Journal of Physics: Conference Series **761** (2016) 012010 doi:10.1088/1742-6596/761/1/012010

$\Delta\kappa'_V$ and ΔQ_V ($V = \gamma,\ Z$) form factors in the Georgi-Machacek model

M.A. Arroyo-Ureña, G. Hernández-Tomé, and G. Tavares-Velasco
Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla, Apartado postal 1152, 72001 Puebla, Pue., México

E-mail: gerardoht@alumnos.fcfm.buap.mx

Abstract. The CP-even static form factors $\Delta\kappa'_V$ and ΔQ_V ($V = \gamma, Z$) associated with the WWV vertex are studied in the context of the Georgi-Machacek model (GMM), which predicts nine new scalar bosons accommodated in a singlet, a triplet and a fiveplet. General expressions for the one-loop contributions to $\Delta\kappa'_V$ and ΔQ_V arising from neutral, singly and doubly charged scalar bosons are obtained in terms of both parametric integrals and Passarino-Veltman scalar functions, which can be numerically evaluated. It is found that the GMM yields 15 (28) distinct contributions to $\Delta\kappa'_\gamma$ and ΔQ_γ ($\Delta\kappa'_Z$ and ΔQ_Z), though several of them are naturally suppressed.

1. Introduction

Models with scalar triplet representations have attracted considerable attention due to their appealing features, such as the possibility of implementing the see-saw mechanism to endow the neutrinos with naturally light Majorana masses (the so called type-II see-saw), the appearance of the $H^\pm W^\mp Z$ coupling at the tree level, and the presence of doubly charged scalar particles. In this respect, the Georgi-Machacek model (GMM) [1, 2] is one of the most attractive Higgs triplet models as it preserves the relationship $\rho = 1$ at the tree level via an $SU(2)$ custodial symmetry. The physical scalar spectrum of the GMM is given by the SM-like Higgs boson h and one extra CP-even singlet H, one scalar triplet H_3 (H_3^0, $H_3^\pm$), and one scalar fiveplet H_5 (H_5^0, $H_5^{\pm\pm}$, $H_5^\pm$). All of these multiplets are mass degenerate as a result of the custodial symmetry. Even if there is not enough energy available to produce the new scalar particles predicted by the GMM, one can search for their virtual effects through some observables. Particular interest has been put on the radiative corrections to the WWV ($V = \gamma, Z$) vertex, which represents a very sensitive scenario to search for any NP effects and test the gauge sector of the SM.

The on-shell WWV vertex can be written in terms of four form factors that define the CP-even and CP-odd static properties of the W boson. The two CP-odd form factors $\Delta\widetilde{\kappa}'_V$ and $\Delta\widetilde{Q}_V$ are absent up to the one-loop level in the SM and are thus expected to be negligibly small. As far as the CP-even form factors $\Delta\kappa'_V$ and ΔQ_V are concerned, they arise at the one-loop level in the SM and any other renormalizable theory, thereby being highly sensitive to NP effects.

The vertex function that determines the WWV CP-conserving coupling can be written as

$$\Gamma_V^{\mu\alpha\beta} = ig_V\Big\{A\left[2p^\mu g^{\alpha\beta} + 4\left(Q^\beta g^{\mu\alpha} - Q^\alpha g^{\mu\beta}\right)\right]$$

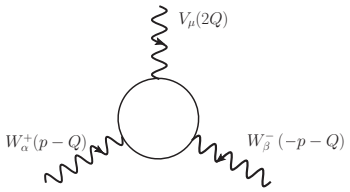

Figure 1. The WWV vertex function. The circle denotes radiative contributions.

$$+2\Delta\kappa'_V\left(Q^\beta g^{\mu\alpha}-Q^\alpha g^{\mu\beta}\right)+\frac{4\Delta Q_V}{m_W^2}\left(p^\mu Q^\alpha Q^\beta-\frac{1}{2}m_V^2 p^\mu g^{\alpha\beta}\right)\Bigg\}, \tag{1}$$

where g_V stand for the WWV tree-level coupling constant (in the SM $g_\gamma = g s_W$ and $g_Z = g c_W$) and we have used the convention employed in [3] for the external momenta, as shown in Fig. 1.

