

Excited State Mass Spectra of Ω_c^0 Baryon

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Abstract. We have calculated the radial and orbital excited states of singly charmed baryon Ω_c using the Hypercentral Constituent Quark Model (hCQM). The confinement potential is assumed as coulomb plus power potential (CPP _{ν}). The ground state and excited state masses are determined with and with out first order correction to the potential. Furthermore, we plot graph between Mass(M) $\rightarrow$ Potential Index(ν). Our calculated results are in good agreement with experimental and other theoretical predictions.

1. Introduction

Baryon spectroscopy is broadly studied by various theoretical approaches and many experimental groups [1]. We know that baryons are strongly interacting fermions and made up of three quarks. Baryons consisting with u and d quarks are called nucleons ($I_3 = \frac{1}{2}$) or Δ ($I_3 = \frac{3}{2}$) resonances. Combination of u, d and s quark are called hyperons. The singly charmed baryon is composed of a charm quark and two light quarks belong to SU(4) multiplets. SU(4) group includes all of the baryons containing zero, one, two or three charmed quarks. The multiplet numerology of the tensor product of three fundamental rendition is: $4 \times 4 \times 4 = 20 + 20'_1 + 20'_2 + \bar{4}$. Representation shows totally symmetric 20-plet, the mixed symmetric $20'$ -plet and the total anti symmetric 4 multiplet [2].

One of the single charmed baryon, Ω_c^0 (*ssc*) belongs to a sextet of flavor symmetric states. It is a combination of two light quark (strange quark) and heavy quark (charm quark). The WA62 collaboration reported the first ground state of Ω_c^0 (Mass = 2740 ± 20 MeV) [3]. Since then, resonances observed simultaneously by a large variety of experiments like Belle, BaBar, CLEO etc.[1, 4, 5, 6]. Ω_c^0 belongs to a sextet but decays weakly [7]. PDG-2014 listed experimental results of ground states $\Omega_c^0(\frac{1}{2}^+)$ and $\Omega_c^0(\frac{3}{2}^+)$ but excited states measurements are still not reported.

In the past decades, various phenomenological Constituent quark models have been used and all models reproduce the baryon spectrum. The non-relativistic as well as relativistic approach like Isgur-Karl model [8], the Capstick-Isgur model [9], the interacting quark-diquark model [2], the Hypercentral Model [10], etc. had been used to calculate the ground state mass spectroscopy of charmed baryons. D. ebert et al. [11] calculated radial and orbital excited states by relativistic quark-diquark model. Moreover, lattice QCD results show the excited state mass spectroscopy of singly as well as doubly and triply charmed baryons [12, 13, 14].



In this paper, we calculate the radial and orbital excited states of Ω_c^0 baryon in Hypercentral Constituent Quark Model (hCQM). This model has been applied for the description of ground state properties of the various baryons [15, 16, 17, 18, 19, 20, 21] previously. The paper organized as follows: after the brief introduction, we give the details of model. Finally we tabulate the results and discuss the outcome of the model.

2. The Model

Baryon is made up of three quarks. The Jacobi Co-ordinates ($\vec{\rho}$ and $\vec{\lambda}$) are used to describe baryon as a bound state of three constituent quarks [15, 16, 17, 18, 19, 20, 21],

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \quad (1)$$

such that

$$m_\rho = \frac{2m_1m_2}{m_1 + m_2} \quad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)} \quad (2)$$

Where $m_1 = 0.500 \text{ GeV}$, $m_2 = 0.500 \text{ GeV}$ and $m_3 = 1.275 \text{ GeV}$ are three different constituent quark masses.

