

Tunneling conductance through normal metal - superconductor junctions : effects of Rashba spin orbit coupling and magnetic field

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Abstract. In a system consisting of a metal-(s-wave) superconductor junction, we study the conductance characteristics in presence of Rashba spin orbit coupling (RSOC) and an external magnetic field applied along the plane of the sample. With a selective inclusion of the Rashba coupling either in the metallic or in both we note that there is a distinct effect with regard to the magnitude of the Andreev peak that occurs at a biasing voltage lower than the superconducting gap energy. The height of the peak is sensitive to the RSOC (increases with increase in RSOC) for RSOC to be present only in the metallic region, (same is true when RSOC is present throughout the junction), while the peak height is fairly independent when RSOC is solely present in the superconducting region. The in-plane magnetic field has very interesting effects which show up in the form of having a conductance peak at zero bias, thereby making it possible to realize a Majorana bound state.

1. Introduction

Transport phenomena in a junction device that constitutes a contact between a superconductor (S) and a normal metal (N) with or without the presence of a thin insulating barrier has been a subject of considerable interest both theoretically and experimentally. Very recently, some works have begun to look at the transport behavior of metal superconductor junctions [1, 2, 3]. When a metal coupled to a superconductor, the transport phenomena are dominated by Andreev reflection [4] that occurs at the interface between the metal and superconductor. It is a process in which an electron transforms into a hole which retraces the path of an incoming particle and transfers a charge $2e$ across the interface, avoiding the single particle transmission within the superconducting energy gap. In the usual metal-superconductor junction, the Andreev reflected hole retraces the path of the incident electron.

Due to breaking of inversion symmetry near the normal metal-superconductor interface, a quasiparticle experiences spin orbit coupling (SOC) of the Rashba type [5], even if both the normal and superconducting crystals have bulk inversion symmetries. The effects of such Rashba spin orbit coupling (RSOC) have mostly been overlooked in the previous studies of tunneling conductance across a $N - S$ junction.

Further It has been suggested that Majorana states [6, 7, 8, 9] could exist in metal-superconductor junction due to SOC and in-plane magnetic field which may be realizable from the proximity effect of a ferromagnet [10]. Majorana fermions in nanowires could be used in future application to quantum information processing [11].



In this paper we carry out the study of tunneling conductance through normal metal-superconductor junction in presence of RSOC. We incorporate RSOC selectively in the metallic regime and also in both the metallic and superconducting leads. The effect of RSOC only in the superconducting lead should be trivial, owing to an uniform superconducting order parameter (s-wave) upon which the RSOC should have negligible effects and hence the Andreev reflection should be by and large unaffected.

Further we include an in-plane magnetic field to probe into the presence of zero mode states. In this section we consider various cases, such as, the in-plane magnetic field (B) is present in both normal and superconducting leads and the magnetic field is only in the superconducting lead, RSOC being present everywhere in the system.

2. Conductance of $N - S$ junction

We consider a system consisting of normal metal coupled to a superconductor denoted by Hamiltonian,

$$H = H_N + H_S \quad (1)$$

where H_N , H_S are the Hamiltonians of the normal metallic lead and superconducting lead respectively. The H_N in the tight binding representation in the presence of RSOC and in-plane magnetic field is given by,

$$H_N = \sum_{\langle i,j \rangle, \sigma, \sigma'} c_{i\sigma}^\dagger t_{ij}^{\sigma\sigma'} c_{j\sigma'} - B \sum_{i\sigma\sigma'} (\sigma_x)_{\sigma\sigma'} c_{i\sigma}^\dagger c_{i\sigma'} \quad (2)$$

and H_S is given by,

$$H_S = \sum_{\langle i,j \rangle, \sigma, \sigma'} c_{i\sigma}^\dagger t_{ij}^{\sigma\sigma'} c_{j\sigma'} - B \sum_{i\sigma\sigma'} (\sigma_x)_{\sigma\sigma'} c_{i\sigma}^\dagger c_{i\sigma'} + \sum_i \Delta_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \sum_i \Delta_i^* c_{i\uparrow} c_{i\downarrow} \quad (3)$$

where $c_{i\sigma}^\dagger$ and $c_{i\sigma}$ are the creation and annihilation operators at the discrete site i , $t_{\sigma\sigma'}$ are the generalized hopping parameters in the σ basis. In $\pm x$ and $\pm y$ directions for square lattice, the hopping parameters are respectively,

