

Study of the semileptonic $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and $\tau^- \rightarrow \pi^- \pi^0 \ell^+ \ell^- \nu_\tau$ ($\ell = e, \mu$) decays.

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Abstract. We analyze the decay modes $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ and $\tau^- \rightarrow \pi^- \pi^0 \ell^+ \ell^- \nu_\tau$, $\ell = e, \mu$. The first one is studied in order to show that the three resonances $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ are necessary to fix the model vector dominance parameters on the two pion system dynamics. We have obtained $\mathcal{BR}(\tau^- \rightarrow \pi^- \pi^0 \nu_\tau) = (25.40 \pm 0.22)\%$ which is in good agreement with the world average reported in the PDG. The second decay, which is the target of our study, is important because it can participate as a background in processes of lepton flavour and lepton number violation and thus its accurate description is essential in searches for such processes. It is also important to test the model dependence of the most important ($\pi\pi$) contribution to the hadron vacuum polarization (HVP) entering the muon anomalous magnetic moment, if evaluated using hadronic tau decay data. In these preliminary results, we have only taken into account the total Inner Bremsstrahlung (IB) contribution to the $\tau^- \rightarrow \pi^- \pi^0 \ell^+ \ell^- \nu_\tau$ decays obtaining $\mathcal{BR}(\tau^- \rightarrow \pi^- \pi^0 e^+ e^- \nu_\tau) = (8.09 \pm 0.26) \times 10^{-5}$ and $\mathcal{BR}(\tau^- \rightarrow \pi^- \pi^0 \mu^+ \mu^- \nu_\tau) = (2.094 \pm 0.004) \times 10^{-6}$. These results have been obtained using the VEGAS routine to integrate numerically the phase space.

1. Introduction

We report on a preliminary study of the $\tau^- \rightarrow \pi^- \pi^0 \ell^+ \ell^- \nu_\tau$ ($\ell = e, \mu$) decays [1] where we have considered only the IB contribution. The inclusion of structure dependent effects is in progress. $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays involve the interplay of strong and weak interactions. Since the latter are under well theoretical control, these processes offer a clean environment to study QCD currents in the GeV region. These decays proceed through the hadronization of the vector piece of the quark current coupling to the W boson. This is dominated by the creation of intermediate particles (resonances) whose masses corresponds to the $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ states. There must be conservation of certain quantum numbers (like isospin, G-parity and C-parity) in the process. In the $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays, the isospin and G-parity have the values $I = 1$, $G = +$, corresponding to the properties of the weak vector current that produces. We note that while the branching ratio can be reproduced just with the $\rho(770)$ resonance contribution, the invariant mass spectrum requires at least three resonances for its proper description, as shown in Figure 2. This decay mode dominates the τ lepton width, with a branching ratio $(25.52 \pm 0.09)\%$ [2]. As a result, if using tau data to evaluate the HVP to a_μ , this decay mode becomes essential in the precise determination demanded by accuracy of the measurement of a_μ . According to the vector dominance model the form factor can be parametrized in terms of Breit-Wigner functions which correspond to the three mentioned resonances. Model parameters are fixed using data on



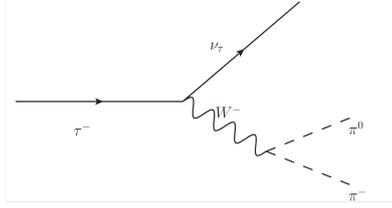


Figure 1. Feynman diagram of $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ decay.

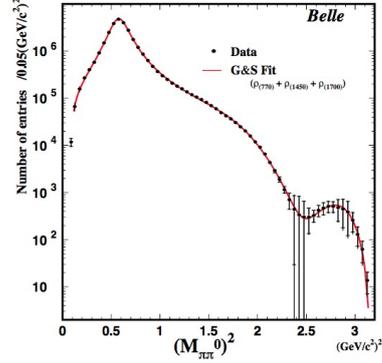


Figure 2. Invariant mass spectrum of the $\pi^- \pi^0$ system. The peaks of the $\rho(770)$, $\rho'(1450)$ and $\rho''(1700)$ resonances are shown [3].

the invariant mass spectrum of $\pi^- \pi^0$ system [3].

As our main result we predict the $\mathcal{BR}(\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \ell^+ \ell^-)$ using the same simplified form factor tested in the $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ channel. The evaluation of structure-dependent contributions to these decays is in progress [1].