The angle of the Hyper spherical coordinates are given by $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$. We define hyper radius x and hyper angle ξ by,

$$x = \sqrt{\rho^2 + \lambda^2} \quad \xi = \arctan\left(\frac{\rho}{\lambda}\right) \quad (3)$$

The hypercentral potential contains three-body effects. The Hamiltonian of three body baryonic system for the hCQM is,

$$H = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(\rho, \lambda) = \frac{P_x^2}{2m} + V(x) \quad (4)$$

Where, $m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}$, is reduced mass. The hyperradial Schrodinger equation corresponds to the Hamiltonian can be written as,

$$\left[\frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2} \right] \Psi_\gamma(x) = -2m[E - V(x)]\Psi_\gamma(x) \quad (5)$$

where $\Psi_\gamma(x)$ is the hypercentral wave function and γ is the grand angular quantum number. We consider transformation, $\phi_\gamma(x) = x^{\frac{5}{2}}\Psi_\gamma(x)$. Thus, the hyperradial Schrodinger equation reduces to,

$$\left[\frac{-1}{2m} \frac{d^2}{dx^2} + \frac{\frac{15}{4} + \gamma(\gamma+4)}{2mx^2} + V(x) \right] \phi_\gamma(x) = E\phi_\gamma(x) \quad (6)$$

For the present study we consider the hypercentral potential $V(x)$ as the color coulomb plus power potential with relativistic correction [22, 23, 24] written as,

$$V(x) = V^0(x) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda} \right) V^{(1)}(x) + V_{spin} \quad (7)$$

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x^\nu \quad (8)$$

$$V^{(1)}(x) = -C_F C_A \frac{\alpha_s^2}{4x^2} \quad (9)$$

Table 1. Masses of radial excited states of Ω_c^0 (in GeV).

(ν)	A	B	Others	A	B	Others
	$1^2S_{\frac{1}{2}}$			$1^4S_{\frac{3}{2}}$		
0.5	2.695	2.695	2.695 ± 0.017 [1]	2.748	2.754	2.766 ± 0.0020 [1]
0.7	2.695	2.695	2.718[2]	2.742	2.752	2.776[2]
0.9	2.695	2.695	2.698[11]	2.748	2.750	2.768[11]
1.0	2.695	2.695	2.681[14]	2.740	2.745	2.764[14]
2.0	2.695	2.695	2.699[26]	2.751	2.757	2.767[26]
	$2^2S_{\frac{1}{2}}$			$2^4S_{\frac{3}{2}}$		
0.5	3.075	3.089		3.113	3.128	
0.7	3.104	3.123	3.088[11]	3.137	3.160	3.123[11]
0.9	3.140	3.154		3.176	3.190	
1.0	3.147	3.164		3.178	3.197	
2.0	3.279	3.306		3.320	3.349	
	$3^2S_{\frac{1}{2}}$			$3^4S_{\frac{3}{2}}$		
0.5	3.349	3.371		3.371	3.394	
0.7	3.421	3.454	3.489[11]	3.441	3.476	3.510[11]
0.9	3.505	3.531		3.527	3.552	
1.0	3.529	3.561		3.548	3.580	
2.0	3.866	3.921		3.892	3.948	
	$4^2S_{\frac{1}{2}}$			$4^4S_{\frac{3}{2}}$		
0.5	3.602	3.633		3.617	3.647	
0.7	3.724	3.769	3.814[11]	3.737	3.783	3.83[11]
0.9	3.863	3.900		3.877	3.914	
1.0	3.907	3.953		3.920	3.966	
2.0	4.491	4.576		4.510	4.595	
	$5^2S_{\frac{1}{2}}$			$5^4S_{\frac{3}{2}}$		
0.5	3.843	3.880		3.853	3.890	
0.7	4.017	4.074	4.102[11]	4.026	4.084	4.114[11]
0.9	4.215	4.263		4.225	4.273	
1.0	4.283	4.343		4.292	4.352	
2.0	5.150	5.267		5.164	5.281	

A= without first order correction. B= with first order correction.

