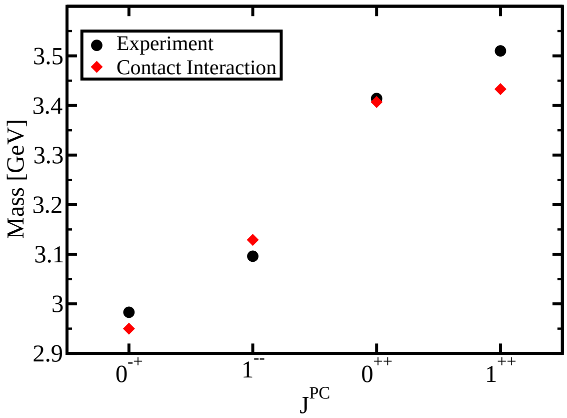

Figure 2. Contact interaction results for the $\bar{c}c$ mass spectrum, see Table 3. PDG-labelled data is taken from Ref. [29].

the parameters are obtained from a best-fit to the mass and decay constant of the pseudoscalar (η_c) and vector (J/Ψ) channels.

One can now readily calculate the masses of the ground state pseudo-scalar, vector, scalar, and axial vector mesons. The results are shown in Table 3 and Fig. 2. They are in very good agreement with experimental values and comparable to the best SDE results with refined truncations.

The decay constants for the $\eta_c(1S)$ and $J/\Psi(1S)$ channels are reported in Table 3. For the pseudoscalar meson, the result aligns nicely with the experimental value. However, this is not exactly the case for the vector channel. Furthermore, we note that the decay constant for $J/\Psi(1S)$ is smaller than that for $\eta_c(1S)$. The correct ordering can be recovered by reducing the interaction strength by a large factor. However, this is something we consider contrived and, therefore, not pursued further. Notice that one of the SDE results yields the $J/\Psi(1S)$ decay constant even smaller than our value [4].

4. Conclusions

We compute the quark model ground state spin-0 and spin-1 charmonia masses and decay constants using a rainbow-ladder truncation of the simultaneous set of SDE and BSE with a CI model of QCD, developed and tested for the light quark sector [5–9]. As the model is non-renormalizable, we employ proper time regularization scheme which ensures confinement is implemented through the absence of quark production threshold. Moreover, the relevant Ward identities and the low energy theorems such as Goldberger-Triemann relations are satisfied. Without parameter readjustment, we find that the masses of the studied mesons are in reasonably good agreement with experimental data and other model calculations. Moreover, the Gell-Mann–Oakes–Renner relation, valid for every current-quark mass in the pseudo-scalar channel, is always satisfied. However, the decay constants of pseudo-scalar as well as vector mesons are significantly underestimated.

We realize that the extension of the CI model to the heavy sector requires a reduction of the effective coupling, which mimics the high momentum tail of the quark mass function obtained in the SDE studies of QCD [30–32]. We only have to ensure that the reduction in the strength of the kernel is appropriately compensated by increasing the ultraviolet cut-off, a natural requirement for studying heavy quarks. We find that with a modified choice of two parameters, not only the masses of the ground state mesons, i.e., pseudo-scalar ($\eta_c(1S)$), vector ($J/\Psi(1S)$), scalar ($\chi_{c_0}(1P)$), and axial vector ($\chi_{c_1}(1P)$), but also their weak decay constants, are in much better agreement with the experiments [29] as well as earlier SDE calculations with QCD based refined truncations [25]. This is an encouraging first step towards a comprehensive study of heavy mesons in this approach. Further steps will involve flavored mesons and baryons. Our goal is to provide a unified description of light and heavy hadrons within the CI model.

Full discussion and details are found in our work [24].

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