

DIS at low x , high gluon densities, BFKL evolution and 3 particle correlations

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Abstract. In this contribution we provide a short overview of some recent results in the study of Deep Inelastic Scattering (DIS) processes at low x . We focus in particular on recent applications of next-to-leading order Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution to inclusive DIS events and to the search for saturation effects in exclusive events. In particular we discuss in some detail the recently calculated 3 particle production process in DIS which – for DIS on a large nuclei – promises to be a suitable candidate for saturation searches.

1. Introduction

Understanding the high energy limit of strong interaction is a long standing challenge which pre-dates the identification of Quantum Chromodynamics (QCD) as the correct theory of strong interactions. While QCD is generally strongly coupled and does not allow for a perturbative description, there exists a class of events where this is actually possible. Such perturbative or ‘hard’ events are characterized by a ‘hard’ scale M much bigger than the QCD confinement scale $\Lambda \sim 200$ MeV which – due to asymptotic freedom of QCD – implies a small strong coupling constant $\alpha_s(M) \ll 1$. With \sqrt{s} the center of mass energy and M the hard scale of the process, the perturbative high energy or Regge limit corresponds then to the limit $x = M^2/s \rightarrow 0$ at fixed M . For such a scenario, even though $\alpha_s(M) \ll 1$, large logarithms $\ln 1/x$ can compensate for the smallness of the strong coupling, $\alpha_s \ln 1/x \sim 1$ and therefore invalidate the naïve perturbative expansion in α_s . To recover predictive power of the perturbative expansion in this kinematic limit, it is therefore necessary to re-arrange the perturbative series through a resummation of terms $(\alpha_s \ln 1/x)^n$, to all orders in α_s . For perturbative QCD amplitudes, this is achieved by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation [1], which is currently known up to next-to-leading order (NLO) w.r.t. an expansion in α_s [2].

2. BFKL evolution and DIS at small x

A first test of BFKL evolution has been possible at the HERA experiments which measured proton structure functions in Deep-Inelastic Scattering (DIS) reactions in ep (electron-proton) scattering, see Fig. 1. In such DIS events, the virtuality Q^2 of the photon, which is exchanged between electron and proton, provides a hard scale and allows therefore for the use of QCD perturbation theory. A first approximate solution of the BFKL evolution equation predicts a strong rise of perturbative cross-sections with energy, $\sigma \sim x^{-\lambda}$, with $\lambda = \alpha_s 12 \ln 2/\pi \simeq 0.52$



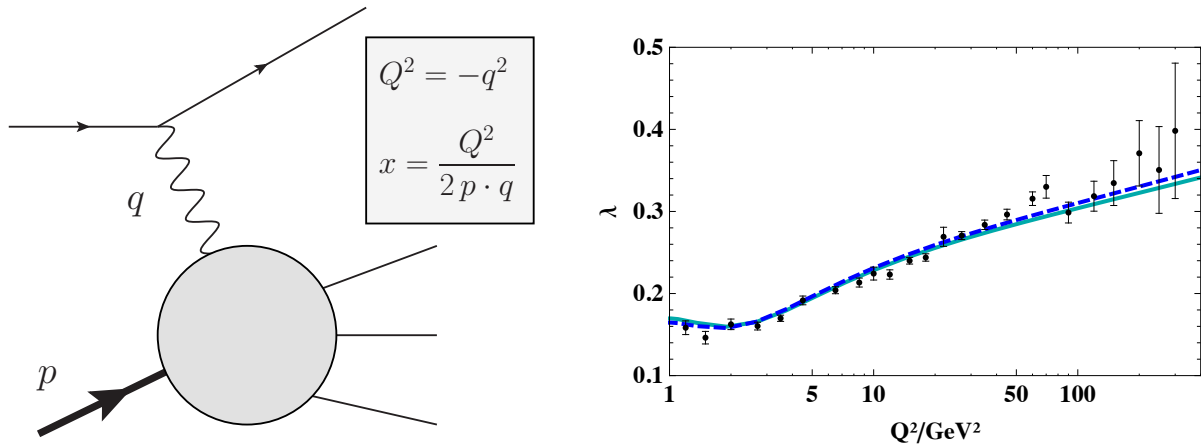


Figure 1. *Left:* Schematic DIS process: a virtual photon, emitted by the scattering electron, probes the proton at high virtuality Q^2 . Bjorken x denotes the ratio of photon virtuality and center-of-mass energy squared of the virtual photon-proton scattering process. *Right:* The effective Pomeron intercept $\lambda(Q^2)$, obtained from combined HERA data, using the following parametrization of the proton structure function $F_2(x, Q^2) = c(Q^2)x^{-\lambda(Q^2)}$. The theory predictions correspond to two different NLO BFKL fits performed in [4] (plot originally published in [4]).