2. The $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decay.

The leading Feynman Diagram contributing to this decay is shown in Figure 1. We use the following momenta convention in this decay:

$$\tau^-(P) \rightarrow \nu_\tau(q) \pi^-(P_-) \pi^0(P_0) \tag{1}$$

The effective Hamiltonian is given by

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} [V_{ud} \bar{d} \gamma_\mu (1 - \gamma_5) u] [\bar{\nu} \gamma^\mu (1 - \gamma_5) \tau] + h.c., \tag{2}$$

where the Cabibbo-Kobayashi-Maskawa matrix element is $|V_{ud}| = 0.97425 \pm 0.00022$ and $G_F = 1.1663787(6) \times 10^{-5} \text{GeV}^{-2}$ is the Fermi constant [2]. Thus the invariant amplitude is given by

$$\mathcal{M}(\tau^- \rightarrow \nu_\tau \pi^- \pi^0) = \frac{G_F V_{ud}^*}{\sqrt{2}} \langle \nu_\tau | \bar{\nu} \gamma_\mu (1 - \gamma_5) \tau | \tau^- \rangle \langle \pi^- \pi^0 | \bar{d} \gamma^\mu (1 - \gamma_5) u | 0 \rangle.$$

The axial part of the hadronic current (γ_5) does not contribute because this term does not conserve G-parity, thus $\langle \pi^- \pi^0 | \bar{d} \gamma^\mu \gamma_5 u | 0 \rangle = 0$. A general parametrization of the hadronic current is $\langle \pi^- \pi^0 | \bar{d} \gamma_\mu u | 0 \rangle = f_+(t)(P_- - P_0)_\mu + f_-(t)(P_- + P_0)_\mu$. However, in this work, we use the alternative form given by

$$\langle \pi^- \pi^0 | \bar{d} \gamma_\mu u | 0 \rangle = f_+(t) \left[(P_- - P_0)_\mu - \frac{\Delta^2}{t} (P_- + P_0)_\mu \right] + f_0(t) \frac{\Delta^2}{t} (P_- + P_0)_\mu, \tag{3}$$

where $\Delta^2 = m_{\pi^-}^2 - m_{\pi^0}^2$. The reason for using this basis is that this is orthogonal since f_+ and f_0 are related with the $J^P = 1^-, 0^+$ configurations, respectively, of the pionic system (while

this does not happen with f_+ and f_-). The invariant mass spectrum distribution of the pionic system is

$$\frac{d\Gamma(\tau^- \rightarrow \pi^- \pi^0 \nu)}{dt} = \frac{G_F^2 |V_{ud}|^2 m_\tau^3 S_{EW}}{6(4\pi)^3} \beta^3(t) \left[1 - \frac{t}{m_\tau^2}\right]^2 |f_+(t)|^2 \left(1 + \frac{2t}{m_\tau^2}\right), \quad (4)$$

where $t = (P_- + P_0)^2 = (P - q)^2$, $\beta = [\lambda^{1/2}(t, m_{\pi^-}^2, m_{\pi^0}^2)]/t$, $\lambda(t, m_{\pi^-}^2, m_{\pi^0}^2) = t^2 + m_{\pi^-}^4 + m_{\pi^0}^4 - 2tm_{\pi^-}^2 - 2tm_{\pi^0}^2 - 2m_{\pi^-}^2 m_{\pi^0}^2$. $S_{EW} = 1.0201 \pm 0.0003$ is electroweak radiative correction factor at short distances [4]. For the form factor we use a parametrization in agreement with the vector dominance model [5]

$$f_+(t) = \frac{1}{1 + \beta e^{i\phi} + \gamma} \left[BW_\rho + \beta e^{i\phi} BW_{\rho'}(t) + \gamma BW_{\rho''}(t) \right], \quad (5)$$

here BW_ρ , $BW_{\rho'}$ and $BW_{\rho''}$ are Breit-Wigner functions for the resonances of the ρ meson and its excitations. β , γ and ϕ are real parameters. For $BW_{\rho'}$ and $BW_{\rho''}$ we have used $\Gamma_{\rho'}$ and $\Gamma_{\rho''}$ as constants. The masses m_τ , m_{π^-} , m_{π^0} and mean life of τ lepton are given in PDG [2] while the parameters m_ρ , Γ_ρ , $m_{\rho'}$, $\Gamma_{\rho'}$, $m_{\rho''}$, $\Gamma_{\rho''}$, β , γ and ϕ are given in reference [6] for the fit to the Belle Spectrum. With this parameters we have obtained the branching ratio

$$\mathcal{B}(\tau \rightarrow \pi^- \pi^0 \nu_\tau) = (25.40 \pm 0.22)\%, \quad (6)$$

Comparing this result with the PDG value [2], $\mathcal{B}(\tau \rightarrow \pi^- \pi^0 \nu_\tau) = (25.52 \pm 0.09)\%$, there is a difference of 0.13 %, which indicates that the result is a very good approximation as expected because the parameters were obtained fitting Belle data. In latter stages of this study, a dispersive representation of this form factor [7] (consistent with elastic unitarity and analyticity) will be used.

