

Electromagnetic corrections to $B^- \rightarrow V^0$ semileptonic transitions

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Abstract. We evaluate the $O(\alpha)$ electromagnetic radiative corrections to the $B^- \rightarrow V^0 \ell^- \bar{\nu}_\ell$ (V is a vector meson and $\ell = e, \mu$) decay rates. Intermediate resonance states in the real-photon amplitude are considered in order to extend the region of validity of the soft-photon approximation. The virtual corrections are regularized by the cutoff scale Λ , which is used to match the long- and short-distance regimes. The radiative corrections to the integrated decay rate are presented and we discuss its impact in the precise determination of the b quark mixing parameters.

1. Introduction

In the Standard Model (SM) of weak interactions, the elements of the Cabbibo-Kobayashi-Maskawa matrix are fundamental parameters to describe the quark-mixing [1]. In particular, improved measurements of the magnitude of V_{ub} and V_{cb} are needed to test the SM mechanism of CP violation [2, 3]. The two methods to extract these elements in B decays are based on the study of inclusive and exclusive semileptonic decays. However, these values differ from each other by $\Delta|V_{xb}| = |V_{xb}|_{\text{incl}} - |V_{xb}|_{\text{excl}} = (1.13 \pm 0.36) \times 10^{-3}$, and $(2.7 \pm 1.1) \times 10^{-3}$ [4], for $x = u$ and c , respectively. The values extracted from exclusive decays are commonly obtained from $B \rightarrow P \ell \nu$ decays (P is a pseudoscalar meson) since they are widely known theoretically [5, 6]. Isospin breaking corrections are useful to test the consistency of experimental data and theoretical calculations, as shown in superallowed Fermi transitions (SFT) and Kaon semileptonic ($K_{\ell 3}$) decays [7]. In this work we are concerned with the calculation of electromagnetic radiative corrections to $B^\pm \rightarrow V^0$ semileptonic transitions, a useful input in testing isospin symmetry in B decays.

2. Tree level amplitude for $B^- \rightarrow V^0$ semileptonic transitions

At the tree-level, the decay amplitude is:

$$\mathcal{M}^0 = \frac{G_F}{\sqrt{2}} V_{qb} c_V W_\nu(P_V, P) L^\nu, \quad (1)$$

where G_F is the Fermi constant, V_{qb} ($q = u, c$) the relevant CKM matrix element, $L^\nu = \bar{u}_\ell \gamma^\nu (1 - \gamma_5) v_\nu$ represents the leptonic current, and $c_V = 1/\sqrt{2}$ is the Clebsh-Gordan coefficient for $V = (\rho^0, \omega)$ mesons. P (P_V) is the four momentum of the decaying (daughter) meson. The



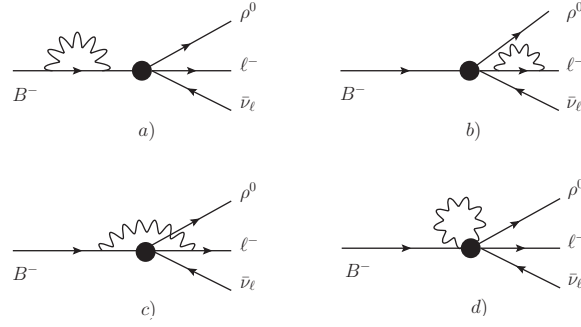


Figure 1. Virtual photon corrections to $B^- \rightarrow \rho^0 \ell^- \bar{\nu}_\ell$. Figs. a) and b) correspond to the self-energies of charged particles and c), d) to vertex contributions.

hadronic matrix element can be parametrized in terms of four $q^2 = (P - P_V)^2$ -dependent form factors, which we choose as (V, A_1, A_2, A)

$$W_\nu(P_V, P) = \frac{2V^{B \rightarrow V}}{M + m_V} \epsilon_{\nu\alpha\beta\gamma} \varphi^\alpha P^\beta P_V^\gamma - i \frac{2m_V A^{B \rightarrow V}}{q^2} q \cdot \varphi q_\nu - i(M + m_V) A_1^{B \rightarrow V} \varphi_\nu + i \frac{A_2^{B \rightarrow V}}{M + m_V} q \cdot \varphi (P + P_V)_\nu. \quad (2)$$

The four-vector φ denotes the vector meson polarization, with the orthogonality condition $P_V \cdot \varphi = 0$. The form factors are specific to the $B \rightarrow V$ transition [8].