for $\alpha_s = 0.2$. This predicted rise is far stronger than the rise of non-perturbative cross-sections such as the total cross-sections for proton-proton scattering, where typical values extracted from experiments are of order $\lambda \sim 0.1$. The strong rise of proton structure functions at large photon virtuality Q^2 , as measured by the HERA experiments, provided therefore first qualitatively evidence for BFKL evolution. While the above discussed approximate solution to BFKL evolution is not sufficient to describe the very precise combined HERA data¹, the agreement with data improves dramatically if a) BFKL evolution is solved exactly, not asymptotically and b) NLO corrections are included, see [3, 4]. In the case of [4] this has been possible due to a resummation of collinear enhanced configurations within the NLO BFKL kernel, together with an optimal scale setting for the scale of the running coupling. To test these fits further, the natural choice is to study LHC cross-sections, which due to their large center-of-mass energies are well suited for such a purpose. Of particular interest are in this context hard production processes in the forward direction of one of the scattering protons, such as production of jets with [5] and without [6–8] rapidity gaps, Z -bosons [9, 10] and bottom quarks [11], since they allow to probe the gluon distribution of the other proton at very small values of x , sometimes down to ultra-small values of $x \simeq 10^{-6}$. A particular interesting class of processes are in this context exclusive photo-production cross-sections of vector meson at the LHC, which are measured in ultra-peripheral pp (proton-proton) and pPb (proton-lead) collisions, where either the lead nucleus (pPb) or a proton (pp) serves as a photon source [12–15]. Combining LHC with HERA data [16–22], measured in ep collisions, such measurements allow to study directly the rise with energy of the photo-production cross-section. For photo-production of J/Ψ and Υ vector mesons, where the heavy charm and bottom masses provide a hard scale, it was recently shown [23] that this rise has an excellent description in terms of NLO BFKL fits of [4].

¹ In particular the asymptotic leading order BFKL ‘intercept’ $\lambda = \alpha_s(Q)12 \ln 2/\pi$ is decreasing with decreasing $\alpha_s(Q)$ and therefore with increasing Q^2 in contrast to data which clearly find an intercept *increasing* with Q^2 , see Fig. 1, right.

3. High gluon densities at small x

Even though NLO BFKL evolution provides a very good description of currently available low x data, there exist unitarity bounds for hadronic cross-sections which indicate that the power-like rise with increasing x cannot continue forever; instead the observed rise must eventually slow down and saturate. A dynamical mechanism which achieves perturbative unitarity of QCD cross-sections at low x is known under the term ‘gluon saturation’ as originally proposed by Gribov, Levin and Ryskin [24]. The framework of the Color Glass Condensate (CGC) provides then an effective action approach to gluon saturation [25]. It gives a description of the high energy hadron or nucleus as a weakly-coupled yet non-perturbative system of gluons with high occupancy number, characterized by a semi-hard scale Q_s , called the saturation scale. As a result one obtains a non-linear generalization of the BFKL evolution equation [26–28], which includes multiple-scattering effects; the latter provide the necessary slow down and saturation of the growth with energy, see [29] and references therein for a detailed discussion.

Since the CGC-framework assumes high occupation number of the gluonic target field, the latter can be no longer treated perturbatively. The resulting formalism represents therefore the DIS scattering process in the low x limit as the scattering of the virtual photon on a strong background color field, which represents the target. Defining light-cone vectors n, \bar{n} through the four momenta of virtual photon and target, *i.e.* $n \sim p$ and $n \cdot \bar{n} = 1$ with $v^- \equiv n \cdot v, v^+ \equiv \bar{n} \cdot v$ for a generic four vector v , theory calculations are performed using quark and gluon propagators in the presence of the background field $A^{+,c}$. In particular one has for the propagators

$$\begin{aligned}
 & \text{Diagram: } p \text{ (quark line) } \rightarrow \text{gluon loop} \rightarrow q \quad = (2\pi)^d \delta^{(d)}(p-q) \tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p) \text{Diagram: } p \text{ (quark line) } \rightarrow \text{gluon loop} \rightarrow q \quad \tilde{S}_F^{(0)}(q), \\
 & \text{Diagram: } p, \mu \text{ (gluon line) } \rightarrow \text{gluon loop} \rightarrow q, \nu \quad = (2\pi)^d \delta^{(d)}(p-q) \tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p) \text{Diagram: } p \text{ (gluon line) } \rightarrow \text{gluon loop} \rightarrow q \quad \tilde{G}_{\alpha\nu}^{(0)}(q), \quad (1)
 \end{aligned}$$

with the free propagators

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0}, \quad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{id_{\mu\nu}(p)}{p^2 + i0}, \quad d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}, \quad (2)$$