2. $\Delta\kappa'_V$ and ΔQ_V form factors in the GMM

The new scalar one-loop contributions from the GMM to the $\Delta\kappa'_V$ and ΔQ_V form factors arise from generic triangle diagrams (the bubble diagrams do not contribute) that can be classified according to the number of distinct particles circulating into the loop. Each type of diagram involves trilinear vertices between gauge bosons only or between scalar bosons and gauge bosons. In Fig. 2 we show a set of Feynman diagrams that contribute to both the $WW\gamma$ and WWZ vertices.

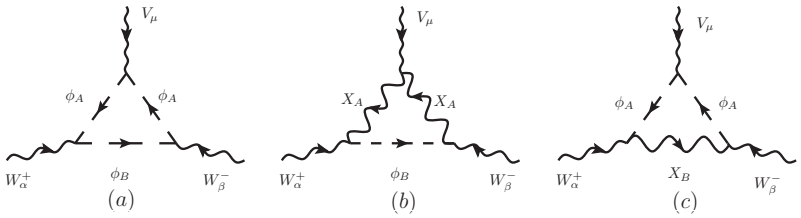

Figure 2. Generic Feynman diagrams for the new scalars contributions to both the $WW\gamma$ and WWZ vertices involving only two distinct virtual particles. The arrows stand for the directions of the 4-momenta. The possible combinations of internal particles are given by the vertices allowed in each particular model. For instance, when $V = \gamma$, the following electric charges of the internal particles are possible in the GMM, in units of the positron charge: if $Q_A = -1$ then $Q_B = 0$, if $Q_A = 1$ then $Q_B = -2$, if $Q_A = -2$ then $Q_B = 1$.

Contrary to the couplings of the photon to a pair of charged scalar bosons, which can only be of diagonal type due to electromagnetic gauge invariance, the Z boson can have nondiagonal couplings to a pair of neutral or charged scalar bosons. Therefore, in addition to the diagrams of Fig. 2, the $\Delta\kappa'_Z$ and ΔQ_Z form factors can receive extra contributions from the Feynman diagrams shown in Fig. 3, which have three distinct particles circulating into the loop. Below we will present the contributions to the $\Delta\kappa'_V$ and ΔQ_V form factors for all these type of diagrams.

Before presenting our results, some remarks about our calculation are in order:

- The Feynman diagrams were evaluated via the unitary gauge. In order to make a cross check of our results we used both, the Feynman parametrization technique and the Passarino-Veltman method to solve the loop integrals.

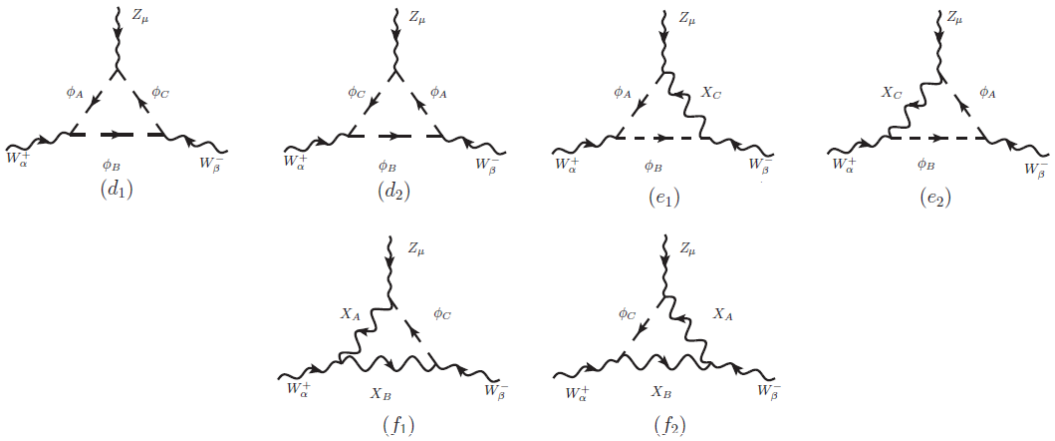

Figure 3. Extra contributions to the $\Delta\kappa'_Z$ and ΔQ_Z form factors from nondiagonal couplings.