$$\begin{pmatrix} -t & t_{so} \\ -t_{so} & -t \end{pmatrix}; \begin{pmatrix} -t & -t_{so} \\ t_{so} & -t \end{pmatrix}; \begin{pmatrix} -t & -it_{so} \\ -it_{so} & -t \end{pmatrix}; \begin{pmatrix} -t & it_{so} \\ it_{so} & -t \end{pmatrix}$$

where t is the nearest neighbor hopping, and $t_{so} = \alpha/2a$ (α : strength of RSOC, a : lattice constant) is the Rashba spin-orbit hopping term. B is strength of in-plane magnetic field, σ_x is Pauli spin matrix, and Δ_i is uniform superconducting order parameter at i th site. This implies $\Delta_i = \Delta_0 \Theta(x)$ ($\Theta(x)$: Heaviside function) where the $N - S$ junction is located at $x = 0$, the left being the normal and the superconducting lead is in the right.

The conductance of a $N - S$ junction is given by the theoretical formalism developed by Blonder - Tinkham - Klapwijk (BTK) [12, 13] which essentially solves the Bogoliubov de Gennes equations selfconsistently using appropriate boundary conditions at the interface. We skip the details here and only present the main formula which is relevant for the conductance calculation. The formula for conductance of a $N - S$ junction is given as,

$$G = (1 - c(E) + d(E)) \quad (4)$$

where $c(E) = |C|^2$, C is the amplitude of specular reflection, $d(E) = |D|^2 \frac{k_N^-}{k_N^+}$, and D is amplitude of Andreev reflection. Now k_N^+ , the momentum of electrons in the N region is given by,

$$k_N^+ = \sqrt{\frac{2m_N}{\hbar^2}} \sqrt{E_{FN} + \epsilon_k} \quad (5)$$

and k_N^- , the momentum of holes in N side is given by,

$$k_N^- = \sqrt{\frac{2m_N}{\hbar^2}} \sqrt{E_{FN} - \epsilon_K} \quad (6)$$

where N side is characterized by mass, m_N and Fermi energy, E_{FN} and energy dispersion, ϵ_K . The energy dispersion, ϵ_K for a two dimension square lattice is given by,

$$E = -2t(\cos k'_x + \cos k'_y) - 2t_{so}\sqrt{\sin^2 k'_x + \sin^2 k'_y} \quad (7)$$

where k'_x, k'_y are wavevectors in the first Brillouin zone.

3. Effect of RSOC in tunneling conductance through normal metal s-wave superconductor junction

For the calculation of conductance, G (in units of $\frac{e^2}{h}$), we have taken a square lattice of dimension $L = 10$ (in units of a) with the superconducting order parameter $\Delta = 0.5$ (in units of t) and $\alpha = 0.5, 1.5$ (in units of $2t$ ($t_{so} = t$)). For calculation of conductance we have used Kwant [14]. We have considered RSOC being present in the normal regime (N) and superconducting regime (S) selectively and in both the normal and superconducting leads.

When RSOC is present only in N , we note that there is distinct effect with regard to magnitude of conductance due to Andreev reflection that occurs at a biasing voltage, E (in units of t) less than gap energy, Δ as expected. The height of Andreev peak is sensitive to the strength of RSOC (α) and it starts gaining weight with increasing α (in units of $2t$). The