and the interaction terms

$$\text{Diagram: } p \text{ (quark line) } \rightarrow \text{gluon loop} \rightarrow q \quad (3)$$

$$= 2\pi\delta(p^- - q^-) \not{n}^- \int d^{d-2}z e^{-iz \cdot (p-q)} \left\{ \theta(p^-) [V(z) - 1] - \theta(-p^-) [V^\dagger(z) - 1] \right\},$$

$$\text{Diagram: } p \text{ (gluon line) } \rightarrow \text{gluon loop} \rightarrow q$$

$$= -2\pi\delta(p^- - q^-) 2p^- \int d^{d-2}z e^{-iz \cdot (p-q)} \cdot \left\{ \theta(p^-) [U(z) - 1] - \theta(-p^-) [U^\dagger(z) - 1] \right\}. \quad (4)$$

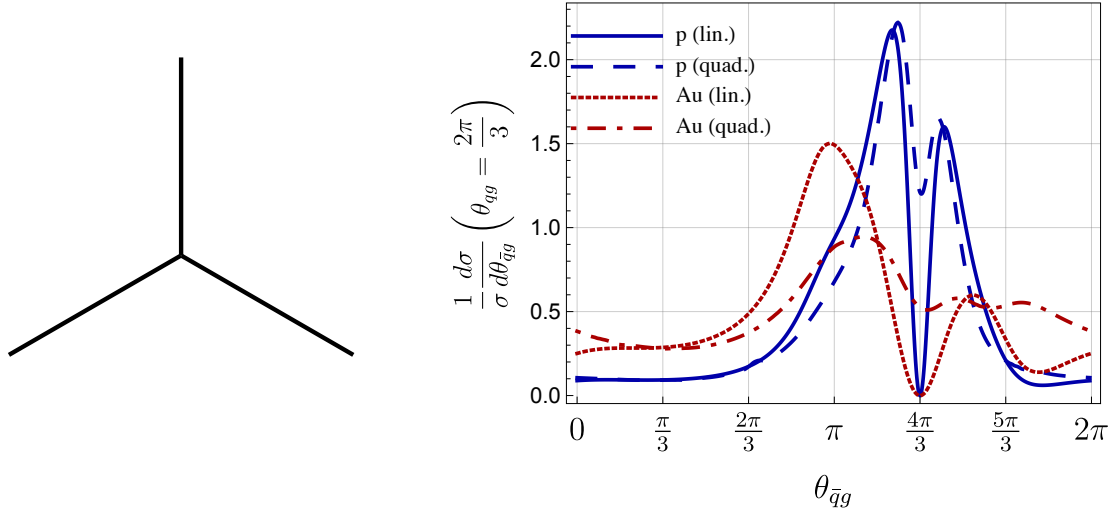


Figure 2. *Left:* Mercedes-Benz star configuration: all transverse momenta have equal absolute value, with relative angle $\Delta = 2\pi/3$. *Right:* Normalized cross-section for three parton production in the presence of large gluon densities against $\Delta\theta_{\bar{q}g}$ with $\Delta_{qg} = 2\pi/3$ for proton and gold up to linear and quadratic order in $N^{(2)}$. For the study the photon momentum fractions of the final state particles have been fixed to $z_1 = z_2 = 0.2$, $z_3 = 0.6$ while $|\mathbf{p}_1| = |\mathbf{p}_2| = |\mathbf{p}_3| = 2$ GeV and $Q^2 = 9$ GeV² (plot originally published in [31]).

The latter contain Wilson lines in the fundamental (V) and adjoint (U) representation which resum the interaction with the background field with

$$V(\mathbf{z}) \equiv V_{ij}(\mathbf{z}) \equiv \text{P exp } ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \mathbf{z}) t^c \quad (5)$$

and $U^{ba}(z_t) = 2 \text{tr}(t^a V(z_t) t^b V^\dagger(z_t))$; P denotes path ordering of the gluon field in the exponent while t^c is a $\text{SU}(N_c)$ generator in the fundamental representation.

Current x values reached at today's collider experiments are most likely still too large to provide definite evidence for the presence of saturation effects. A first possibility to increase gluon densities and therefore the sensitivity to saturation is to consider scattering on large nuclei, where gluon densities are expected to be enhanced by a factor $A^{\frac{1}{3}}$, with A the mass number of the nucleon. A second possibility is to consider exclusive final states: while for inclusive observables the presence of high gluon densities leads mainly to a modification of the x dependence of gluon densities [30], exclusive production processes can provide matrix elements which differ for high gluon densities from their low density counter parts. They are therefore more sensitive to effects related to high gluon densities and provide a tool for their exploration. Such a study has been carried out in [31], where the differential cross-section for the process

$$\gamma^*(q) + \text{target}(p) \rightarrow q(p_1) + \bar{q}(p_2) + g(p_3) + X, \quad (6)$$