- We verified that all the contributions of bubble diagrams to the $\Delta\kappa'_V$ and ΔQ_V form factors involving quartic vertices with two scalar bosons and two gauge bosons vanish, and thus the only contributions arise from triangle diagrams.
- The mass shell and transversality conditions for the gauge bosons enabled us to make the following replacements

$$Q^2 = \frac{m_V^2}{4}, \qquad p\cdot Q = 0, \qquad p^2 = m_W^2 - \frac{m_V^2}{4}, \tag{2}$$

and

$$p_\alpha \to Q_\alpha, \quad p_\beta \to -Q_\beta, \quad p_\mu \to 0, \tag{3}$$

which results in a considerable simplification of the calculation.
- Instead of dealing with the calculation of the $WW\gamma$ and WWZ vertices separately, we performed instead the calculation of the general WWV vertex, with V a massive neutral gauge boson. We have exploited the fact that there are only three generic trilinear vertices involved in the one-loop contributions to the WWV vertex and thus a model independent calculation was done using the generic Feynman rules of Fig. 4. The result for the contribution of each type of Feynman diagram will be presented in terms of loop functions, given as parametric integrals and also in terms of Passarino-Veltman scalar integrals, times a factor involving all the generic coupling constants associated with each vertex participating in the particular diagram. The contribution to the form factors of the $WW\gamma$ and WWZ vertices follow easily from our general expressions after taking the appropriate mass limits and substituting the corresponding coupling constants of the GMM or any other extension model.
- We corroborated all the contributions to the $\Delta\kappa'_V$ and ΔQ_V form factors are free of ultraviolet divergences.

Once the amplitude for each Feynman diagram is written down, the Feynman parametrization technique and the Passarino-Veltman method can be applied straightforwardly, along with some cumbersome algebra. Thereafter one can express the contributions to the $\Delta\kappa'_V$ and ΔQ_V form factors for each type of Feynman diagram of Fig. 2 as follows

$$\Delta\kappa_V'^{i} \;=\; -\frac{C_V^i}{16\pi^2} I_\kappa^{V-i}(x_A, x_B, x_V), \tag{4}$$

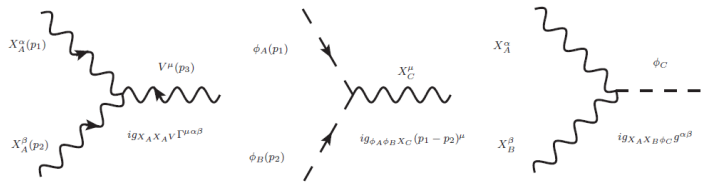

Figure 4. Generic Feynman rules for the relevant vertices involved in our calculation. The arrows stand for the direction of the 4-momenta and $\Gamma_{\mu\alpha\beta} = g_{\mu\alpha}(p_1 - p_3)_\beta + g_{\alpha\beta}(p_2 - p_1)_\mu + g_{\beta\mu}(p_3 - p_2)_\alpha$. $V = \gamma, Z$, ϕ_I ($I = A$, B, C) denote a neutral singly, or doubly charged scalar boson, and X_J ($J = A$, B) stands for a neutral or charged gauge boson.