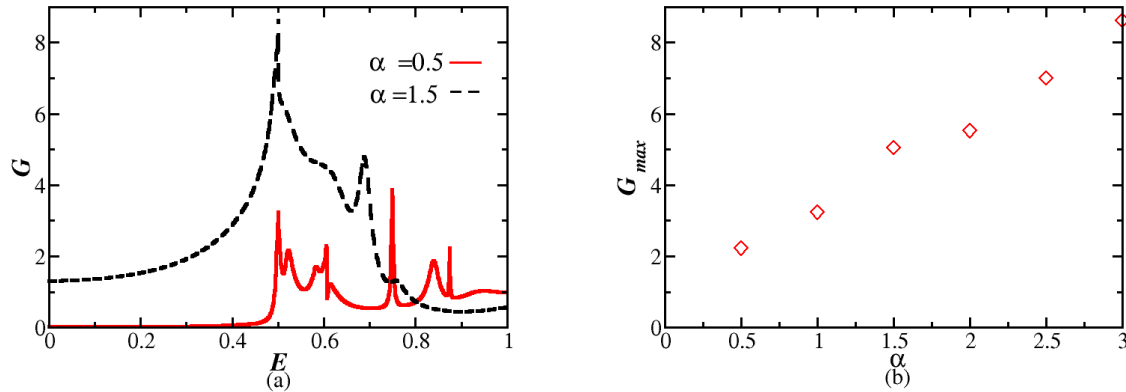


Figure 1. Red (solid) line denotes conductance for $\alpha = 0.5$ where the black (dashed) line denotes conductance for $\alpha = 1.5$. (a) The variation of conductance, G (in units of $\frac{e^2}{h}$) as a function of biasing energy, E (in units of t) for different strengths of RSOC (α) (in units of $2t$) when RSOC is present in N . (b) The variation of conductance peak, G_{max} (in units of $\frac{e^2}{h}$) as the function of strengths of RSOC (α) (in units of $2t$) when RSOC is present in N .

variation of the conductance (G) as a function of energy (E) for different strengths of RSOC is shown in Fig. (1a). Fig. (1a) shows the signature of an Andreev peak at $E \leq \Delta$ and the enhancement of conductance at biasing energy lower the superconducting gap amplitude with increasing α

In Fig. (1b) we have plotted the height of conductance peak (G_{max}) as the function of strengths of RSOC (α). From Fig. (1b) it is well understood that the magnitude of the Andreev peak increases with α . We can understand this results as follows. With increasing strength of α , the momentum of the electrons (Eq. (5)) in the N side decreases while the momentum of the holes (Eq. (6)) in the N side increases, since the energy, ϵ_k decreases (see Eq. (7)). So we can conclude that the term $d(E)$ which is responsible for Andreev reflection in Eq. (4) increases with α . Thus RSOC helps in enhancing Andreev reflection.

Next we consider the presence of RSOC both in the metallic and superconducting regimes. Here also the Andreev reflection which occurs at $E < \Delta$ increases with α . Fig. (2a) and Fig. (2b) shows the variation of conductance (G) as the function of biasing energy (E) for different strengths of RSOC and the variation of conductance peak (G_{max}) as the function of α when RSOC is present in both N and S leads. Similar to Fig. (1b), Fig. (2b) also reveals that the Andreev peak is responsive to the RSOC and it increases with the increasing strength of RSOC. The same argument as given earlier applies here.

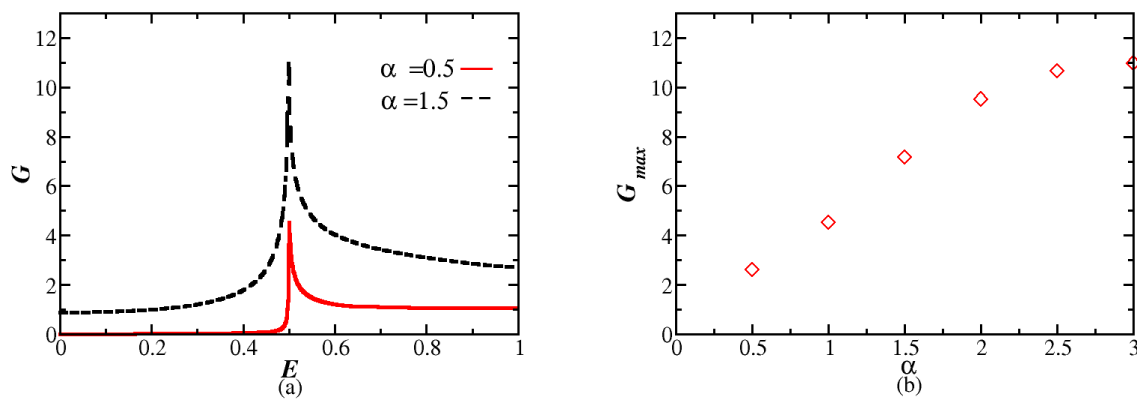


Figure 2. Red (solid) line denotes conductance for $\alpha = 0.5$ where the black (dashed) line denotes conductance for $\alpha = 1.5$. (a) The variation of conductance, G (in units of $\frac{e^2}{h}$) as a function of biasing energy, E (in units of t) for different strengths of RSOC (α) (in units of $2t$) when RSOC is present in both N and S leads. (b) The variation of conductance peak, G_{max} (in units of $\frac{e^2}{h}$) as the function of strengths of RSOC (α) (in units of $2t$) when RSOC is present in both N and S leads.