has been calculated for first time within the CGC formalism. For details on the calculation we refer to [31]. The results of a first numerical study are shown in Fig. 2. With the differential cross-section for the process Eq. (6) expressed in terms of dipole and quadrupole operators of Wilson lines Eq. (5),

$$S_{(\mathbf{x}_1 \mathbf{x}_2)}^{(2)} \equiv \frac{1}{N_c} \text{tr} [V(\mathbf{x}_1) V^\dagger(\mathbf{x}_2)], \quad S_{(\mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_3 \mathbf{x}_4)}^{(4)} \equiv \frac{1}{N_c} \text{tr} [V(\mathbf{x}_1) V^\dagger(\mathbf{x}_2) V(\mathbf{x}_3) V^\dagger(\mathbf{x}_4)], \quad (7)$$

this study uses the large N_c and Gaussian approximation [33] to express the quadrupole in terms of the dipole operator. For the dipole profile itself a model motivated by recent rcBK² fits to HERA data [32] has been used,

$$S_{(\mathbf{x}_1\mathbf{x}_2)}^{(2)} = \int d^2\mathbf{l} e^{-i\mathbf{l}\cdot\mathbf{x}_{12}} \Phi(\mathbf{l}^2) = 2 \left(\frac{Q_0|\mathbf{x}_{12}|}{2} \right)^{\rho-1} \frac{K_{\rho-1}(Q_0|\mathbf{x}_{12}|)}{\Gamma(\rho-1)},$$

$$\text{where} \quad \Phi(\mathbf{l}^2) = \frac{\rho-1}{Q_0^2\pi} \left(\frac{Q_0^2}{Q_0^2 + \mathbf{l}^2} \right)^\rho, \quad (8)$$

and Q_0 is a scale proportional to the saturation scale. To justify the dilute limit associated with the Gaussian approximation, we study the cross-section at large photon virtuality $Q^2 = 9 \text{ GeV}^2$ and expand the differential cross-section up to quadratic order in $N^{(2)} = 1 - S^{(2)}$. We further use $\rho = 2.3$ and $Q_0^{\text{proton}} = 0.69 \text{ GeV}$, as motivated by inclusive DIS fits at $x = 0.2 \times 10^{-3}$. Since we want to search for effects due to large gluon densities, we further simulate scattering on a gold nucleus through the re-scaling $Q_0^{\text{Au}} = A^{1/6} \cdot Q_0^{\text{proton}} = 1.67 \text{ GeV}$. At linear order in $N^{(2)}$, the cross-section is proportional to the Fourier transform of the dipole, $\Phi((\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)^2)$ and therefore gives direct access to the gluon distribution in the target. At this order the cross-section is – up to the detailed shape and x -dependence of the dipole factor Eq. (8) – identical to a corresponding BFKL calculation. The configuration dominant in the ‘collinear’ limit corresponds on the other hand to the vanishing of the sum of transverse momenta $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$, *i.e.* the transverse momentum transfer between projectile and target is zero. To study deviations from this configuration we first chose $|\mathbf{p}_1| = |\mathbf{p}_2| = |\mathbf{p}_3|$. For this choice the ‘collinear’ limit $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$ is then given by the Mercedes-Benz star configuration, see Fig. 2. We observe vanishing of the partonic cross section at these ‘collinear’ configurations, Fig. 2, with a strong double peak. This vanishing is explained by the vanishing of the partonic matrix element at leading order in $N^{(2)}$ for zero momentum transfer between projectile and target, which itself is expected from gauge invariance. For a complete study at hadronic level, we expect the double peak to turn into a single peak. While in the case of the proton we find (besides the double-peak structure) mainly a smearing of the collinear peak at $\Delta_{\bar{q}g} = 4\pi/3$, the effect of a larger gluon saturation scale for a gold nucleus can be clearly seen: while the shape of the linear gold curve is similar to the proton curve, it is wider and depleted with respect to the former. Including further quadratic terms in $N^{(2)}$, we start to probe effects due to high gluon densities at the level of the matrix element beyond the mere modification of Eq. (8) and beyond the corresponding BFKL matrix element. In the case of the proton, these corrections are small, as expected for $x = 0.2 \cdot 10^{-3}$ and a relatively large photon virtuality $Q^2 = 9 \text{ GeV}^2$. For a highly saturated gold nucleus, they are on the other hand sizable. While this apparently puts doubts on the reliability of the current expansion up to quadratic order in $N^{(2)}$, it clearly demonstrates the sensitivity of this processes to high and saturated gluon densities and identifies it as suitable tool in searching for such effects already at currently accessible collider energies.

Acknowledgments

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² The running coupling Balitsky-Kovchegov equation (rcBK) provides a non-linear generalization of the BFKL equation as discussed above, see [27, 32] for details

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