$$\Delta Q_V^i = -\frac{C_V^i}{16\pi^2} I_Q^{V-i}(x_A, x_B, x_V), \tag{5}$$

for $V = Z$, γ and $i = a, b, c$. We have introduced the scaled variable $x_I = m_I^2/m_W^2$ ($I = A$, B), with m_A and m_B denoting the masses of the particles circulating into each type of diagram. A word of caution is in order here as m_A and m_B, and thereby x_A and x_B, are distinct for each type of contribution, which in turn is denoted by the superscript i ($i = a$, b, c). As for the loop functions I_κ^{V-i} and I_Q^{V-i}, they are in terms of parametric integrals and Passarino-Veltman scalar integrals and can be found in [5], while the C_V^i factors are given in term of the coupling constants of the vertices involved in each Feynman diagram.

As explained above, the $\Delta\kappa_\gamma'^i$ and ΔQ_γ^i form factors can be obtained from the general expressions (4) -(5) by taking the $m_V \to 0$ limit. We have verified that these expressions are in agreement with the results presented in Ref. [4], where the $WW\gamma$ vertex was studied in the context of little Higgs models.

As far as the Feynman diagrams of Fig. 3 are concerned, they only contribute to the WWZ vertex and the respective form factors depend now on three distinct internal masses. They can be written as follows

$$\Delta\kappa_Z'^i = -\frac{C_Z^i}{16\pi^2} I_\kappa^{Z-i}(x_A, x_B, x_C, x_Z), \tag{6}$$

$$\Delta Q_Z^i = -\frac{C_Z^i}{16\pi^2} I_Q^{Z-i}(x_A, x_B, x_C, x_Z). \tag{7}$$

This time the superscript $i = d, e, f$ stands for the whole contributions of diagrams i_1 and i_2, and again the loop functions I_κ^{Z-i} and I_Q^{Z-i} can be found in [5].

Once the general expressions for the different kinds of contributions are obtained, we can compute the total contribution of the scalar sector of a given model by simple adding up all the partial contributions. We will present below a numerical analysis of the contributions of the GMM. For the numerical evaluation we computed the parametric integrals via the Mathematica numerical routines. A cross check was done using the results obtained by evaluating the results given in terms of Passarino-Veltman scalar functions with the help of the LoopTools routines.

2.1. $\Delta\kappa_\gamma'$ and ΔQ_γ form factors

Excluding the pure SM contributions, in the GMM, the $\Delta\kappa_\gamma'$ and ΔQ_γ form factors receive 10 contributions of the type-(a) diagrams, 3 of the type-(b) diagrams, and 2 of the type-(c) diagrams. Notice that all the new scalar bosons participate in the type-(a) diagrams, whereas the type-(b) diagrams only receive contributions from the singlet and the fiveplet scalar bosons,

and the type-(c) diagrams from the fiveplet scalar bosons only. We examine the general behavior of $\Delta\kappa'_\gamma$ and ΔQ_γ as functions of the masses of the scalar bosons. For the type-(b) and type-(c) contributions we show in Fig. 5 the form factors as a function of the mass of the scalar boson circulating into the loop, whereas for type-(a) diagram we consider two scenarios: when both scalar bosons are degenerate and when one scalar boson mass is fixed and the other one is variable.