4. Effect of an in-plane magnetic field

To realize a zero mode state, which are the much talked about Majorana states, we have included an in-plane magnetic field. We have considered two different situations, the magnetic field (B) is present throughout the sample and the magnetic field is present only in the superconducting lead. The RSOC is present everywhere in the sample. Since our focus is to clarify the presence of zero mode states, we have used two representative values for B , namely $B = 1, 3$ in units of t . For both two cases we have found conductance peak at zero biasing energy which points towards the presence of Majorana zero mode states. Fig. (3) shows the variation of conductance as function of biasing energy when in-plane magnetic field and RSOC both are present in throughout the sample

Fig. (4) shows the variation of conductance as function of biasing energy when in-plane magnetic field is present in S . Both figures reveal the distinct presence of a zero mode state.

The presence of the zero mode peak in the conductance data is shown clearly in the inset. Thus the Andreev peak is shifted from $E = \Delta$ to $E = 0$. The above results should be very interesting to the conductance experiments for a $N - S$ junction. The robustness of the zero mode peak can be tested in presence of impurity, disorder or other kind of perturbations.

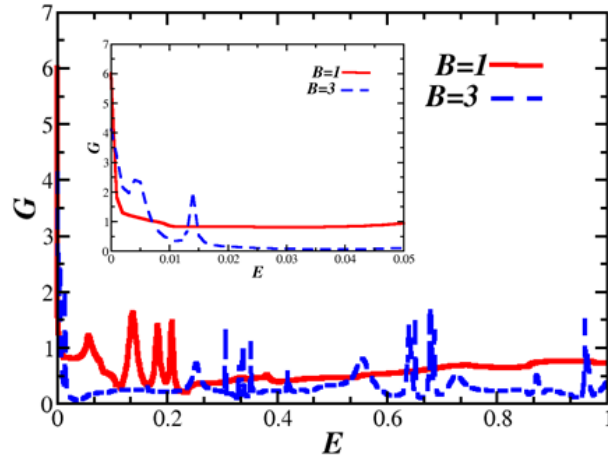


Figure 3. Red (solid) line denotes conductance for $B = 1$ where the blue (dashed) line denotes conductance for $B = 3$. The variation of conductance, G (in units of $\frac{e^2}{h}$) as a function of energy, E (in units of t) for different strengths of in-plane magnetic field (B) (in units of t) when magnetic field and RSOC both are present in both leads.

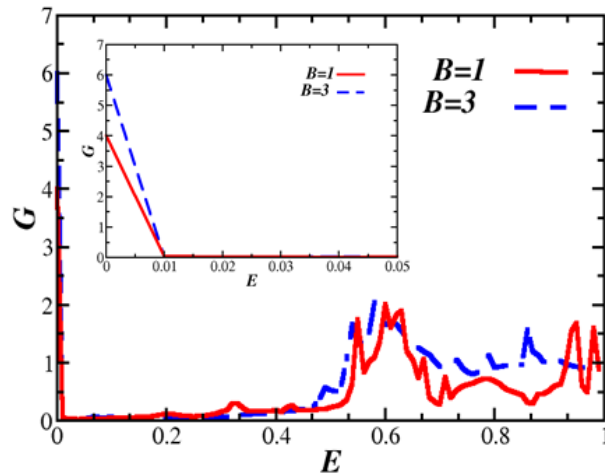


Figure 4. Red (solid) line denotes conductance for $B = 1$ where the blue (dashed) line denotes conductance for $B = 3$. The variation of conductance, G (in units of $\frac{e^2}{h}$) as a function of energy, E (in units of t) for different strengths of in-plane magnetic field, B (in units of t) when magnetic field is present in S lead but RSOC is present in both leads.

5. Conclusions

We conclude saying that the Andreev peak is sensitive to the strength of RSOC. Further, the conductance increases with increasing RSOC at lower biasing energies. Also there are exciting possibilities of realizing zero mode state in presence of in-plane magnetic field in the conduction characteristics of a $N - S$ junction.

6. References

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