3. Virtual radiative corrections

The charged meson and lepton self-energies (Fig. 1 a and b) contribute to the decay amplitude as

$$\mathcal{M}_{a+b}^1 = \mathcal{M}^0 \frac{1}{2} (\delta Z_\ell + \delta Z_B), \quad (3)$$

where

$$\begin{aligned} \delta Z_\ell &= \frac{\alpha}{4\pi} \left(2 - B_0[m^2, 0, m^2] + 4m^2 B'_0[m^2, \lambda^2, m^2] \right) \\ \delta Z_B &= \frac{\alpha}{4\pi} \left(2B_0^M[M^2, 0, M^2] + 4M^2 B_0'^M[M^2, \lambda^2, M^2] \right), \end{aligned}$$

which contribute to the wave function renormalization of the charged particles. $B_0^{(M)}[\dots]$ and $B_0'^{(M)}[\dots]$ are the Passarino-Veltman functions corresponding to the scalar two-point integral and its derivative [9]. We will isolate the infrared (IR) singularity in $B_0^{(M)}$ by providing a fictitious mass λ to the photon, and regulate the ultraviolet (UV) singularity of B_0 by using the cutoff Λ as the maximum scale at which the long-distance approximation is expected to be valid. By long-distance (LD) corrections we mean, emission/absorption of real and virtual photons with small momenta ($k < \Lambda$) such that they cannot resolve the structure of hadrons; at these low photon momenta, the point-like approximation for hadrons and the use of scalar QED for describing photon-meson interactions should be a good approximation.

We will follow Sirlin's prescription [10] to separate the model-independent and model-dependent contributions from Figure 1c,d. For the model-independent part of the virtual corrections we obtain (here $u = (P - p)^2$)

$$\mathcal{M}_{c+d, \text{MI}}^1 = \frac{\alpha}{4\pi} \mathcal{M}^0 \left\{ B_0^M[M^2, 0, M^2] + 4p \cdot PC_0[m^2, M^2, u, m^2, \lambda^2, M^2] \right\}$$

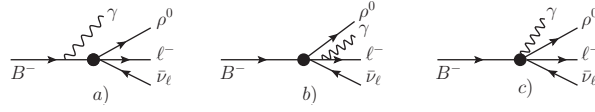


Figure 2. Feynman diagrams for real photon emission.

$$\begin{aligned}
 & +2M^2 C_1[m^2, M^2, u, m^2, 0, M^2] + 4P \cdot p C_2[m^2, M^2, u, m^2, 0, M^2] \Big\} \\
 & - \frac{\alpha}{4\pi} \mathcal{M}_{\text{NF}} C_2[m^2, M^2, u, m^2, 0, M^2] ,
 \end{aligned} \tag{4}$$

where $C_i[m^2, M^2, u, m^2, \lambda^2, M^2]$ ($i = 0, 1, 2$) denote the Passarino-Veltman functions. We have used again λ to isolate the IR-divergence contained in C_0 , while $C_{1,2}$ are evaluated in the limit $\lambda \rightarrow 0$. The last term in Eq.(4) involves $\mathcal{M}_{\text{NF}} = \frac{G_F}{\sqrt{2}} m V_{ub} W^\mu(P_V, P) \bar{u}_\ell \not{P} \gamma_\mu (1 - \gamma_5) v_{\bar{\nu}_\ell}$, which corresponds to a non-factorizable (NF) amplitude, it is IR safe and is suppressed for light charged leptons.

The short distance (SD) corrections to semileptonic decays are finite in the electroweak theory [11, 12]. In the dominant logarithmic approximation the $O(\alpha)$ corrections depend upon the separation scale Λ as follows (m_Z is the Z boson mass)

$$\delta_{\text{SD}}^1 = \frac{2\alpha}{\pi} \ln \left(\frac{m_Z}{\Lambda} \right) . \tag{5}$$

It is customary to set the lower cutoff of SD corrections as the mass of the decaying particle; however, we will keep it explicitly to study the cutoff scale dependence of the full radiative corrections.