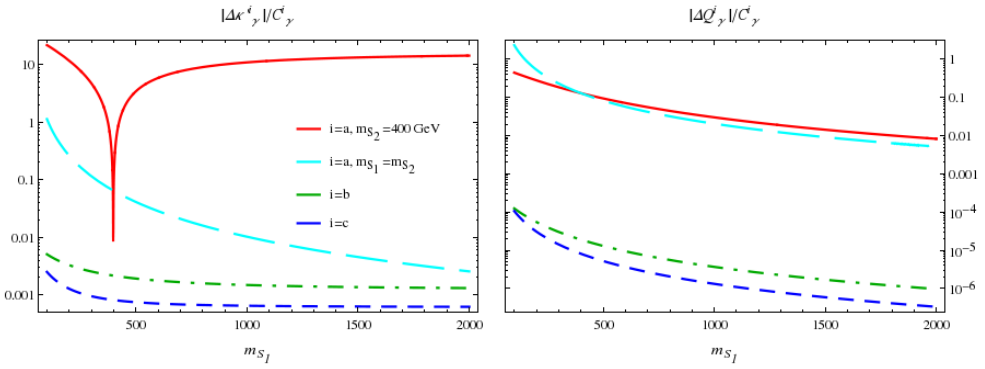

Figure 5. Behavior of the $\Delta\kappa'_\gamma$ and ΔQ_γ form factors as functions of the masses of the scalar bosons circulating into the loops of each type of contribution divided by the C^i_γ coefficient and in units of $a = g^2/(96\pi^2)$. While type-(a) contribution depends on two scalar boson masses m_{S_1} and m_{S_2}, type-(b) and type-(c) diagrams depends on only one scalar boson mass m_{S_1}.

2.2. $\Delta\kappa'_Z$ and ΔQ_Z form factors.

We will now analyze the $\Delta\kappa'_Z$ and ΔQ_Z form factors, for which we will follow a similar approach to that used above. We thus start by studying the general behavior of the distinct types of contributions. Apart from the diagrams of Fig. 2, there is additional contributions due to the diagrams of Fig. 3. As for the contributions of type (a), (b) and (c), their behavior is quite similar to that observed in Fig. 5, so here we will thus focus on the analysis of the extra contributions, whose behavior will turn out to be rather similar to that of contributions type (a), (b) and (c). There are 7 contributions of type (d), 4 of type (e), and 3 of type (f) to the $\Delta\kappa'_Z$ and ΔQ_Z form factors. Although our general results allow us to calculate type-(d) contributions with three distinct scalar boson masses m_{S_1}, m_{S_2} and m_{S_3}, in the GMM all the masses of the same multiplet are degenerate. It means that type-(d) contributions arise only from diagrams with al least two degenerate scalar bosons. Also, although type-(e) contribution arise from diagrams that can have two distinct scalar bosons, their masses are degenerate and there is dependence on one mass only, and this is also true for type-(f) contributions. Therefore, we expect that type-(d) contributions will be dominant as long as there is a large mass splitting between the scalar boson masses, whereas type-(e) and type-(f) contributions will only be important for a relatively light scalar boson mass. This is depicted in Fig. 6, where we show the behavior of the $\Delta\kappa'_Z$ and ΔQ_Z form factors for all the scenarios allowed in the GMM. For type-(d) contributions we consider three scenarios: m_{S_3} fixed and $m_{S_2} = m_{S_1}$ variables, $m_{S_3} = m_{S_2}$ fixed and m_{S_1} variable, and the three scalar boson masses degenerate $m_{S_3} = m_{S_2} = m_{S_1}$. On the other hand, for type-(e) contributions we only consider the case when the two scalar bosons are degenerate. In Fig. 6 we observe that $\Delta\kappa_Z$ and ΔQ_Z have a similar behavior to that of the $\Delta\kappa'_\gamma$ and ΔQ_γ form factors. In particular, the largest contributions to $\Delta\kappa'_\gamma$ are reached when there is a large mass splitting or when all the scalar bosons masses circulating into each loop are relatively light.

However, the decrease of $\Delta\kappa'_Z$ for large m_{S_1} is now less quick than in the case of $\Delta\kappa'_\gamma$. Again, the C_Z^i factor is proportional to v, so the values shown in the plots for type-(e) and type-(f) contributions will increase by two orders of magnitude. As for ΔQ_Z, it will reach its large value for the smallest allowed scalar boson masses as in the case of ΔQ_γ. When the scalar bosons are very heavy, they will be approximately degenerate, in which case ΔQ_Z will decrease significantly. Extra suppression for both form factors can arise from the C_Z^i coefficients and from potential cancellations between the distinct contributions.