A hyper-Coulomb, $\tau = -\frac{2}{3} \alpha_s$ related to strong coupling constant; $\frac{2}{3}$ is the color factor for baryon, β corresponds to the string tension of the confinement. Potential index, ν varies from

0.5 to 2.0. $C_F = \frac{4}{3}$ and $C_A = 3$ are the Casimir charges of the fundamental and adjoint representation. Coupling constant α_s is given by,

$$\alpha_s = \frac{\alpha_s(\mu_0)}{1 + \frac{33-2n_f}{2\pi}\alpha_s(\mu_0)\ln\left(\frac{\mu}{\mu_0}\right)} \quad (10)$$

The spin-dependent part of the three-body interaction potential of eq. (7) is taken as

$$V_{spin}(x) = -\frac{1}{4}A\alpha_s \frac{e^{-x/x_0}}{xx_0^2} \sum_{i>j} \frac{\vec{\sigma}_i \vec{\sigma}_j}{6m_i m_j} \vec{\lambda}_i \vec{\lambda}_j \quad (11)$$

where x_0 is the hyperfine parameter of the model. The spin-dependent part, $V_{SD}(x)$ contains three types of the interaction terms, such as the spin-spin term $V_{SS}(x)$, the spin-orbit term $V_{\Gamma S}(x)$ and tensor term $V_T(x)$ given by [25],

$$V_{SD}(x) = V_{SS}(x) \left[S(S+1) - s_\rho(s_\rho+1) - \frac{3}{4} \right] + V_{\Gamma S}(x)(\vec{\Gamma} \cdot \vec{S}) \\ + V_T(x) \left[S(S+1) - \frac{3(\vec{S} \cdot \vec{x})(\vec{S} \cdot \vec{x})}{x^2} \right] \quad (12)$$

The Spin-orbit and the tensor term describe the fine structure of the states, while the spin-spin term gives the spin singlet triplet splittings. The coefficient of these spin-dependent terms of (12) can be written in terms of the vector, $V_V(x)$ and scalar, $V_S(x)$ parts of the static potential as

$$V_{\Gamma S}(x) = \frac{1}{2m_\rho m_\lambda x} \left(3 \frac{dV_V}{dx} - \frac{dV_S}{dx} \right) \quad (13)$$

$$V_T(x) = \frac{1}{6m_\rho m_\lambda} \left(3 \frac{d^2 V_V}{dx^2} - \frac{1}{x} \frac{dV_V}{dx} \right) \quad (14)$$

$$V_{SS}(x) = \frac{1}{3m_\rho m_\lambda} \nabla^2 V_V \quad (15)$$

The baryon spin average mass in this hypercentral model is given by

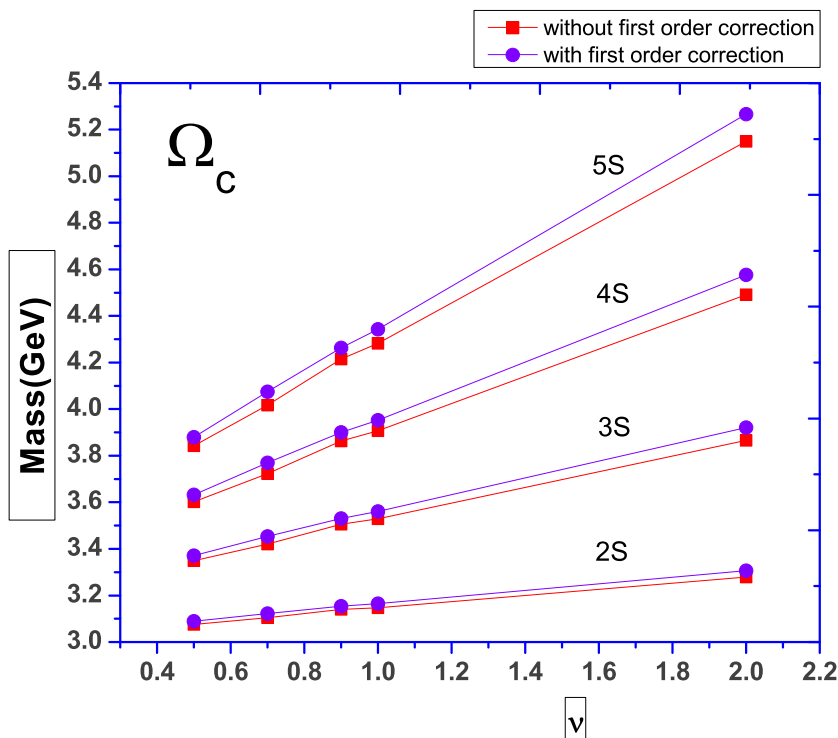
$$M_B = \sum_{i=1} m_i + BE + \langle V_{SD}(x) \rangle \quad (16)$$