3. The $\tau^- \rightarrow \pi^- \pi^0 \ell^+ \ell^- \nu_\tau$ decay.

This decay is interesting for at least three reasons: it is a prominent background in searches of Lepton Flavour Violation in the charged sector, and of Lepton Number Violation ($\tau^- \rightarrow \pi^- \pi^0 \ell^+ \ell^- \nu_\tau$ can emulate the signal of processes of the kind $L^- \rightarrow \ell^- \ell'^+ \ell'^-$). It is also very relevant to verify the radiative corrections used in the computation of the HVP contribution to a_μ using τ hadronic decay data. We use the following convention for the involved momenta

$$\tau(P) \rightarrow \pi^-(p_-) + \pi^0(p_0) + \ell^+(p_{\ell^+}) + \ell^-(p_{\ell^-}) + \nu_\tau(q), \quad (7)$$

where $\ell = e, \mu$. The matrix element is given by

$$T = eG_F V_{ud} \frac{l^\mu}{k^2} \left\{ F_\nu \bar{u}(q) \gamma^\nu (1 - \gamma_5) (M_\tau + \not{P} - \not{k}) \gamma_\mu u(P) + (V_{\mu\nu} - A_{\mu\nu}) \bar{u}(q) \gamma^\nu (1 - \gamma_5) u(P) \right\}, \quad (8)$$

with $l^\mu = \bar{u}_{\ell^-}(p_{\ell^-}) \gamma^\mu v_{\ell^+}(p_{\ell^+})$ the leptonic current, $k = p_{\ell^-} + p_{\ell^+}$ the momentum of the virtual photon. The first term describes the Bremsstrahlung off the initial τ^- . In this part $F_\nu = [(p_- - p_0)_\nu f_+(t)]/(k^2 - 2P \cdot k)$ and $t = (p_- + p_0)^2$. The form factor $f_+(t)$ rules the non-radiative decay (its parameters have been fixed in the preceding section). The second part of (8) describes the vector and axial-vector model-dependent contributions to the process

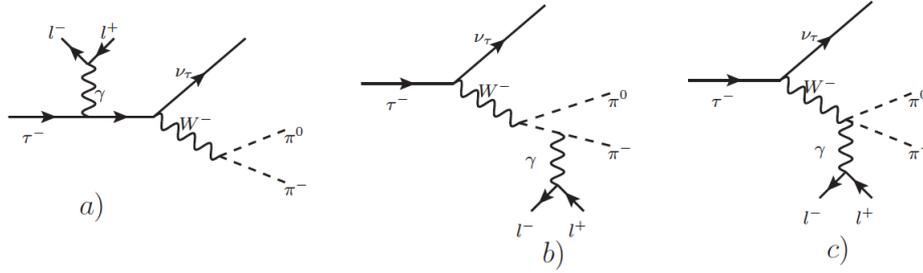


Figure 3. Leading diagrams contributing to the $\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ decays. The a) part corresponds to Inner Bremsstrahlung off the τ^- lepton. The b) diagram correspond to Inner Bremsstrahlung off the π^- meson. The c) diagram contains both and inner Bremsstrahlung and a structure-dependent part.

$W^-(P - q) \rightarrow \pi^-(p_-)\pi^0(p_0)\gamma(k)$. The hadronic tensor $V_{\mu\nu}$ contains a part that corresponds to Bremsstrahlung off the π^- in the final state. With respect to the influence of the model-dependent part (V_μ and A_μ) on the branching ratio, it is expected to be minimal for $\ell = e$, but dominant for $\ell = \mu$ [8]. The evaluation of this structure-dependent part is work in progress. Leading Feynman diagrams of the process are shown in Figure 3. The total Inner Bremsstrahlung amplitude is given by [9]

$$\begin{aligned} \mathcal{M}_{IB} = & \frac{e^2}{k^2} G_F V_{ud}^* l^\mu \left\{ \frac{(p_- - p_0)_\nu}{k^2 - 2P \cdot k} f_+(t) \bar{u}_\nu(q) \gamma^\nu (1 - \gamma_5) (\not{P} - \not{k} + M_\tau) \gamma_\mu u_\tau(P) \right. \\ & + \left[f_+(t') \frac{2p_{-\mu}}{2p_- \cdot k + k^2} (p_- + k - p_0)_\nu - f_+(t') g_{\mu\nu} + \right. \\ & \left. \left. 2 \left(\frac{f_+(t') - f_+(t)}{2(p_0 + p_-) \cdot k + k^2} \right) (p_- + p_0)_\mu (p_0 - p_-)_\nu \right] L^\nu \right\}, \end{aligned} \quad (9)$$