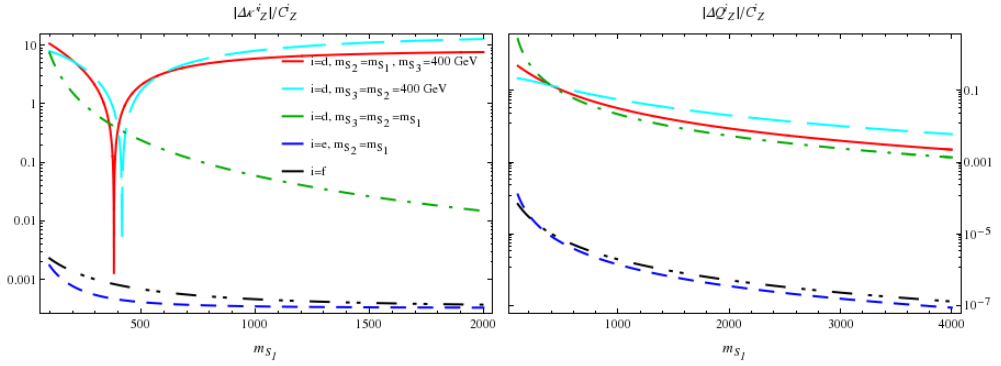

Figure 6. Behavior of the $\Delta\kappa'_Z$ and ΔQ_Z form factors as a function of the masses of the scalar bosons circulating into the loops of each type of contribution divided by the C_γ^i coefficient. We only consider the possible scenarios arising in the GMM.

3. Conclusions

A model independent calculation was done via both the Feynman parameter technique and the Passarino-Veltman reduction scheme. Our general results are expressed in terms of a six generic contributions to $\Delta\kappa'_V$ and ΔQ_V from scalar bosons that can be used to calculate the corrections arising from models with an extended scalar sector predicting new neutral, singly, and doubly charged scalar bosons. For the numerical analysis we have focused on the GMM, this model predicts 9 new scalar bosons accommodated in a singlet, a triplet and a fiveplet, which yield 15 new contributions to $\Delta\kappa'_\gamma$ and ΔQ_γ, whereas $\Delta\kappa'_Z$ and ΔQ_Z receive 28 distinct contributions. The general behavior of the $\Delta\kappa'_V$ and ΔQ_V form factors was analyzed and it was found that $\Delta\kappa'_V$ reaches values of the order of $a = g^2/(96\pi^2)$, with the largest values arising from the diagrams with two nondegenerate scalar bosons provided that there is a large splitting between their masses. On the other hand ΔQ_V reaches values of the order of $10^{-2}a$, with the largest contributions arising from diagrams with relatively light degenerate scalar bosons. Both form factors decrease rapidly when all the scalar boson masses are heavy. The values for $\Delta\kappa'_V$ and ΔQ_V predicted by the GMM are competitive with the ones predicted by other weakly coupled SM extensions, but a very high precision still would be necessary to detect such effects.

References

[1] H. Georgi and M. Machacek, Nucl. Phys. B **262**, 463 (1985). doi:10.1016/0550-3213(85)90325-6
[2] J. F. Gunion, R. Vega and J. Wudka, Phys. Rev. D **42**, 1673 (1990). doi:10.1103/PhysRevD.42.1673
[3] W. A. Bardeen, R. Gastmans and B. E. Lautrup, Nucl. Phys. B **46**, 319 (1972). doi:10.1016/0550-3213(72)90218-0
[4] A. Moyotl and G. Tavares-Velasco, J. Phys. G **37**, 105012 (2010) doi:10.1088/0954-3899/37/10/105012 [arXiv:1003.3230 [hep-ph]].
[5] M.A. Arroyo-Ureña, G. Hernández-Tomé, and G. Tavares-Velasco, WWV ($V = \gamma, Z$) vertex in the Georgi-Machacek model, Work in progress.