We take $A = A_0/(\omega + \gamma + \frac{3}{2})^2$, where A_0 is arbitrary constant. We fix potential parameter β and hyperfine parameter τ for each choices of ν using ground state experimental masses of $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ single charmed flavor baryons (Ω_c^0). For radial excited states, potential parameter β is found to vary as $\beta = \beta_0 \sqrt{\omega + \gamma + \frac{3}{2}}$ and for orbital excited states, potential parameter $\beta = \beta_0 \sqrt{\omega + \gamma + 1}$. where β_0 correspond to ground state.

Table 2. Masses of Orbital excited states of Ω_c^0 (in GeV).

Ω_c^0	0.5		0.7		0.9		1.0		2.0		[11]	[2]
	A	B	A	B	A	B	A	B	A	B		
$(1^2P_{1/2})$	2.985	3.000	2.997	3.019	3.022	3.037	3.022	3.041	3.164	3.221	3.055	2.977
$(1^2P_{3/2})$	2.982	2.996	2.991	3.012	3.010	3.024	3.007	3.024	3.067	3.110	3.054	2.986
$(1^4P_{1/2})$	2.987	3.001	3.000	3.023	3.028	3.043	3.030	3.050	3.212	3.277	2.966	2.977
$(1^4P_{3/2})$	2.984	2.998	2.994	3.015	3.016	3.030	3.014	3.033	3.115	3.165	3.029	2.959
$(1^4P_{5/2})$	2.979	2.993	2.986	3.006	3.000	3.013	2.994	3.010	2.986	3.017	3.051	3.014
$(2^2P_{1/2})$	3.254	3.277	3.308	3.342	3.377	3.402	3.394	3.427	3.740	3.839	3.435	
$(2^2P_{3/2})$	3.251	3.273	3.301	3.334	3.364	3.389	3.377	3.408	3.622	3.703	3.433	
$(1^2P_{1/2})$	3.256	3.278	3.312	3.346	3.383	3.408	3.403	3.436	3.800	3.907	3.384	
$(2^4P_{3/2})$	3.253	3.275	3.305	3.338	3.370	3.396	3.386	3.417	3.681	3.771	3.415	
$(2^4P_{5/2})$	3.248	3.270	3.295	3.328	3.353	3.378	3.363	3.393	3.523	3.589	3.427	

A= without first order correction. B= with first order correction.

**Figure 1.** Variation of mass with potential index in S states

3. Result and Discussion

Using Hypercentral Constituent Quark Model(hCQM), we have calculated the mass spectra of the single charmed baryon Ω_c^0 . We calculate the radial excited states upto 5S states and orbital excited states upto 2P states. Here, we have incorporated the first order correction in the hypercentral power potential, for potential index 0.5 to 2. Our results are close to the D. Ebert et al. (quark-diquark model) [11] results at the potential index 0.7 for S states, but for P-states the results are close at potential index 1.0 (see Tables (1-2)). The result obtained using the first order corrections are higher (few MeV) than without correction. As we are moving from lower excited states to higher excited states these correction effects increasing.

We have shown the variation of mass with potential index in Fig-1. We have plotted the masses of S states without correction to the potential and with first order correction to potential ($J^P = \frac{1}{2}^+$ only). One can see the result obtained from with correction lies over the without correction. We have observed same trend in the $J^P = \frac{3}{2}^+$. We will apply our model to calculate doubly as well as triply charmed Ω 's in future.

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