4. Real photon emission

By taking into account the contributions in Fig. 2 one gets the decay amplitude in the Low's expansion [13]

$$\begin{aligned}
 \mathcal{M}^{\text{Low}} = & e \frac{G_F}{\sqrt{2}} V_{qb} c_V \left\{ W_\mu(P_V, P) \bar{u}_\ell \left[-\frac{P \cdot \varepsilon}{P \cdot k} + \frac{2p \cdot \varepsilon + \not{\varepsilon} \not{k}}{2p \cdot k} \right] \gamma^\mu (1 - \gamma_5) v_{\nu_\ell} \right. \\
 & \left. + L^\mu D^\lambda W_{\mu\lambda} - 2P_V \cdot D L^\mu \frac{\partial W_\mu(P_V, P)}{\partial q^2} \right\} ,
 \end{aligned} \tag{6}$$

where ε denotes the polarization four-vector of the real photon, with $k \cdot \varepsilon = 0$. We have defined $D^\lambda = (\varepsilon \cdot P / k \cdot P) k^\lambda - \varepsilon^\lambda$, L and W are the same as in Eq. (1), and

$$\begin{aligned}
 W_{\mu\lambda} = & -\frac{2}{M + m_V} V \epsilon_{\mu\lambda\nu\alpha} \varphi_V^\nu P_V^\alpha + i \frac{q \cdot \varphi}{M + m_V} A_2 \delta_{\mu\lambda} \\
 & + i \frac{A_2}{M + m_V} (P + P_V)_\mu \varphi_\lambda - 2i \frac{m_V A}{q^2} q_\mu \varphi_\lambda - 2i \frac{m_V A}{q^2} q \cdot \varphi \delta_{\mu\lambda} .
 \end{aligned} \tag{7}$$

As required by soft photon theorems [13], Eq. (6) contains terms up to $\mathcal{O}(k^0)$ which are fixed by gauge-invariance and depend only upon the form factors of the non-radiative decay amplitude. The partial derivative in the last term of Eq. (6) must be carried over explicit q^2 -dependent form factors in W . We will consider the evaluation of the last two terms in Eq. (6) as a part of the model-dependent contribution.

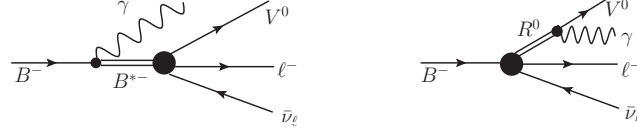


Figure 3. Model-dependent real photon corrections; R^0 denotes a pseudoscalar resonance.

4.1. Model-dependent contribution

For relatively low photon energies, [6] points out that the diagrams shown in Figure 3 must be taken into account in $B \rightarrow D\ell\nu_\ell\gamma$ decays since on-shell $D^*(\rightarrow D\gamma)$ intermediate states can have an important contribution as non-radiative events. For the decays considered in this paper $B^\pm \rightarrow (\rho^0, \omega, D^{*0})\ell^\pm\nu_\ell$, there are not nearby narrow intermediate states decaying into $V^0\gamma$ states that can produce a similar important effect. In our study, we evaluate the model-dependent (MD) contributions in Fig. 3 for moderately energetic photons following closely Ref. [6].

The general form of the MD decay amplitude becomes

$$\mathcal{M}^{MD} = e \frac{G_F}{\sqrt{2}} V_{qb} c_V \epsilon^\mu (V_{\mu\nu}^{MD} - A_{\mu\nu}^{MD}) L^\nu. \quad (8)$$