here $t' = (P - q)^2$ and $L^\nu = \bar{u}_\nu(q) \gamma^\nu (1 - \gamma_5) u_\tau(P)$. In order to get an easier calculation we have divided the square amplitude summed over polarizations as $|\overline{\mathcal{M}_{IB}}|^2 = |\overline{\mathcal{M}_{IB\tau}}|^2 + |\overline{\mathcal{M}_{IB\pi}}|^2 + 2Re[\overline{\mathcal{M}_{IB\tau}} \overline{\mathcal{M}_{IB\pi}}^*]$. Using these amplitudes we have obtained in $|\overline{\mathcal{M}_{IB\pi}}|^2$ and $2Re[\overline{\mathcal{M}_{IB\tau}} \overline{\mathcal{M}_{IB\pi}}^*]$ terms of the form $\epsilon_{\mu\nu\rho\sigma} q^\mu p_-^\nu p_0^\rho p_{\ell^-}^\sigma$, which we have expressed in terms of Lorentz invariants. For the kinematics of this five body decay we have used the reference [10]. Following this reference we can write all the scalar products $p_i \cdot p_j$ in terms of eight independent variables which we have defined as $s_1 = (P - q)^2$, $s_2 = (P - q - p_-)^2$, $s_3 = (P - q - p_- - p_0)^2$, $u_1 = (P - p_-)^2$, $u_2 = (P - p_0)^2$, $u_3 = (P - p_{\ell^-})^2$, $t_2 = (P - p_- - p_0)^2$, $t_3 = (P - p_- - p_0 - p_{\ell^-})^2$, and the auxiliary variables $s_0 = M^2$, $s_4 = m_{\ell^+}^2$, $u_0 = s_1$ and $t_1 = u_1$.

Using the PDG data [2] for the masses and considering the isospin limit $m_{\pi^-} = m_{\pi^0} = m_\pi$ (since we are considering a radiative process, their inequality is a next-to-leading order effect in $SU(2)$ breaking). Therefore, we take $m_\pi = [(2m_{\pi^-}^2 + m_{\pi^0}^2)/3]^{1/2}$. As the main result of this analysis, we have obtained the branching fractions shown in Table 1. As it is seen, the IB off the τ yields in both cases the smallest (but non-negligible) contribution to the branching ratios. Interference effects between IB off the τ and off the π are very relevant in both decay channels.

4. Conclusions

In this work, we have checked a simple description of the vector form factor of the pion against data. In this way, we have shown that the invariant mass spectrum of the $\pi^- \pi^0$ system is well

Table 1. Results for $\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ decay.

Contribution	$\tau^- \rightarrow \nu_\tau \pi^- \pi^0 e^+ e^-$	$\tau^- \rightarrow \nu_\tau \pi^- \pi^0 \mu^+ \mu^-$
$ \mathcal{M}_{IB\tau} ^2$	$(5.960 \pm 1.39) \times 10^{-6}$	$(4.947 \pm 0.021) \times 10^{-7}$
$ \mathcal{M}_{IB\pi} ^2$	$(3.416 \pm 0.211) \times 10^{-5}$	$(7.243 \pm 0.139) \times 10^{-7}$
$2\text{Re}[\mathcal{M}_{IB\tau}\mathcal{M}_{IB\pi}^*]$	$(4.075 \pm 0.723) \times 10^{-5}$	$(8.756 \pm 0.220) \times 10^{-7}$
Total	$(8.087 \pm 0.260) \times 10^{-5}$	$(2.094 \pm 0.004) \times 10^{-6}$

described by three resonances $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ and we have fixed their parameters in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays. The obtained branching ratio agrees with the world average PDG value.

For the $\tau^- \rightarrow \pi^- \pi^0 \ell^+ \ell^- \nu_\tau$ decays, the IB contributions to the branching fractions are of the order of magnitude expected. Consequently, these branching ratios are at a measurable level in Belle-II, and therefore they constitute an important background in the search for lepton flavor and lepton number violation in related processes. The considered decays can mimic the $\tau^- \rightarrow \mu^- \mu^+ \mu^-$, $\tau^- \rightarrow \mu^- e^+ e^-$, $\tau^- \rightarrow e^- \mu^+ \mu^-$, $\tau^- \rightarrow e^- e^+ e^-$, $\tau^- \rightarrow e^- \pi^+ \pi^-$, $\tau^- \rightarrow \mu^- \pi^+ \pi^-$, $\tau^- \rightarrow e^+ \pi^- \pi^-$, $\tau^- \rightarrow \mu^+ \pi^- \pi^-$ decays, with current limits of order 10^{-8} . The results using the dispersive pion form factor and including the structure-dependent contributions will be presented elsewhere.

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