From Figure 3 we get [6]

$$V_{\mu\nu}^{MD} - A_{\mu\nu}^{MD} = \frac{\langle V^0 | (V_\nu - A_\nu) | B^{*-}(P_1) \rangle \langle B^{*-}(P_1) | J_\mu | B^- \rangle}{P_1^2 - m_{B^*}^2 + i\epsilon} + \frac{\langle V^0 | J_\mu | R^0(P_2) \rangle \langle R^0(P_2) | (V_\nu - A_\nu) | B^- \rangle}{P_2^2 - m_R^2 + i\epsilon}. \quad (9)$$

where $P_1 = P - k$ and $P_2 = P_V + k$ denote the four momenta of the corresponding resonances, and $m_{B^*}(m_R)$ the mass of $B^{*-}(R^0)$ (if the R resonance is produced on-shell, we have to use $\epsilon \rightarrow m_R \Gamma_R$). In the above expression J_μ denotes the electromagnetic current and $(V_\nu - A_\nu)$ denotes the usual V-A weak currents [6]. Details about the expressions for the matrix elements in Eq. (9) can be found in [14].

5. Results

In this section we present the results of radiative corrections to $B^- \rightarrow V^0 \ell^- \bar{\nu}_\ell$, where $V^0 = (\rho^0, \omega, D^{*0})$. The integrals involved are evaluated numerically, and we include the LD radiative corrections obtained in the previous sections and the SD correction in Eq. (5).

In the case of $V = \rho^0$ meson, the total radiative corrections for the two leptonic channels are

$$\delta_T^1(\ell) = \begin{cases} (1.62 \pm 0.10 \pm 0.04)\%, & \text{for } \ell = \mu \\ (1.63 \pm 0.11 \pm 0.04)\%, & \text{for } \ell = e \end{cases}. \quad (10)$$

The first uncertainty is associated to the variation in the scale Λ between $m_\rho/2$ and 2 GeV, while the central value is taken for $\Lambda = m_\rho$. The remaining uncertainty corresponds to the error in the numerical evaluation and the use of different models for the form factors [8]. We do not found any relevant difference between this case and $V^0 = \omega$, given the near equality of the masses and the matrix elements.

A similar analysis can be applied to the case with $V^0 = D^{*0}$, where we obtain

$$\delta_T^1(\ell) = \begin{cases} (1.53 \pm 0.06 \pm 0.04)\%, & \text{for } \ell = \mu \\ (1.53 \pm 0.08 \pm 0.04)\%, & \text{for } \ell = e \end{cases}. \quad (11)$$

The values are taken for $\Lambda = m_{D^{*0}}$, the first uncertainty corresponds to variations between $m_{D^{*0}}/2$ and $2m_{D^{*0}}$, and the second one to the numerical estimation as in the previous case.

We have found that the total radiative corrections of Eqs. (10) and (11) are dominated by the SD radiative corrections since the LD are 4 (7) times smaller, for $V^0 = \rho^0$ (D^{*0}). This is due to partial cancellations of the contributions coming from the three and four body region of the phase space. The MD corrections play a subleading role because their contributions were estimated to be at most ten times smaller than the MI ones, which is in agreement with the results obtained for radiative $K_{\ell 3}$ decays [15]. The effect of LD radiative corrections to specific points of the Dalitz plot has been studied in [14].

6. Conclusions

We have evaluated numerically the long-distance (LD) electromagnetic corrections to the charmless and charmfull semileptonic $B^- \rightarrow V^0 \ell^- \bar{\nu}_\ell$ decays involving the ground state vector mesons V^0 and $\ell = \mu$ and e flavored leptons. We have associated an uncertainty to the radiative corrections in the decay rate due to the cutoff dependence by allowing it to vary between $m_V/2$ and approximately $2m_V$. The effects of using different models for the weak form factors are estimated to play a subleading role compared to the dependence upon the separation scale Λ . We have found that there exist large cancellations between the contributions of LD corrections to the integrated decay rates coming from the three- and four-body regions of the Dalitz plot. We found that the MD contributions are negligible since they are smaller than the quoted error. Our results can be useful for future/improved measurements of the different charged decay channels of $B \rightarrow V \ell^- \bar{\nu}_\ell$ branching ratios in order to test consistency of data with isospin symmetry. Actually, the determination of the V_{ub} and V_{cb} CKM elements at the few percent level requires the consideration of the full radiative corrections of $O(\alpha)$.

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